ALTERNATING CURRENTS

THEIR THEORY, GENERATION, AND TRANSFORMATION

BY

ALFRED HAY, D.Sc., M.I.E.E.,

LATELY PROFESSOR OF ELECTRICAL TECHNOLOGY, INDIAN INSTITUTE OF SCIENCE, BANGALORE, INDIA

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PREFACE TO FIFTH EDITION

AMONG the more important alterations and additions in the present edition may be mentioned the treatment of polyphase commutator motors, and the theory of unbalanced polyphase systems, instrument transformers and phase converters.

A. H.

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ALTERNATING CURRENTS

THEIR THEORY, GENERATION, AND TRANSFORMATION

CHAPTER I

§ 1. Alternating currents. Frequency and wave-shape. Form factor and amplitude factor—§ 2. Simple sine waves. Vector diagrams—§ 3. Relations connecting amplitude, r.m.s., and arithmetic mean values of simple sine wave—§ 4. Impressed and induced e.m.f.s. Self-inductance—§ 5 Fundamental equation for a circuit in which the current is variable—§ 6. Sine waves in circuits containing resistance, self-inductance, and capacity—§ 7. Power in alternating-current circuit.

§ 1. Alternating Currents. Frequency and Wave-shape. Form Factor and Amplitude Factor

An alternating electric current is a current which periodically passes through a definite cycle of changes, the cycle consisting of two half-cycles, during one of which the current is positive, and during the other negative; the negative half-cycle being, however, an exact reproduction of the positive half-cycle, and the only difference being one of algebraic sign.

The graph of an alternating current would therefore consist of a curve such as that shown in Fig. 1. We may speak of the complete cycle of changes as a complete *wave* of current; and of the two half-cycles as the positive and negative half-waves.

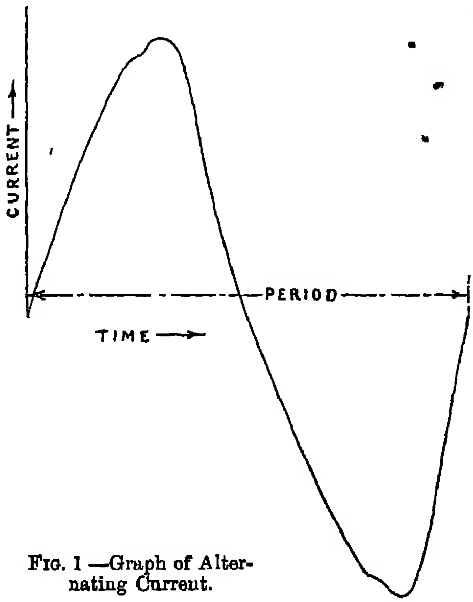


Fig. 1 —Graph of Alternating Current.

The time taken by the current to run through a complete cycle of changes is spoken of as the *period* of the alternating current; and

the number of cycles per second is the *frequency* (sometimes also termed *periodicity*) of the current.

In the practical applications of alternating currents, we have to deal with currents whose graphs differ widely from one another. The shape of the graph is spoken of as the *wave-shape* or *wave-form* of the current, and is in many problems a matter of considerable importance. In a later chapter, we shall explain how a record of the wave-form or graph of an alternating current may be obtained experimentally.

Since an alternating current varies from instant to instant, it is obvious that in deciding on a system of measurement, we must define exactly what we mean by the *numerical value* of an alternating current.

Now in connection with the most important applications of alternating currents—electric lighting, electric power transmission and distribution, electric furnaces—the useful effect produced by the current at any instant depends on the value of the *square* of the current at that instant. Hence, in order that an alternating current may be equivalent (as regards its effect) to a given continuous current, the mean value of the square of the alternating current over a period * must be equal to the square of the continuous current. Otherwise, the two currents are equivalent if the square root of the mean square value of the alternating current is equal to the continuous current. This root-mean-square value is generally termed the r.m.s. value of the current, and when we speak of the numerical value of an alternating current, we mean its r.m.s. value.

In some few cases, we have to consider the *arithmetic mean value* of a current; in others, we have to take into account its *maximum* value.

The relations connecting the r.m.s., the arithmetic mean, and the maximum values of an alternating current depend on its wave-form. It is usual to consider the ratios of the r.m.s. value to the other two, and in this connection two terms introduced by Dr. Fleming, and known as the *form factor* and *amplitude factor*, are convenient.

$$\text{The } \textit{form factor} \text{ of a given wave} = \frac{\text{r.m.s. value}}{\text{mean value}}$$

$$\text{The } \textit{amplitude factor} \dagger \text{ ,, ,, ,,} = \frac{\text{r.m.s. value}}{\text{maximum value}}$$

Hitherto, we have spoken of alternating *currents;* but all that has been said applies equally to alternating p.d.s or e.m.f.s.

* Or half period, since the numerical values during a negative half-wave are identical with those during a positive half-wave.

† It is generally more convenient to consider the reciprocal of the amplitude factor, which Dr Kapp has proposed to call the *crest factor.* We thus have *crest factor* $= \frac{\text{maximum value}}{\text{r.m.s value}}$.

§ 2. Simple Sine Waves. Vector Diagrams

In some cases (as, *e.g.*, in incandescent lighting, or electric furnace work) the results obtained by using an alternating current of given r.m.s. value are entirely independent of its *wave-form;* so long as the r.m.s. value is unaltered, we may use a current of any wave-form we please without in any way affecting the results. In other cases, however, the effects produced will, for a given r.m.s. value, depend, to a greater or less extent, on the *wave-form* (as, *e.g.*, in connection with motors, transformers, arc lights, and *especially* in cases where *capacity* is present, as in concentric cables).

Now in order to simplify the theoretical treatment of the subject as much as possible, it is an obvious advantage to select a wave-form which shall lend itself readily to mathematical treatment; and of all possible wave-forms, the simplest and easiest to deal with is that known as a *simple harmonic* or *sine wave.* The equation to such a wave is—

$$y = \mathrm{Y} \sin \omega t\,*$$

and its graph is shown in Fig. 2.

In the classical theory of alternating currents, it is usual to assume that the waves dealt with are simple sine waves. Since there are, as mentioned above, cases in which the results obtained are independent of the wave-form, the assumption of the simplest wave-form in such cases is perfectly justifiable, and leads to correct results while considerably simplifying the treatment. Unfortunately, there are other cases in which the assumption of sine waves is no longer admissible, and leads to results more or less erroneous, and at times entirely misleading.

For this reason, the use of sine waves has been severely criticized by some writers. But there is nothing else that can be usefully substituted for them, and since in a large number of problems they yield results sufficiently accurate for practical purposes, their use is certainly justifiable. It must be remembered, however, that results deduced on the sine-wave hypothesis must be used with due caution when applied to certain problems.

There is, however, a still further justification for the assumption of sine waves in alternating-current theory, and this is due to the recognition of the fact that for most practical purposes also the sine wave is a *desideratum.* Especially is this the case in connection with long-distance power transmission. Successful attempts have been made to construct alternators capable of giving pure sine waves of e.m.f.; and although some of the older machines gave irregular

* The angle ωt, where t is the time, is expressed in *radians.*

wave-forms, the e.m.f.s of modern machines depart but slightly from the pure sine form.

A sine wave may be represented either analytically, by means of an equation of the form—

$$y = Y \sin \omega t,$$

or graphically as in Fig. 2, t being plotted horizontally and y verti-

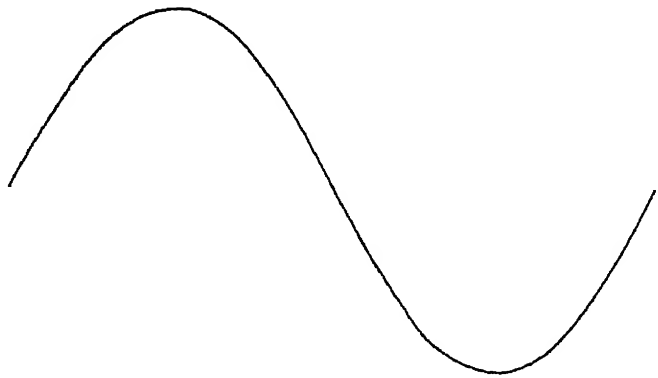


FIG. 2.—Sine Wave.

cally. There is still another mode of representation, however, very generally used, and known as the *vector* or *clock-face* diagram method. In this, we suppose a straight line of constant length OP (Fig. 3) = Y to revolve with constant angular velocity ω (radians per sec.) about one of its extremities O as centre. Such a rotating line of constant length may be termed a *rotating vector*. Suppose that at the time $t = 0$ the line is in the horizontal position (shown dotted in Fig. 3). After t secs., it will have swept out an angle ωt, so that its projection on the vertical will be OP $\cos\left(\frac{\pi}{2} - \omega t\right) = Y \sin \omega t = y$. As, therefore, the line OP rotates, its projection on the vertical at any instant gives us the magnitude and sign of the alternating current at that instant. The extremity of this projection moves with a simple harmonic motion, and the projection itself is spoken of as an *alternating vector*.

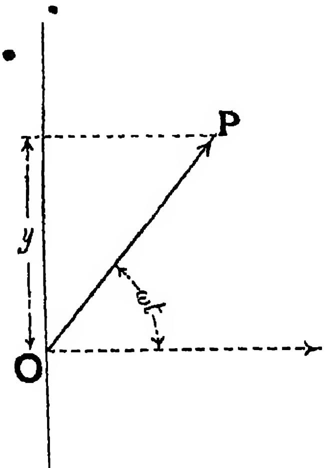


FIG. 3.—Vector or Clock-face Diagram

The maximum value Y of the sine wave is termed its *amplitude.* A complete cycle of changes clearly corresponds to a complete revolution of OP, or to the time T taken by OP to sweep out an angle of 2π radians. Since the angular velocity is ω, we have $\omega T = 2\pi$, or $\omega = \frac{2\pi}{T}$. We have already (§ 1) termed T the *period.* Now if the *frequency* or number of complete cycles per second be denoted by n, we have $T = \frac{1}{n}$, so that $\omega = 2\pi n$; i.e. *the angular velocity of the rotating vector in the vector diagram is equal to* 2π *times the frequency.*

If the equation of the sine wave is given in the form—

$$y_1 = Y_1 \sin (\omega t + \theta)$$

then y_1 may as before be represented by the projection on the vertical of a rotating vector of length Y_1, the only difference now being that at the time $t = 0$ this rotating vector is not horizontal, but makes an angle $+ \theta$ with the horizontal.

Since the rotating vectors from which y and y_1 are derived have

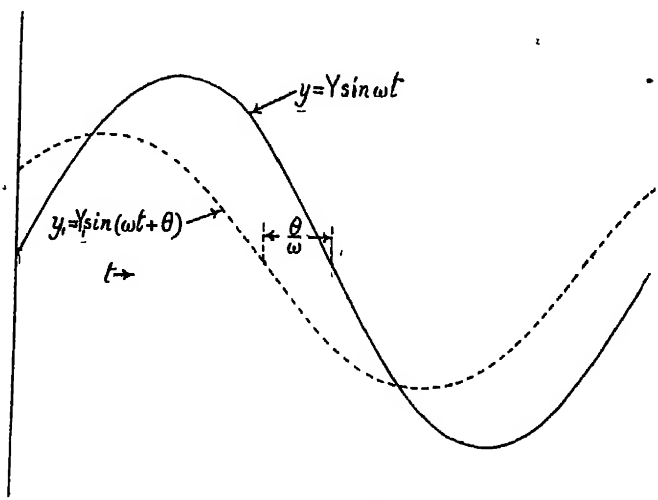


FIG. 4.—Graphs of Two Sine Waves, with a Phase Difference θ.

the same angular velocity ω, the angle between them must remain constant and equal to θ, which is the angle they make with each other at the time $t = 0$. This constant angle θ is the *phase difference* between the two sine waves y and y_1. The wave y is said to *lag* behind y_1, and y_1 is said to *lead* with respect to y, the angle θ being

referred to as the *angle of lag* and *lead* respectively in the two cases.

The ordinary graphs of the two waves y and y_1 are shown in Fig. 4, the full-line curve representing y and the dotted one y_1.

It is, of course, obvious that two sine waves whose equations are $y_3 = Y_3 \sin(\omega t + \theta_3)$ and $y_4 = Y_4 \sin(\omega t + \theta_4)$ have a phase difference $\theta_3 - \theta_4$; this being the fixed angle between the two corresponding rotating vectors (of lengths Y_3 and Y_4) in the clock diagram.

Instead of assuming an axis fixed in space, and considering the projection on it of a rotating vector, the direction of rotation of which is counter-clockwise, we may suppose the vector to remain fixed, and consider its projection on a *rotating axis*, the angular velocity being *clockwise* and numerically equal to ω. It is evident that both these methods yield identical results.

What we are concerned with in practice when dealing with a number of sine waves of the same frequency is to know (1) the r.m.s. value of each simple sine wave; and (2) their phase differences. Hence in constructing a vector diagram it is generally convenient to make the length of each vector correspond to the *r.m.s. value* of the sine wave, instead of, as in the original mode of representation, to its maximum value.

§ 3. Relations connecting Amplitude, r.m.s., and Arithmetic Mean Values of Simple Sine Wave

In order to enable us to pass from the maximum value or amplitude of a sine wave to its r.m.s. value, we have to find the relation connecting these two quantities. The r.m.s. value is the square root of the mean value of $y^2 = Y^2 \sin^2 \omega t$ over a period. Now the mean value of $y^2 = Y^2 \sin^2 \omega t$ may be found by determining the area of the graph of this function over a period, and dividing this area by the length of the base-line, *i.e.* by T. The area is, however, represented by the definite integral $\int_0^T Y^2 \sin^2 \omega t \,.\, dt$, the value of which may be obtained as follows:—

$$\begin{aligned}
\int_0^T Y^2 \sin^2 \omega t \quad dt &= \tfrac{1}{2}Y^2\int_0^T (1 - \cos 2\omega t)dt \\
&= \tfrac{1}{2}Y^2\left\{\int_0^T dt - \int_0^T \cos 2\omega t \,.\, dt\right\} \\
&= \tfrac{1}{2}Y^2\left\{\Big[t\Big]_0^T - \Big[\tfrac{1}{2} \sin 2\omega t\Big]_0^T\right\} \\
&= \tfrac{1}{2}Y^2 T
\end{aligned}$$

since $\sin 2\omega T = \sin 4\pi = 0$. Dividing the area by T, we find that *the mean value of the square of a sine function over a period equals* $\frac{1}{2}Y^2$, i.e. *equals half the square of the amplitude.* Hence the r.m.s. value is $\frac{Y}{\sqrt{2}} = 0{\cdot}707$ times the amplitude. The *amplitude factor* (§ 1) for a simple sine wave is thus $\frac{1}{\sqrt{2}} = 0{\cdot}707$.

By a similar method, we can find the arithmetic mean value and the form factor of a sine wave. To find the arithmetic mean value, we may determine the area of a half-wave, *i.e.* the value of the integral $\int_0^{\frac{T}{2}} Y \sin \omega t \,.\, dt$, and divide this by $\frac{T}{2}$. Now—

$$\int_0^{\frac{T}{2}} Y \sin \omega t \,.\, dt = Y\left[-\frac{1}{\omega}\cos \omega t\right]_0^{\frac{T}{2}} = \frac{2}{\omega}Y$$

Hence the mean value is $\frac{4}{\omega T}Y$, or, since $\omega T = 2\pi$, the mean value is $\frac{2}{\pi}Y$, and the *form factor* (§ 1) of a sine wave is thus—

$$\frac{\frac{Y}{\sqrt{2}}}{\frac{2Y}{\pi}} = \frac{\pi}{2\sqrt{2}} = 1{\cdot}11$$

§ 4. Impressed and Induced e.m.f.s. Self-inductance

In order to maintain an alternating current in a circuit, it becomes necessary to introduce into the circuit a source of alternating e.m.f. The e.m.f. provided by such a source is spoken of as the *impressed e.m.f.*, in order to distinguish it from other e.m.f.s which are generally called into play as soon as the alternating current begins to flow.

A current flowing in a circuit gives rise to a definite number of lines of magnetic induction, which become linked with the circuit. By the great principle discovered by Faraday in 1831, any change in the magnetic flux linked with a circuit is accompanied by the induction of an e.m.f. around the circuit; *the direction of the induced e.m.f. being always such as to oppose the change which gives rise to it* (Lenz's law).

Now, since an alternating current is changing from instant to

instant, the magnetic flux which it produces will similarly change, and will thus give rise to an *induced e.m.f.*, which becomes superposed on the *impressed e.m.f.* The resultant e.m.f., which is instrumental in maintaining the current through the resistance of the circuit, is at every instant equal to the algebraical sum of the impressed and induced e.m.f.s.

It is therefore obvious that, for given values of the resistance and the impressed e.m.f., the magnitude of the current will be determined by the *magnetic flux* to which the current gives rise. In other words, *resistance* is, in the case of an alternating current circuit, not the only factor determining the value of the current, which also depends on the *total flux linked with the circuit when conveying a unit current.*

This latter quantity—the flux linked with the circuit when conveying unit current—is defined to be the *self-inductance*,* or *inductance* simply, of the circuit.

§ 5. Fundamental Equation for a Circuit in which the Current is Variable

Let us suppose, in the first instance, that all the quantities are expressed in C.G.S. units. If L = self-inductance of circuit, and i = value of current at time t, then the total flux linked with the circuit at time t is Li. The induced e.m.f. is numerically equal to the rate of change of the magnetic flux, but since it always opposes the changes which give rise to it, it must be taken with a negative sign. Thus the induced e.m.f. at time t is given by—

$$-\frac{d}{dt}(\mathrm{L}i)$$

Now, although $\mathrm{L} = \dfrac{\text{magnetic flux}}{\text{current}}$ is constant for coreless coils, it is no longer so in the case of coils provided with iron cores. It may, however, be assumed to be approximately constant even in this latter case, so long as the magnetization is well below the knee of the B-H curve. Assuming, then, L to be constant, we have for the induced e.m.f. the value—

$$-\mathrm{L}\frac{di}{dt}$$

If e = impressed e.m.f. at time t, then the resultant e.m.f. at the same instant is given by—

$$e - \mathrm{L}\frac{di}{dt}$$

* The older term is *self-induction*, or *coefficient of self-induction.*

and it is this resultant e.m.f. which maintains the current against the resistance r of the circuit. Hence—

$$i = \frac{e - L\frac{di}{dt}}{r}$$

This equation may be written in the form—

$$e = ri + L\frac{di}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the physical interpretation of which is, that the impressed e.m.f. at any given instant may be regarded as employed in two ways: (1) in balancing the drop of potential ri due to the resistance of the circuit; and (2) in balancing the opposing e.m.f. of self-inductance.

Equation (1) is of great importance, and we shall frequently have to make use of it.

Let us now suppose that the practical units—the volt, the ohm, and the ampere—are substituted for the C.G.S. units, and let us choose the practical unit of self-inductance so that equation (1) will also hold for the practical units, without being complicated by the introduction of any constants. Let the practical unit of self-inductance contain 10^x C.G.S. units. In order to avoid unnecessary constants, we must choose x so that—

$$L \times 10^x \times 10^{-1}\frac{di}{dt} = 10^8 . L\frac{di}{dt}$$

or $x = 9$, so that the corresponding practical unit of self-inductance, termed the *henry*, is equal to 10^9 C.G.S. units.

The corresponding practical unit of magnetic flux is equal to 10^8 C.G.S. units.

In some cases, the self-inductance of a circuit is so small as to be practically negligible in comparison with its resistance. As an example of such a circuit, we may consider an insulated wire which has been doubled on itself and then wound into a coil—as in the usual method of winding standard resistance coils. Such a circuit is termed a *non-inductive* one. An incandescent lamp—whose resistance is very high and self-inductance very low—is another example of a non-inductive circuit.

§ 6. Sine Waves in Circuits containing Resistance, Self-inductance, and Capacity

Consider a non-inductive circuit of resistance r in which there is an alternating current represented by—

$$i = I \sin \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Since L is negligible, the impressed e.m.f. is, by equation (1) equal to—

$$e = ri = r\mathrm{I} \sin \omega t$$

The impressed e.m.f. is thus *in phase* with the current, and the vector corresponding to the impressed e.m.f. and the current in a vecto diagram lie along the same straight line, as shown in Fig. 5. Furthe considering r.m.s. values of e.m.f. and current, we see that—

$$\text{current} = \frac{\text{e.m.f.}}{\text{resistance}}$$

Let us next consider another extreme case—that, namely, i which the self-inductance of a circuit is so high that its resistanc may be neglected in comparison. An example of such a circuit i furnished by a coil consisting of a very large number of turns of fairl thick wire. Let L stand for the inductance of the circuit, the curren being as before given by (2). Then since r is by supposition negligibl (1) gives—

$$e = \mathrm{L}\frac{di}{dt} = \mathrm{L}\frac{d}{dt}(\mathrm{I} \sin \omega t)$$
$$= \omega\mathrm{LI} \cos \omega t$$

Let OP = I in Fig. 6 represent the current vector. Draw OR a right angles to OP, making OR = ωLI. At any instant t, the pro

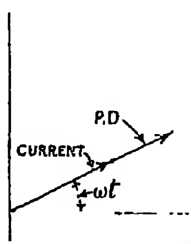


Fig. 5.—Vector Diagram for Pure Resistance.

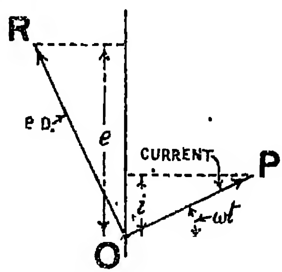


Fig. 6—Vector Diagram for Pure Inductance.

jection of OP on the vertical, gives the value of the current, while a the same instant the projection of OR, which is given by OR co $\omega t = \omega$LI cos $\omega t = e$, represents the value of the impressed e.m.f Thus OR is the e.m.f. vector, and we see that there is a phase differ ence of $\frac{\pi}{2}$ or 90° between the impressed e.m.f. and the current, th current *lagging behind the impressed e.m.f.* When two sine wave differ in phase by $\frac{\pi}{2}$, they are said to be *in quadrature* with eac

other, and we see that in the case considered the impressed e.m.f. and the current are in quadrature with each other. Further, again considering r.m.s. values, we see that—

$$\text{current} = \frac{\text{e.m.f.}}{\omega L}$$

The quantity ωL is termed the *reactance* of the circuit.

Consider a condenser of capacity C, across whose terminals there is an impressed p.d. given by—

$$v = V \sin \omega t$$

Let q = instantaneous quantity or charge in the condenser. Then—

$$q = Cv = CV \sin \omega t$$

If i = instantaneous current, then clearly $i = \frac{dq}{dt}$, or—

$$i = \omega CV \cos \omega t$$

In a vector diagram, therefore, the current would be represented by a vector which is $\frac{\pi}{2}$ or 90° *ahead* of the p.d. vector, as in Fig. 7. The p.d. and current are *in quadrature* with each other, and the p.d. *lags behind the current.*

The r.m.s. values of the p.d. and current are connected by the relation—

$$\text{current} = \text{p.d.} \times C\omega = \frac{\text{p.d.}}{\frac{1}{C\omega}}$$

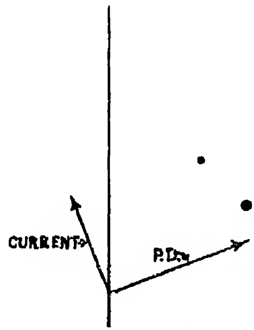


FIG. 7.—Vector Diagram for Pure Capacity.

and $\frac{1}{C\omega}$ is spoken of as the *reactance* of the condenser.

Let the resistance of a circuit be r, and its self-inductance L. In order to maintain a current—

$$i = I \sin \omega t$$

in the circuit, we must, by equation (1), provide an impressed e.m.f. of amount—

$$e = ri + L\frac{di}{dt}$$
$$= rI \sin \omega t + \omega LI \cos \omega t$$

The first component of the impressed e.m.f., viz. $rI \sin \omega t$, may be represented, in a vector diagram, by the projection of a rotating

vector OA, Fig. 8, of length rI, on the vertical axis; while the second component, $\omega LI \cos \omega t$, may be represented by the projection on the same axis of a rotating vector $OB = L\omega I$. The impressed e.m.f. at any instant is the algebraical sum of these two projections; but this is clearly the same as the projection of the diagonal OE of the rectangle constructed on OA and OB as sides. Hence OE will be the rotating vector corresponding to the impressed e.m.f. But since—

$$OE = \sqrt{OA^2 + OB^2} = \sqrt{r^2I^2 + \omega^2L^2I^2} = I\sqrt{r^2 + \omega^2L^2}$$

we see that—

$$\text{current} = \frac{\text{e.m.f}}{\sqrt{r^2 + \omega^2L^2}}\text{*}$$

and that the current *lags* behind the e.m.f. by an angle AOE such that—

$$\tan AOE = \frac{L\omega}{r}$$

The quantity $\sqrt{r^2 + \omega^2L^2}$ is termed the *impedance* of the circuit, and the angle $AOE = \tan^{-1} \frac{L\omega}{r}$ is the *angle of lag* of the current behind the impressed e.m.f.

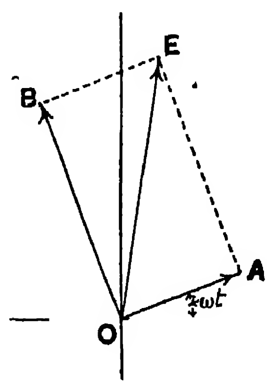


FIG. 8.—Vector Diagram for Inductive Resistance.

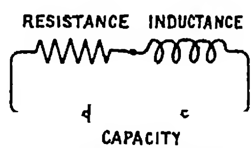


FIG. 9.—Arrangement of Resistance, Inductance, and Capacity in Series.

We may now consider a circuit containing resistance, inductance, and capacity. Such a circuit is diagrammatically represented in Fig. 9.* The impressed e.m.f. may be regarded as made up of the

* This equation holds good whether maxima or r.m.s. values be considered.

† We shall, in the diagrammatic representation of a circuit, use a coiled line to indicate an inductive resistance, and a zig-zag line to indicate non-inductive resistance

following three components, represented by the vectors OA, OB, and OC in Fig. 10:—

(1) The component $OA = rI$, in phase with the current.

(2) The component $OB = L\omega I$, 90° in advance of the current.

(3) The component $OC = \frac{1}{C\omega}I$, 90° behind the current.

The lengths of the vectors representing the amplitudes of the components, their projections at any instant correspond to the instantaneous values of the components, and the algebraical sum of the projections gives the instantaneous value of the impressed e.m.f.

Now the vectors OB and OC lying in the same straight line and being oppositely directed, the algebraical sum of their projections is equal to the projection of a single vector OD of length equal to $OB - OC = I\left(L\omega - \frac{1}{C\omega}\right)$. Finally, the sum of the projections of OA and OD is equal to the projection of the single vector OE, which is the diagonal of the rectangle constructed on OA and OD as sides, and which represents the impressed e.m.f. Now—

$$OE = \sqrt{OA^2 + OD^2} = \sqrt{r^2I^2 + \left(L\omega - \frac{1}{C\omega}\right)^2 I^2} = I\sqrt{r^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

Hence we see that—

$$\text{current} = \frac{\text{impressed e.m.f.}}{\sqrt{r^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

and the current lags behind the impressed e.m.f. by an angle $AOE = \theta$ such that—

$$\tan\theta = \frac{L\omega - \frac{1}{C\omega}}{r}$$

The quantity $L\omega - \frac{1}{C\omega}$ is the *reactance* of the circuit, while $\sqrt{r^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$ is termed the *impedance* of the circuit.

It is easy to see that the impedance, resistance, and reactance of a circuit are capable of being represented by the three sides of a right-angled triangle, one of whose angles corresponds to $\theta = \tan^{-1}\frac{L\omega - \frac{1}{C\omega}}{r}$ (the triangle OAE, in Fig. 10, may, to a suitable scale, be taken to represent these three quantities).

The diagram of Fig. 10 has been drawn on the supposition that the reactance $L\omega$ due to inductance exceeds the reactance $\frac{1}{C\omega}$ due to capacity. In this case, θ is positive, *i.e.* the current lags behind the impressed e.m.f.; θ is frequently spoken of as the *angle of lag*. It may, however, happen that $\frac{1}{C\omega} > L\omega$, θ then becoming negative, *i.e.* the current *leading* instead of lagging.

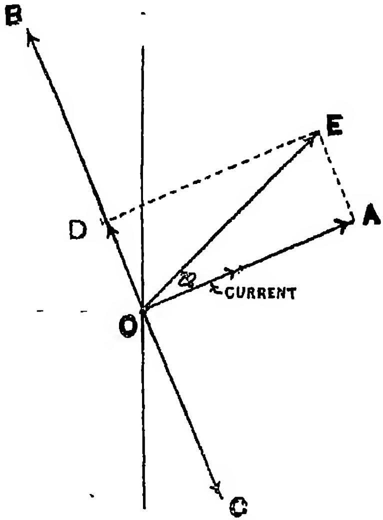


FIG 10.—Vector Diagram for Inductive Resistance in Series with Capacity.

We may, further, consider the special case in which $L\omega = \frac{1}{C\omega}$; θ now vanishes, *i.e.* the current comes into phase with the impressed e.m.f., and the impedance becomes simply equal to the resistance. The current is, therefore, the same as that which would be obtained with the same impressed e.m.f. in a non-inductive circuit of resistance r. Under these very special circumstances, the circuit is said to exhibit *electrical resonance* or *syntony*; the effect due to self-inductance being completely neutralized by that due to capacity.

When electrical resonance occurs in a highly inductive circuit, *i.e.* one whose self-inductance is very large in comparison with its resistance, we have the very remarkable result that the p.d.s across the inductive resistance and the condenser may very largely exceed the impressed e.m.f. This is at once evident from the vector diagram

of Fig. 10, since in this case each of the vectors OB and OC becomes very large in comparison with OA. The impressed e.m.f. and the p.d.s across the two portions of the circuit are given by—

$$\text{impressed e.m.f.} = \text{current} \times \text{resistance}$$
$$\text{p.d. across inductive resistance} = \text{current} \times \sqrt{r^2 + \omega^2 L^2}$$
$$\text{p.d. across condenser terminals} = \frac{\text{current}}{C\omega}$$

§ 7. Power in Alternating Current Circuit

Let the current in a circuit be given by—

$$i = I \sin \omega t$$

and the impressed e.m.f. by—

$$e = E \sin (\omega t + \theta)$$

The power w at any instant is—

$$\begin{aligned} w = ei &= EI \,.\, \sin(\omega t + \theta) \,.\, \sin \omega t \\ &= \tfrac{1}{2}EI \,.\, 2 \sin(\omega t + \theta) \,.\, \sin \omega t \\ &= \tfrac{1}{2}EI \{\cos\theta - \cos(2\omega t + \theta)\} \\ &= \tfrac{1}{2}EI \cos\theta - \tfrac{1}{2}EI \,.\, \cos(2\omega t + \theta) \end{aligned}$$

We therefore see that the expression for the instantaneous power consists of two terms, one of which, $\frac{1}{2}EI \cos \theta$, is a constant, while the other is a cosine (or sine) wave of frequency equal to *double* the frequency of the e.m.f. and current waves.

Now when we speak, without in any way qualifying the expression, of the *power* in an alternating current circuit, we understand by this term the *mean value* of the power over a complete period. The mean value of the second term in the expression for the *instantaneous power* w is, however, zero over any whole number of periods. Thus the mean value of the power becomes equal to the first or constant term $\frac{1}{2}EI \cos \theta$. This may be written in the form $\frac{E}{\sqrt{2}} \cdot \frac{I}{\sqrt{2}} \cdot \cos \theta$, and since $\frac{E}{\sqrt{2}}$ = r.m.s. value of e.m.f., and $\frac{I}{\sqrt{2}}$ = r.m.s. value of current, we see that

$$\text{mean power} = \text{e.m.f.} \times \text{current} \times \cos \theta,$$

r.m.s. values of e.m.f. and current being understood. The power, therefore, is not simply equal to the product of the e.m.f. and current, but is equal to this product multiplied by $\cos \theta$. The multiplier which converts *volt-amperes* or *apparent power* into watts or *true power* is

termed the *power factor* of the circuit. In the case considered, the power-factor is cos θ, but the term "power factor" is used generally, even in connection with circuits in which the waves are no longer of the simple sine form, to express the ratio $\frac{\text{true power}}{\text{volt-amperes}}$.

The expression for the mean power may be written in the form $\frac{E}{\sqrt{2}} \cdot \frac{I}{\sqrt{2}} \cos \theta$. Now I cos θ is the projection of the current vector on the impressed e.m.f. vector, and since the power depends on the magnitude of this projection, we speak of $\frac{I}{\sqrt{2}} \cos \theta$ (*i.e.* r.m.s. current $\times$ cos θ) as the *power component*, *load component* or *wattful component* of the current, while $\frac{I}{\sqrt{2}} \sin \theta$ is termed the *idle* or *wattless* component of the current. It is frequently convenient to regard the current in a circuit as split up into these two components, as shown in Fig. 11, where the vectors are taken to represent r.m.s. values of the current.

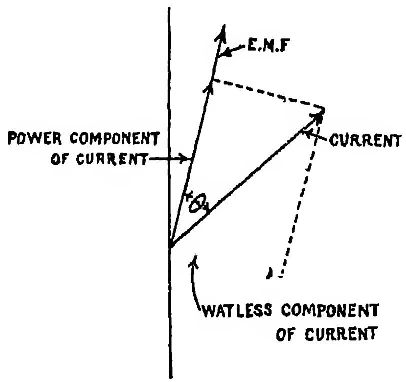


Fig. 11.—Decomposition of Current into Power and Wattless Components.

CHAPTER II

§ 8. Series arrangement of impedances—§ 9. Parallel arrangement of impedances—§ 10. Numerical example—§ 11. Mutual inductance—§ 12. General problem of two circuits having mutual inductance—§ 13. Skin effect—§ 14. Method of dealing with hysteresis and eddy current losses.

§ 8. Series Arrangement of Impedances

LET two impedances, AB and BC, be connected in series as shown in Fig. 12 (*a*), and let it be required to find their combined impedance.

An impedance is not completely specified by its numerical value. For with the same numerical value of the impedance we may have widely differing values of the ratio $\frac{\text{reactance}}{\text{resistance}}$, which determines the phase difference between e.m.f. and current. The impedance is, however, uniquely determined if in addition to the ratio $\frac{\text{p.d.}}{\text{current}}$ we know the angle of phase difference θ between p.d. and current.

Since the two impedances in Fig. 12 (*a*) are arranged in series with each other, it follows that the current must have the same value

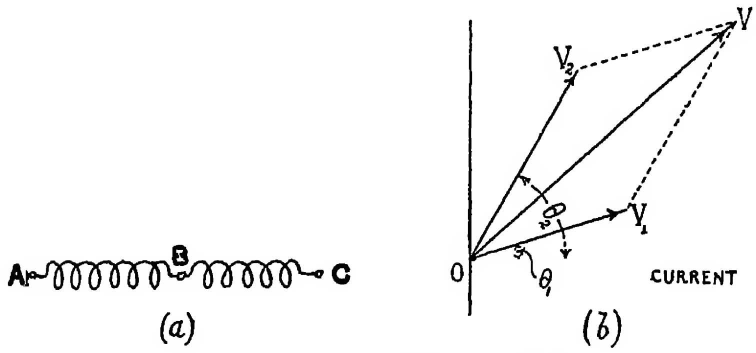


FIG. 12.— Two Impedances in Series.

in each of them at every instant. For this reason, we choose the current vector as our vector of reference in the vector diagram, and

we may conveniently lay off this current vector along the horizontal axis, as in Fig. 12 (*b*).

Let θ_1, θ_2 be the phase differences corresponding to the two given impedances, and let Z_1, Z_2 be their numerical values.

In the vector diagram, assume the current to have a value of *unity*. Then the p.d. across AB is given by a vector OV_1, Fig. 12 (*b*), of length Z_1, making an angle θ_1 with the current vector. Similarly, the p.d. across BC is given by a vector OV_2 of length Z_2, making an angle θ_2 with the current vector. From this it follows that the p.d. across AC is given by the diagonal OV of the parallelogram constructed on OV_1 and OV_2 as sides, and the angle made by this diagonal with the current vector is the angle by which the current lags behind the p.d. across AC.

Since, however, we have assumed the current to be unity, it follows that the length of OV will correspond to the numerical value of the total impedance, and the angle which OV makes with the current vector will correspond to the angle of phase difference for the total impedance. Thus the required impedance is completely determined.

It is now evident that the rule for the composition of two impedances in series with each other is identical with the rule for the composition of two forces acting at a point—we have simply to apply the *parallelogram law.*

The same rule may be extended to any number of impedances connected in series, and by applying the *polygon law* we can easily find, by a purely graphical method, both the magnitude of, and the angle of phase difference corresponding to, the total impedance.

Although a purely graphical method enables us to deal with this problem, yet where accuracy is required it may be preferable to have recourse to calculation. The most convenient method is then as follows.

Let Z_1, Z_2, Z_3 . . . be the given impedances, and θ_1, θ_2, θ_3 . . . the angles of phase difference, or the *phase angles*,* as we may briefly term them, between the current and the p.d. across each impedance. Resolve each impedance into two components, one of which lies along the horizontal, and the other along the vertical axis. Find the sum of all the horizontal and the sum of all the vertical components, square the two sums, add them, and extract the square root; this gives the total impedance. Thus—

total impedance =

$$\sqrt{(Z_1\cos\theta_1 + Z_2\cos\theta_2 + Z_3\cos\theta_3 + \ldots)^2 + (Z_1\sin\theta_1 + Z_2\sin\theta_2 + Z_3\sin\theta_3 + \ldots)^2}$$

* The phase angle is to be reckoned *positive* if the current *lags* behind the p.d. and *negative* if it *leads*

and the phase angle of the total impedance is given by—

$$\tan^{-1}\frac{Z_1 \sin \theta_1 + Z_2 \sin \theta_2 + Z_3 \sin \theta_3 + \ldots}{Z_1 \cos \theta_1 + Z_2 \cos \theta_2 + Z_3 \cos \theta_3 + \ldots}$$

If, instead of being given the impedances Z_1, Z_2, etc., and their phase angles θ_1, θ_2 . . . , we are given the values of the resistances r_1, r_2, r_3 . . . , and of the reactances V_1, V_2, V_3 . . . , then clearly—

$$\text{total impedance} = \sqrt{(r_1 + r_2 + r_3 + \ldots)^2 + (V_1 + V_2 + V_3 + \ldots)^2}$$

With a series arrangement of impedances, it is, therefore, permissible to add the resistances arithmetically in order to obtain the total resistance; and to add the reactances algebraically in order to obtain the total reactance. But it is not permissible to add the impedances arithmetically; this latter addition must be carried out *vectorially, i.e.* in accordance with the polygon law.

As a result, we find that in the case of a series circuit such as the one shown in Fig. 13, if V_1, V_2, V_3 denote the r.m.s. values of the p.d.s across AB, BC, and CD respectively, and V the r.m.s. value of the p.d. across the entire circuit AD, then in general $V_1 + V_2 + V_3 > V$, since the sum of the sides of any open polygon is in general greater than the closing side of the polygon. In the special case in which $\theta_1 = \theta_2 = \theta_3$, we have $V_1 + V_2 + V_3 = V$, the polygon in this case degenerating into a straight line.

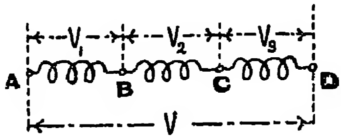


FIG. 13.—Series Arrangement of Impedances.

It may be well to point out that although the sum of the r.m.s. values of the p.d.s in the case considered is in general different from the r.m.s. value of the total p.d., yet at every instant the sum of the *instantaneous values* of the p.d.s must necessarily equal the *instantaneous* value of the total p.d. (the sum of the projections of the sides of an open polygon being always equal to the projection of the closing side of the polygon).

§ 9. Parallel Arrangement of Impedances.

Let a number of impedances, Z_1, Z_2, Z_3 . . . , having phase angles θ_1, θ_2, θ_3 . . . , be connected in parallel between two points, and let it be required to find their joint or parallel impedance.

The quantity which is common to all the branch circuits is the p.d. It is, therefore, suggested to take the p.d. vector as the vector

of reference in our vector diagram; let this p.d. vector be laid off horizontally as in Fig. 14.

The value of the joint impedance being the same for all values of the p.d., we may, for the sake of convenience, assume the p.d. to have a value of *unity*. Since—

$$\text{current} = \frac{\text{p.d.}}{\text{impedance}} = \text{p.d.} \times \frac{1}{\text{impedance}}$$

we see that, on the assumption of unit p.d., the currents in the various branches are given by the *reciprocals of the impedances*. The reciprocal of the impedance of a circuit is termed its *admittance*. Let A_1, A_2, A_3 . . . be the admittances corresponding to the various branches, so that $A_1 = \frac{1}{Z_1}$, $A_2 = \frac{1}{Z_2}$, $A_3 = \frac{1}{Z_3}$ . . .; then, with unit p.d. across the terminals, the currents in the various branches are equal to A_1, A_2, A_3. . . .

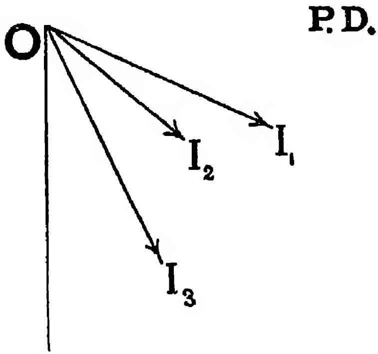


FIG. 14.—Vector Diagram of Currents in Branched Circuit.

An admittance, like an impedance, is a directed or vector quantity, so that it is not completely determined unless in addition to its magnitude we are also given its phase angle.

The phase angles of the admittances A_1, A_2, A_3 . . . are equal in magnitude to θ_1, θ_2, θ_3 . . .

Let us now, in our vector diagram, lay off vectors OI_1, OI_2, OI_3 . . . (Fig. 14) of lengths A_1, A_2, A_3 . . . making angles θ_1, θ_2, θ_3 . . . with the p.d. vector. These vectors will be the current vectors for the various branches, and since the total current between the two points is *at any instant* equal to the sum of the instantaneous currents in the various branches, it is clear that the vector of total current is obtained by compounding the vectors according to the polygon law, as in Fig. 15, the closing side OI of the polygon giving the vector of total current. Since, however, the p.d. was assumed to have a value of unity, it follows that OI represents the *joint admittance* of the various branches, and its phase angle is the angle which it makes with the p.d. vector.

The *joint impedance* is at once obtained by taking the reciprocal of the joint admittance.

With a parallel arrangement of impedances, therefore, the admittances of the various branches are compounded according to the same

rule as that which governs the composition of impedances in a series circuit.

It will be noticed that the sum of the r.m.s. values of the branch currents is in general greater than the r.m.s. value of the total current;

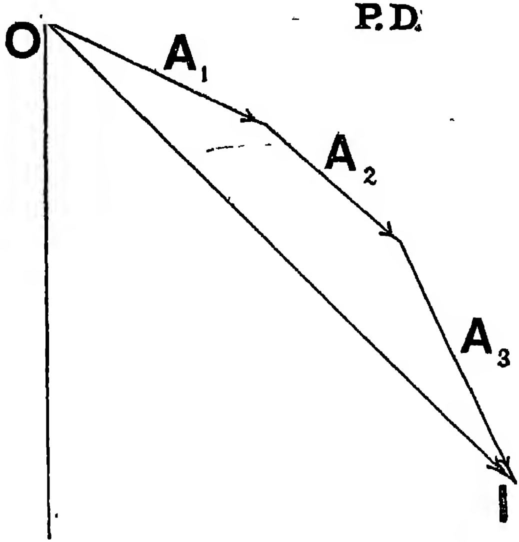


FIG. 15.—Vectorial Addition of Admittances.

but may become equal to it in the special case in which all the phase angles θ_1, θ_2, θ_3 . . . are equal.

In order to secure accuracy, we may as before *calculate* the joint impedance. The joint admittance is given by—

$$\sqrt{(A_1\cos\theta_1+A_2\cos\theta_2+A_3\cos\theta_3+\ldots)^2+(A_1\sin\theta_1+A_2\sin\theta_2+A_3\sin\theta_3+\ldots)^2}$$

and the reciprocal of this is the joint impedance. The phase angle θ corresponding to the joint impedance is such that—

$$\tan\theta = \frac{A_1 \sin\theta_1 + A_2 \sin\theta_2 + A_3 \sin\theta_3 + \ldots}{A_1 \cos\theta_1 + A_2 \cos\theta_2 + A_3 \cos\theta_3 + \ldots}$$

§ 10. Numerical Example.

In order to illustrate the application of the above principles, we shall work out in detail a fairly complicated numerical example.

In Fig. 16 is shown an arrangement of a number of resistances,

inductive and non-inductive, and condensers. The magnitudes of these are marked in the diagram. An alternating p.d. of 500 volts, having a frequency of 50, is applied across the extreme terminals

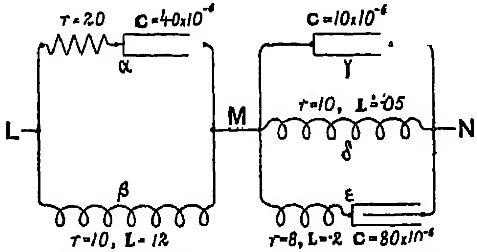


FIG. 16.—Circuit containing Resistances, Inductances, and Capacities.

L and N. The problem is to find the currents in the various branches.

We begin by finding the joint impedance of the branches α and β between L and M, and the joint impedance of the branches γ, δ, and ϵ between M and N.

The frequency being 50, we have $\omega = 2\pi \times 50 = 314$, say. Hence the capacity reactance in the branch α is $\frac{1}{\omega C} = \frac{1}{314 . 40 . 10^{-6}}$ $= 80{\cdot}4$ ohms; and since the resistance is 20, the impedance of the branch α is $\sqrt{20^2 + 80{\cdot}4^2} = 82{\cdot}8$ ohms, and its phase angle is $\tan^{-1} \frac{-80{\cdot}4}{20} = -76° \ 2'$. The *admittance* of this branch is $\frac{1}{82{\cdot}8}$ $= 0{\cdot}0121$, say.

Considering next the branch β, we have for its inductive reactance $\omega L = 314 \times 0{\cdot}12 = 37{\cdot}7$. The impedance is $\sqrt{10^2 + 37{\cdot}7^2} = 39$, the admittance $\frac{1}{39} = 0{\cdot}0256$, and the phase angle $\tan^{-1} \frac{37{\cdot}7}{10}$ $= + 75° \ 8'$.

Applying the rule for the composition of parallel admittances, we have for the joint admittance of α and β—

$$\sqrt{(0{\cdot}0121 \cos 76° 2' + 0{\cdot}0256 \cos 75° 8')^2 + (-0{\cdot}0121 \sin 76° 2' + 0{\cdot}0256 \sin 75° 8')^2} = \sqrt{0{\cdot}00953^2 + 0{\cdot}013^2} = 0{\cdot}01612$$

Hence the joint impedance between L and M is 62 ohms, and the joint phase angle is $\tan^{-1} \frac{0{\cdot}013}{0{\cdot}00953} = + 53° \ 45'$.

Taking next the branches γ, δ, and ϵ between M and N, and proceeding as before, we find—

		Impedance.	Admittance.	Phase Angle.
γ	...	319	0·00314	− 90°
δ	...	18·6	0·0537	+ 57° 30′
ϵ	...	24·35	0·0411	+ 70° 49′

Applying, as before, the rule for the composition of parallel admittances, we have—

$$0{\cdot}00314 \cos 90° + 0{\cdot}0537 \cos 57° \, 30' + 0{\cdot}0411 \cos 70° \, 49' = 0{\cdot}0424$$

and—

$$-0{\cdot}00314 \sin 90° + 0{\cdot}0537 \sin 57° \, 30' + 0{\cdot}0411 \sin 70° \, 49' = 0{\cdot}081$$

Thus the joint admittance is $\sqrt{0{\cdot}0424^2 + 0{\cdot}081^2} = 0{\cdot}0914$, the joint impedance 10·94, and the joint phase angle $\tan^{-1}\dfrac{0{\cdot}081}{0{\cdot}0424}$ $= +62° \, 22'$.

Having obtained the joint impedance between L and M, and also that between M and N, and the corresponding joint phase angles, we next proceed to compound these two impedances, according to the law for the composition of series impedances. We thus find—

Joint resistance between L and M = 62 cos 53° 45′ = 36·66
" " " M " N = 10·94 cos 62° 22′ = 5·08

Total resistance between L and N = 41·72

Similarly—

Joint reactance between L and M = 62 sin 53° 45′ = 50·00
" " " M " N = 10·94 sin 62° 22′ = 9·69

Total reactance between L and N = 59·69

We now find for the total impedance between L and N the value $\sqrt{41{\cdot}7^2 + 59{\cdot}7^2} = 72{\cdot}82$ ohms, and for the total phase angle $\tan^{-1}\dfrac{59{\cdot}7}{41{\cdot}7} = +55° \, 3'$.

The remaining part of the problem presents but little difficulty. The total current is $\dfrac{500}{72{\cdot}82} = 6{\cdot}87$ amperes. The p.d. across LM is obtained by multiplying the total current by the impedance between L and M, and similarly the p.d. across MN is found by multiplying the total current by the impedance between M and N.

P.d. across LM = 6·87 × 62 = 426 volts
P.d. across MN = 6·87 × 10·94 = 75 volts

We may note, in passing, that the sum of the r.m.s. voltages across LM and MN, namely, $426 + 75 = 501$, is very nearly equal to the voltage of 500 across LN. This close coincidence is purely accidental, and is due to the fact that the joint phase angle between L and M (53° 45′) is not very different from that between M and N (62° 22′).

Having determined the voltages across LM and MN, we at once obtain the branch currents by multiplying the admittance of each branch by the voltage across it. We thus find—

$$\begin{aligned}
\text{Current in } \alpha &= 426 \times 0{\cdot}0121 = 5{\cdot}15 \\
\text{,, \quad ,, } \beta &= 426 \times 0{\cdot}0256 = 10{\cdot}9 \\
\text{,, \quad ,, } \gamma &= 75 \times 0{\cdot}00314 = 0{\cdot}235 \\
\text{,, \quad ,, } \delta &= 75 \times 0{\cdot}0537 = 4{\cdot}03 \\
\text{,, \quad ,, } \epsilon &= 75 \times 0{\cdot}0411 = 3{\cdot}08
\end{aligned}$$

It will be noticed that the current in the branch β is considerably greater than the total current.

§ 11. Mutual Inductance

In the practical applications of alternating currents, we are constantly coming across instances of two circuits so placed relatively to each other that a current sent through one of them gives rise to a magnetic flux which becomes partly linked with the other. Two circuits so arranged are said to possess *mutual inductance*.

Let the relative positions of the circuits be such that when a unit (continuous) current is sent through the first circuit, it produces a total flux* F_2 through the second; and that when a unit current is sent through the second circuit, it gives rise to a total flux F_1 through the first. Assuming unit current to be flowing in each circuit, let us suppose the first circuit to be fixed in position, while the second is displaced in such a manner as to produce a change δF_2 in the flux F_2. The work spent (or gained) during this displacement = current in second circuit × change of flux† $= 1 \times \delta F_2 = \delta F_2$. Suppose next that, instead of moving the second circuit, we keep this circuit fixed, and displace the first circuit in an opposite direction, so that the final relative positions of the two circuits are the same as before. If δF_1 stand for the change in the flux F_1 during this displacement, the work done = current in first circuit × change of flux $= 1 \times \delta F_1 = \delta F_1$. But since the force and couple exerted by the first circuit on the second are, by the principle of equality of action

* By the "total flux" is meant the sum of the fluxes through the various elementary loops or turns of which the circuit may be supposed to consist.

† Hay, *Continuous Current Engineering*, p. 4.

and reaction, always equal to those exerted by the second circuit on the first, it follows that the work done during the displacement of the first circuit is equal to that done during the displacement of the second circuit. Hence $\delta F_1 = \delta F_2$, i.e. *if the circuits are displaced relatively to each other, the change of flux through the second circuit is equal to the change of flux through the first* (unit current being supposed to flow in each circuit; a similar result obviously holds if the circuits convey equal currents of any magnitude).

Let now the circuits be separated from each other to an infinite distance. Then since both F_1 and F_2 vanish as a result of the separation, it follows that F_1 and F_2 represent the corresponding changes of flux through the two circuits, and hence by the principle just established we must have $F_1 = F_2$, i.e., *the flux through the first circuit due to unit current in the second is equal to the flux through the second circuit due to unit current in the first.* This flux is defined to be the *mutual inductance* of the two circuits. Being a quantity of the same nature as self-inductance, it is measured in henries. When two circuits having mutual inductance are traversed by alternating currents, they mutually react on each other, and owing to this reaction each circuit behaves in a manner different from that which would occur if the other circuit conveyed no current. In considering such circuits, two distinct modes of treatment are available. These may be briefly termed the *hypothetical flux* and the *resultant flux* method. The essential difference between the two methods consists in the way in which the induced e.m.f. in either circuit is supposed to be made up of two components.

In the *hypothetical flux* method, we make use of the self and mutual inductances of the two circuits. By the self-inductance of either circuit is in this case to be understood the magnetic flux which becomes linked with the circuit when conveying unit current, the other circuit being supposed to carry no current or else to be removed to an infinite distance. Were there no mutual inductance between the circuits, the e.m.f. induced in each would be simply that due to its self-inductance. But in addition to this, we have to take into account the e.m.f. induced by the flux of mutual inductance. Thus the total induced e.m.f. in either circuit is regarded as made up of (1) the e.m.f. of self-inductance, which is in quadrature with the imaginary or hypothetical flux that would result if the other circuit either conveyed no current or were removed to infinity; and (2) the e.m.f. of mutual inductance, which is in quadrature with the hypothetical flux that would traverse the circuit under consideration if its own current were annulled, and the current in the other circuit maintained at its original value. Since (neglecting hysteresis effects) the hypothetical fluxes mentioned are in phase with the currents producing them, we see that in this method the total induced e.m.f.

is regarded as split up into two components which are in quadrature respectively with the currents in the two circuits, and whose phase difference is therefore the same as the phase difference of the currents.

In the *resultant flux* method, a somewhat different analysis of the actually existing magnetic field into a number of components is adopted. Considering the hypothetical field due to the primary * current—*i.e.* the field which would be produced by this current if acting alone, we may regard it as made up of (1) a portion f_l consisting of lines linked with the primary only, and (2) another portion f_m, consisting of lines linked with both circuits. Similarly, the secondary hypothetical field may be regarded as made up of two corresponding amounts f'_l and f'_m. Since f_m and f'_m represent fields linked with both circuits, their superposition yields a resultant field f which is also linked with both circuits, and which we may term the *main field*. Since the main field is acted on by the ampere-turns of both circuits, it is (neglecting hysteresis) in phase with the resultant of the ampere-turns of the two circuits, and hence the e.m.f. induced by it in either circuit is in quadrature with this resultant. The fields f_l and f'_l are termed the primary and secondary *leakage fields* respectively, and since each of them is acted on by the ampere-turns of its own circuit only, they are (neglecting hysteresis) in phase with the primary and secondary turns respectively. The self-inductances arising from the leakage fields are termed the *leakage self-inductances* of the two circuits. It will now be evident that the total induced e.m.f. in either circuit may be regarded as made up of the following two components:—(1) the e.m.f. induced by the main field, which is in quadrature with the resultant of the primary and secondary ampere-turns; and (2) the e.m.f. induced by the leakage field, which is in quadrature with the current in the circuit under consideration.

Let the two circuits be represented by two coils of wire, the primary coil consisting of S_1 turns, and the secondary of S_2 turns. Let L_1, L_2 stand for the self-inductances, and M for the mutual inductance of the coils. When unit current circulates in the primary, it produces a flux M through the secondary, and since the secondary consists of S_2 turns, the mean flux *per turn* is $\frac{M}{S_2}$. Thus of the total flux linkage L_1 with the primary an amount $\frac{S_1 M}{S_2}$ is due to lines which are also linked with the secondary, so that the remainder, viz. $L_1 - \frac{S_1}{S_2}M$, represents the primary leakage flux due to unit current, or *primary leakage self-inductance* l_1. Similarly, the *secondary leakage self-inductance* l_2 is given by $l_2 = L_2 - \frac{S_2}{S_1}M$.

* It is convenient to speak of the two circuits as the *primary* and

§ 12. General Problem of Two Circuits having Mutual Inductance.

Consider the two circuits diagramatically represented in Fig. 17. The first circuit, which we shall term the *primary*, is of resistance r_1, self-inductance L_1, and has an alternating e.m.f. E_1 impressed on it from an external source. The second circuit or *secondary* is of resistance r_2, self-inductance L_2, and has an e.m.f. E_2 impressed upon it from an external source. The mutual inductance of the two circuits is M.

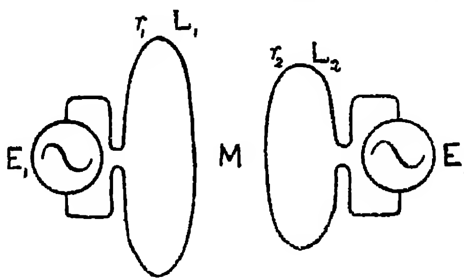


FIG. 17.—Diagram of Circuits having Mutual Inductance.

Let us suppose that there is a current I_1 in the primary and a current I_2 in the secondary, and that the phase difference between them is known. Then the problem of finding the impressed e.m.f.s E_1 and E_2 may be solved graphically by constructing a vector diagram, and in doing so we may employ either the hypothetical flux or the resultant flux method.

The hypothetical flux diagram is shown in Fig. 18 (*a*), and is constructed as follows. We draw two lines making an angle with each other which is equal to the known angle of phase difference between I_1 and I_2. Along one of these we lay off a length r_1I_1 to represent the resistance drop in the primary, and along the other a length r_2I_2 to represent the resistance drop in the secondary. Considering first the secondary circuit, we lay off, at right angles to r_2I_2, a length ωL_2I_2 to represent the self-inductance component of E_2, and from the end of ωL_2I_2 a length ωMI_1 normal to r_1I_1 to represent the mutual inductance component of E_2. The resultant of r_2I_2, ωL_2I_2 and ωMI_1 determines E_2. A similar mode of construction when applied to the primary enables us to find E_1, as shown in the diagram.

The resultant flux diagram is shown in Fig. 18 (*b*). The first step in constructing the diagram consists in drawing two vectors to represent the primary and secondary ampere-turns, and finding their resultant. This part of the construction is indicated by the dotted lines of Fig. 18 (*b*). From the resultant ampere-turns and the known data of the magnetic circuit we then calculate the flux common to the primary and secondary. Let l_1 and l_2 denote the leakage self-inductances of the primary and secondary respectively. We lay off

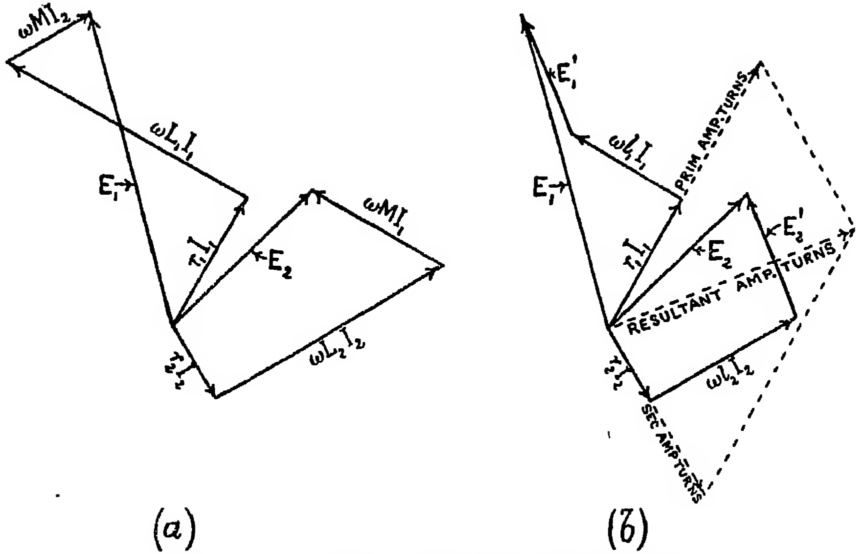


FIG 18.—Vector Diagrams of Mutually Inductive Circuits.

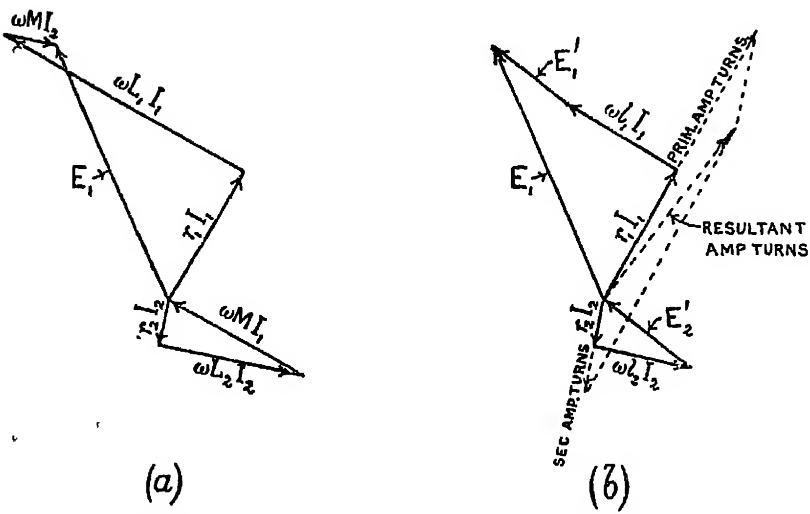


FIG 19.—Vector Diagrams of Mutually Inductive Circuits when there is no impressed e.m.f. in secondary.

a vector r_2I_2 to represent the resistance drop in the secondary, at right angles to this a vector ωl_2I_2 to represent the leakage self-inductance drop, and a third vector E'_2, normal to the flux (assumed to be in phase with the resultant ampere-turns) common to the two circuits, to represent the component of E_2 required to balance the e.m.f. induced in the secondary by this common flux. The resultant of $r_2I_2, \omega l_2I_2$ and E'_2 gives us E_2. A similar construction applied to the primary yields E_1.

A case of special importance arises when there is no external e.m.f. impressed on the secondary, *i.e.* when $E_2 = 0$. The hypothetical flux and resultant flux diagrams for this case are shown in Figs. 19 (*a*) and (*b*) respectively. An examination of Fig. 19 (*a*) shows that the presence of a closed secondary in the neighbourhood of the primary—which introduces the vector ωMI_2 into the primary diagram—always results in bringing the impressed e.m.f. more nearly into phase with the primary current. This effect is equivalent to *a decrease in the self-inductance and an increase in the resistance of the primary.* If M is large, the vector ωMI_2 may become nearly equal in length to ωL_1I_1, and so *the self-inductance of the primary may be nearly wiped out* by the effect of mutual inductance.

§ 13. Skin Effect

In the case of a conductor of large cross-section conveying a current, there is an appreciable difference between the magnetic flux linked with the surface layers of the conductor and the flux linked with its central portion. For if we imagine the conductor split up into a large number of small parallel filaments, then the current along any filament at a considerable depth below the surface will be linked not only with the lines external to the conductor, but also with an appreciable number of lines in the substance of the conductor; whereas a surface filament is only linked with the external lines. As a consequence, the e.m.f. induced along a surface filament when an alternating current is sent through the conductor is less than that induced along a central filament, and since the various filaments may be regarded as different branches of a parallel circuit, it is obvious that, for a given cross-section of filament, the current will be less in the case of a filament near the axis than in one near the surface of the conductor. Thus the current density will be greater in the surface layers, the current being *unevenly distributed* over the cross-section of the conductor. This uneven distribution of current is equivalent to a reduction of cross-section, or to an increase of resistance, of the conductor, and is frequently spoken of as the *skin* effect.

The skin effect with cylindrical *copper* conductors is, at ordinary frequencies, inappreciable until a diameter of about ½ inch is reached; it then increases very rapidly with the diameter. But with conductors constructed of magnetic materials, such as iron or steel, a very marked effect occurs even with conductors of small cross-section.

This effect is of considerable practical interest in cases where an alternating-current system is used for working electric railways or tramways, and where the ordinary track rails are used as one of the conductors of the system. In the case of a steel conductor of large cross-section, the effective area of cross-section over which the current is distributed is confined to a comparatively thin surface layer—not exceeding ⅛ inch at all ordinary frequencies. Hence the loss occurring with a given current in such a conductor is very much greater for an alternating than for a continuous current.

§ 14. Method of Dealing with Hysteresis and Eddy Current Losses

The method explained in § 7 of splitting up the total current in any circuit into a wattful and a wattless component is useful in dealing with circuits containing iron cores, as it simplifies the treatment of the hysteresis and eddy-current losses occurring in such circuits. Consider a coil provided with a laminated core. The losses occurring in such a coil are:—(1) the loss due to the resistance of the coil; (2) that caused by hysteresis of the core; and (3) that due to eddy currents. The behaviour of the iron-cored coil may be imitated by the arrangement of non-inductive resistances R, r and pure (*i.e.* resistanceless) inductance L shown in Fig. 20. The resistance R is equal to that of the copper winding, and conveys the total current. The inductance L is such that the maximum magnetic flux through it equals that through the winding. Finally, the resistance r is supposed to have a value such that the loss occurring in it is equal to the sum of the hysteresis and eddy-current losses. The current in the branch r gives the wattful component of the total current, while that in L corresponds to the wattless component.

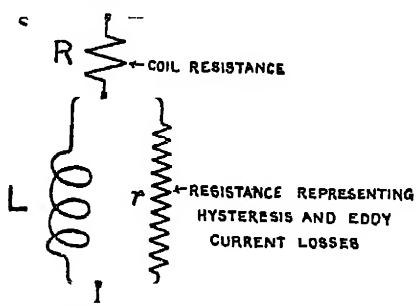


Fig. 20.—Arrangement of Circuits equivalent to Electromagnet.

In order to exhibit as clearly as possible the part played by the

magnetic properties of the core, we shall consider the case of a number of similar electromagnets, but having cores of different materials, connected (1) in series with each other and (2) in parallel.

In the series arrangement of magnets, each exciting coil necessarily conveys the same current, and the value of the magnetic flux in any core will be unaffected by the resistance of the exciting coil. On the other hand, the value of the maximum flux and counter-e.m.f., and of the angle of lag between the current and the p.d. across any coil, will be determined by the permeability, hysteresis and eddy-current losses of the core, as well as by the resistance of the coil. If the resistance drop be negligible in comparison with the counter-e.m.f., then the magnetic flux and angle of lag for any magnet will depend solely on the physical properties of its core.

Consider next a parallel arrangement of electromagnets. If the resistances of the exciting coils are different, the value of the counter-e.m.f. will to some extent depend on the resistance. In many cases, the resistance drop may be neglected. This is equivalent to wiping out the resistance R in Fig. 20, so that we now have the inductance L connected in parallel with r and directly across the mains. It is obvious that any changes in r cannot affect the maximum flux through L so long as the p.d. is maintained constant; now since the value of r depends on the hysteresis and eddy-current losses in the core, the above statement is equivalent to saying that the magnetic flux is entirely independent of the properties of the core. This is also otherwise obvious from the following considerations. Since the p.d. must (owing to the fact that the resistance drop is negligible) be at every instant equal and opposite to the counter-e.m.f., and since the latter is determined by the rate of change of magnetic flux, it follows that the rate of change of magnetic flux, and hence also the magnetic flux itself, have perfectly definite values at every instant (determined by the wave-form of the p.d.), and that these values are quite independent of the properties of the core. We thus arrive at the result that the *magnetic flux in a shunt electromagnet is entirely independent of the properties of the core*, if the resistance drop be negligible. Further, the flux is in this case in quadrature with the p.d.*

The above results are of great importance in connection with iron-cored measuring instruments for alternating current circuits (§ 40).

* Two periodic functions of the same frequency are said to be in quadrature with each other if their mean product over a period vanishes. Let, in the case considered, v = instantaneous p.d.; Φ = instantaneous total flux; T = period. Then since $v = \frac{d\Phi}{dt}$, we have mean value of $v\Phi = \frac{1}{T}\int_0^T \Phi\frac{d\Phi}{dt}dt = \frac{1}{T}\left[\frac{1}{2}\Phi^2\right]_0^T = 0$, or v and Φ are in quadrature with each other.

CHAPTER III

§ 15. Polyphase currents—§ 16 Rotating field produced by polyphase currents—§ 17. Connections of polyphase systems—§ 18. Equivalence of Y and Δ loads in balanced three-phase systems—§ 19. Comparison of single-, two-, and three-phase systems—§ 20. Simple alternating wave of magnetic flux—§ 21. Analysis of alternating into two rotating waves, and *vice versâ*—§ 22. Production of rotating waves of magnetic flux by means of polyphase currents.

§ 15. Polyphase Currents

IN connection with many important practical applications, the alternating currents employed consist not of simple or *single-phase* currents, but of a system of several currents of the *same frequency* but differing in phase. Such a system is termed a *polyphase system.*

The only two polyphase systems of practical importance are those known as the *two-phase* and the *three-phase* system.*

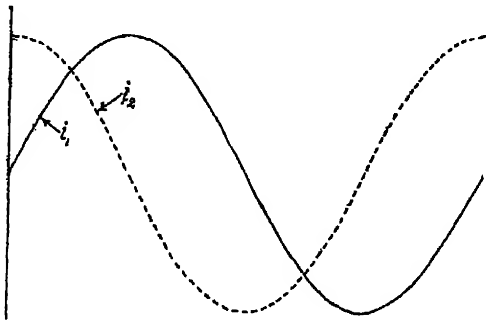


FIG. 21.—Graphs of Two-phase Currents.

A two-phase system consists of two currents of the same frequency differing in phase by 90°.

A three-phase system consists of three currents of the same frequency differing 120° in phase.

Fig. 21 shows the graphs of the currents (sine waves being

* A six-phase system is frequently used in the armature windings of rotary converters. But the transmission of power to the converter takes place by means of three-phase currents.

assumed) forming a two-phase, and Fig. 22 the graphs of the currents forming a three-phase system.

The vector diagram for a two-phase system consists, as shown in

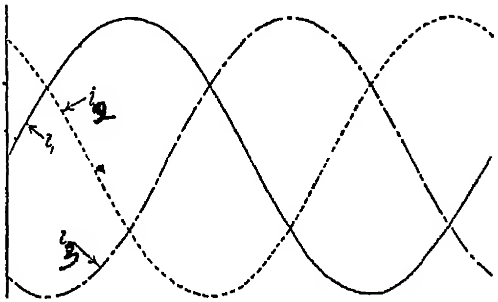


FIG 22.—Graphs of Three-phase Currents.

Fig. 23 (*a*), of two vectors at right angles to each other; while that for a three-phase system, Fig. 23 (*b*), consists of three vectors making angles of 120° with each other.

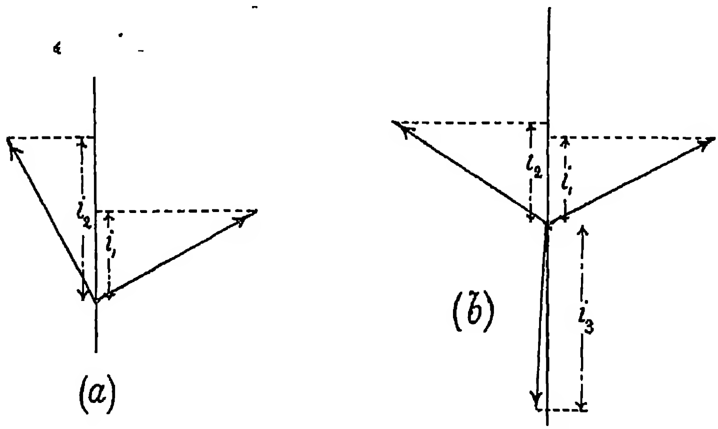


FIG 23.—Vector Diagrams of Two- and Three-phase Systems.

Algebraically, the expressions for the currents forming a two-phase system may be written—

$$i_1 = I_1 \sin \omega t$$

$$i_2 = I_2 \sin\left(\omega t + \frac{\pi}{2}\right) = I_2 \cos \omega t$$

and those for the currents forming a three-phase system—

$$i_1 = I_1 \sin \omega t$$
$$i_2 = I_2 \sin \left(\omega t + \frac{2\pi}{3}\right)$$
$$i_3 = I_3 \sin \left(\omega t + \frac{4\pi}{3}\right)$$

It may be pointed out that the amplitudes of the currents forming a polyphase system need not necessarily be equal; if they are, the system is said to be *balanced.*

§ 16. Rotating Field produced by Polyphase Currents

Polyphase systems present several advantages * over the single-phase system, one of the most important being the possibility of producing a *rotating magnetic field* without the aid of any mechanical rotation.

In order to explain the production of such a field by means of two-phase currents, we may consider two similar coils placed at right angles to each other, as in Fig. 24; at any given point of space each coil will produce, when conveying a current, a field proportional to the current. Hence, if we suppose that the coils are traversed by the two currents of equal amplitude—

$$i_1 = I \sin \omega t$$

and—

$$i_2 = I \cos \omega t$$

respectively, the magnetic fields at the common centre O of the two coils due to the currents may be written (Fig. 24)—

$$x = M \sin \omega t, \text{ along the horizontal axis,}$$

and—

$$y = M \cos \omega t, \text{ along the vertical axis,}$$

and since the fields are at right angles to each other, the resultant field OR is given by the square root of the sum of their squares, *i.e.* the magnitude of the resultant field is—

$$OR = \sqrt{x^2 + y^2} = M$$

The magnitude of the resultant field is thus *constant.* In order to

* These are considered in § 19.

find its *direction* at the time t, we notice that if θ be the angle which it makes with the vertical axis—

$$\tan \theta = \frac{x}{y} = \tan \omega t$$

so that—

$$\theta = \omega t$$

The angle therefore changes at a constant rate, *i.e.* the field rotates with the constant angular velocity ω.

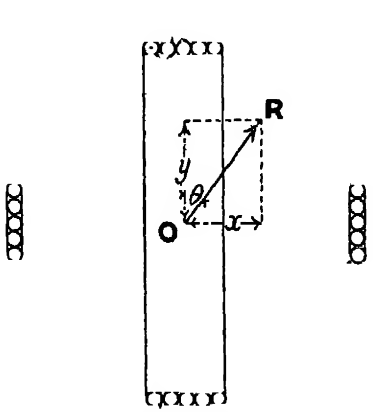


FIG. 24.—Production of Rotating Field by Two-phase Currents.

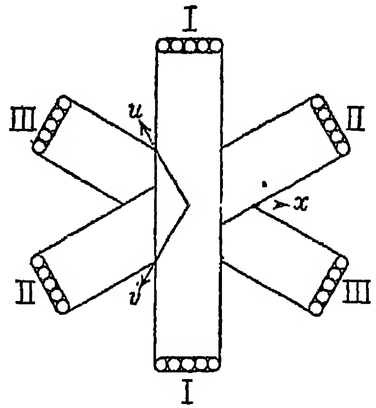


FIG. 25.—Rotating Field produced by Three-phase Currents.

Consider next three coils, I, II, and III, arranged at 120° to each other, as shown in Fig. 25, and conveying the three currents of equal amplitude—

$$i_1 = \mathrm{I} \sin \omega t$$

$$i_2 = \mathrm{I} \sin\left(\omega t + \frac{2\pi}{3}\right) = -\tfrac{1}{2}\mathrm{I} \sin \omega t + \frac{\sqrt{3}}{2}\mathrm{I} \cos \omega t$$

$$i_3 = \mathrm{I} \sin\left(\omega t + \frac{4\pi}{3}\right) = -\tfrac{1}{2}\mathrm{I} \sin \omega t - \frac{\sqrt{3}}{2}\mathrm{I} \cos \omega t$$

The directions of the magnetic fields produced by the three coils at their common centre are indicated by x, u, and v in Fig. 25. The values of these magnetic fields are given by—

$$x = \mathrm{M} \sin \omega t$$

$$u = -\tfrac{1}{2}\mathrm{M} \sin \omega t + \frac{\sqrt{3}}{2}\mathrm{M} \cos \omega t$$

$$v = -\tfrac{1}{2}\mathrm{M} \sin \omega t - \frac{\sqrt{3}}{2}\mathrm{M} \cos \omega t$$

In order to find the magnitude of the resultant field, we may determine the sum of all the horizontal components, then the sum of all the vertical components, and finally take the square root of the sum of the squares of the total horizontal and vertical components.

Now, the total horizontal component at time t is given by (Fig. 25)—

$$X = x - (u + v) \cos 60° = M \sin \omega t + \tfrac{1}{2}M \sin \omega t = \tfrac{3}{2}M \sin \omega t$$

and the total vertical component by—

$$Y = (u - v) \sin 60° = \tfrac{3}{2}M \cos \omega t$$

so that the magnitude of the resultant field is—

$$\sqrt{X^2 + Y^2} = \tfrac{3}{2}M$$

i.e. the resultant field is of constant magnitude. If ϕ be the angle which it makes with the vertical axis at time t, then—

$$\tan \phi = \frac{X}{Y} = \tan \omega t$$

so that the resultant field rotates with constant angular velocity ω.

In this possibility of producing a rotating magnetic field without any mechanical rotation lies, as already mentioned, one of the main advantages of polyphase currents.

For the production of two-phase currents, two independent sources of e.m.f. are required, having the same frequency but a phase displacement of 90°. These two sources of e.m.f. are represented by two independent windings in the armature of a two-phase generator.

Similarly, the three sources of e.m.f. required for the production of three-phase currents are represented by three independent armature windings in the generator.

§ 17. Connections of Polyphase Systems

In the case of two-phase systems, the two phases or circuits of the generator, motor, or other receiving apparatus forming the system are, as a rule, kept entirely separate, as there is no advantage in electrically linking them. Sometimes, however, such linkage is resorted to, the arrangement adopted being that shown in Fig. 26.

If we suppose the system balanced, then the current in the common wire will be represented, in the vector diagram of Fig. 23 (a), by the diagonal of the square constructed on the two current vectors there shown as sides. Hence the current in the common wire is $\sqrt{2}$ or 1·414 times the phase current (*i.e.* the current in either phase),

and if the current density in all the wires be the same, the cross-section of the common wire will have to be $\sqrt{2}$ times the cross-section of one of the outer wires.

Three-phase circuits are *invariably* linked together, as this effects a saving in the amount of copper required in the mains. There are two methods of coupling such circuits. One of these, shown in Fig. 27, is known as the *star* or Y coupling. In this, it will be noticed, the *line* current is the same as the *phase* current, and there is a point—known as the *neutral point*, marked N in Fig. 27 (*a*)—common to the three phases. The possibility of so coupling the circuits is due to the fact, at once evident from Fig. 23 (*b*), or from an inspection of the equations of § 15, that the algebraical sum of the three currents vanishes at every instant. With the star method of connection, the line currents are equal to the phase currents, but the line p.d. is much higher than the phase p.d. If we assume for the positive directions of the three p.d.s the directions *away from* the neutral point N,

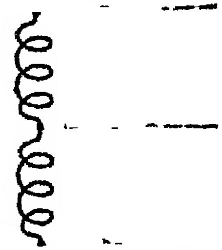
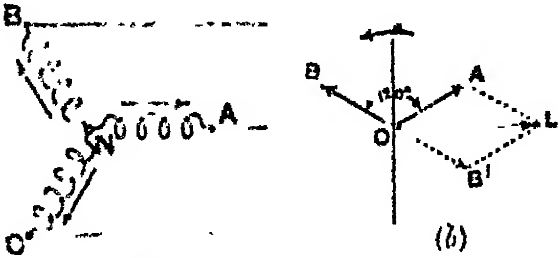


FIG. 26.—Interconnected Two-phase System.



(*a*) (*b*)

FIG. 27.—Star-connected Three-phase System.

as indicated by the arrows in the figure, then since in proceeding from B to N we move in the negative direction, and in proceeding from N to A in the positive direction, it is evident that the *line* p.d., *i.e.* the p.d. across BA, will at every instant be equal to the *difference* of the instantaneous p.d.s across NA and NB. Now, if the instantaneous p.d. across NA be represented by the projection of OA in the vector diagram of Fig. 27 (*b*), and that across NB by the projection of OB, then the difference of these two projections, which gives the instantaneous value of the p.d. across BA, is equal to the projection of OA, plus the projection of OB *reversed*. In Fig. 27 (*b*), OB' is OB reversed, so that the p.d. across BA is equal to the sum of the projections of

OA and OB′, *i.e.* to the projection of OL. But since OL = 2OA . cos 30° $= \sqrt{3}$. OA, we see that, with a balanced system having a star coupling and supplied with sine waves of e.m.f., *the line p.d. is equal to* $\sqrt{3}$ *or* 1·732 *times the phase p.d.*

Another method of coupling three-phase circuits is that shown in Fig. 28, and known as the *mesh* or *triangle* or *delta* method. It is immediately obvious that the line p.d. is now equal to the phase p.d. The line *currents*, however, are not equal to the phase currents, each line current being at any given instant equal to the difference of the adjacent phase currents—as is at once evident from Fig. 28. Hence the vector diagram of Fig. 27 (*b*) is now applicable to the *currents*, and we see that with a mesh grouping and simple sine waves *the line current is equal to* $\sqrt{3}$ *or* 1·732 *times the phase current.*

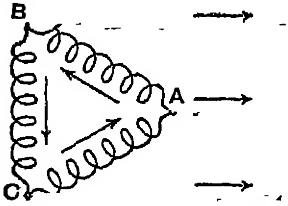


FIG. 28.—Delta-connected Three-phase System.

The advantage of coupling the circuits will now be readily understood. For if in Fig. 28 the three circuits had been kept separate, six line wires, each, we shall suppose, of cross-section *a*, would have been required (two wires for each phase or circuit). By coupling the circuits delta fashion, so as to maintain the original p.d. across each pair of mains unaltered, the number of wires is reduced to three and the cross-section of each is, for the same current density as before, $\sqrt{3} . a$. Thus the ratio of the amount of copper in the coupled circuits to that in the uncoupled ones is $\dfrac{3 . \sqrt{3} . a}{6a} = \dfrac{\sqrt{3}}{2}$ $= \dfrac{1{\cdot}732}{2} = 0{\cdot}866$, representing a saving of about 13 per cent., and this saving is a clear gain, the coupling of the circuits not being attended with any disadvantages such as result, for example, from the coupling of two-phase circuits. In the later case, the maximum p.d. between the two outer line wires is $\sqrt{2}$ or 1·414 times the phase p.d., which either increases the risk of a breakdown or else necessitates better and hence more expensive insulation for the line than would be necessary if the circuits were kept separate.

As regards the practical use of the two methods of coupling three-phase circuits, it may be said that the coils of generators and motors are generally coupled star fashion, while transformers are, for reasons to be explained later, in most cases delta-connected. Rotary converters are, from the nature of the case, also delta-connected ; and so is an ordinary lamp load.

§ 18. Equivalence of Y and Δ Loads in Balanced Three-phase Systems

By the aid of the relations just established between the phase p d.s and currents and the corresponding line p.d.s and currents for a three-phase load, we can readily deduce the condition of equivalence of a star-connected and a delta-connected load. With a given line p.d., the loads will be equivalent if the line current has the same value in each case, and if its phase relation to the line p.d. remains unaltered. Now the latter condition will be fulfilled if the phase angle between the star current and its corresponding star p.d. in the

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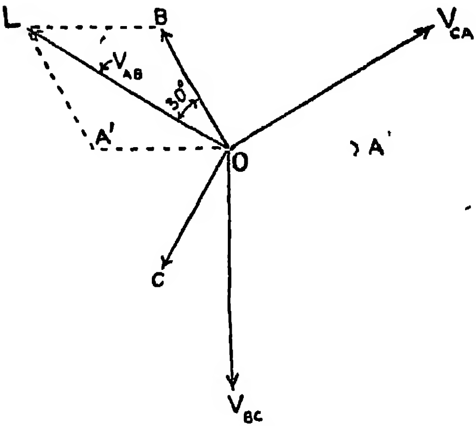


Fig. 29.—To illustrate relation of Star and Mesh p.d s.

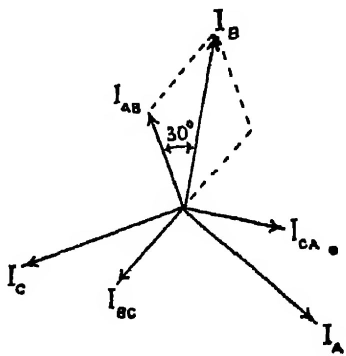


Fig. 30.—To illustrate relation of Star and Mesh Currents.

star-connected load be the same as that between the Δ-current and its Δ or line p.d. in the Δ-connected load. This is easily seen by referring to the vector diagrams of Figs. 29 and 30,* when it will be noticed that in order to derive the line p.d.s from the star p.d.s in the diagram for the star load (Fig. 29), the vectors representing the star p.d.s have to be advanced in phase by 30° (and lengthened in the ratio $\sqrt{3}:1$); whereas in order to derive the line current from the Δ-current in a Δ-connected load the current vectors have (Fig. 30) to be retarded in phase by 30° (and lengthened in the ratio $\sqrt{3}:1$). Now, since a forward rotation of the p.d. vector has the same effect on

* The vector diagrams of Figs. 29 and 30 correspond to the circuit diagrams of Figs. 27 and 28 respectively.

the phase angle as an equal backward rotation of the current vector, it follows that the change of phase is the same in each of the diagrams of Figs. 29 and 30, and hence the phase relations of the line p.d.s and line currents will remain unaffected by the substitution of a Δ for a Y load, or *vice versâ*, provided the power factor of each side of the Δ is the same as that of each arm of the Y.

Next, considering the magnitudes of the p.d.s and currents, we have, if V_s, I_s denote the star p.d. and star current in a Y-connected load, and V_Δ, I_Δ the line p.d. and Δ-current in a Δ-connected load, star impedance (or impedance of each ray of the star) $= \frac{V_s}{I_s}$; and Δ-impedance (or impedance of each side of the Δ) $= \frac{V_\Delta}{I_\Delta}$. Hence we have

$$\frac{\text{Y-impedance}}{\Delta\text{-impedance}} = \frac{V_s/I_s}{V_\Delta/I_\Delta} = \frac{V_s}{V_\Delta} \times \frac{I_\Delta}{I_s} = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3},$$

or in order that the loads may be equivalent *the Y-load must have an impedance equal to ⅓rd that of the Δ-load* (and, as shown above, the power factors of the loads must be equal).

§ 19. Comparison of Single-, Two-, and Three-Phase Systems

The relative advantages and disadvantages of the single-, two-, and three-phase systems of power generation and transmission may be briefly summarized as follows. The single-phase system is the simplest, since it requires only two conductors, with a correspondingly simple arrangement of switch-gear. Where the number of conductors is a matter of importance—as in connection with electric tramways and railways—the single-phase system possesses an important advantage over its polyphase rivals. On the other hand, polyphase generators are, for a given speed and power, lighter and cheaper than single-phase ones; induction motors and rotary converters of the polyphase type are also superior to single-phase machines.* The cost of the conductors to transmit a given amount of power with a given p.d. between any two conductors and a given loss in transmission is least in the case of a three-phase system,† and it is for this reason that the three-phase system has been much more generally adopted than the

* The performance of a single-phase rotary converter is so poor that this type of machine is never used in modern practice.

† Hay, *Electrical Distributing Networks and Transmission Lines*, p. 30.

two-phase one. The latter has been mostly used to replace single-phase systems, in which case the existing single-phase concentric mains may be utilized; a three-phase system, requiring three-core or triple concentric mains, would in such a case involve the scrapping of the existing system of single-phase mains.

§ 20. Simple Alternating Wave of Magnetic Flux

In § 16 we explained how a rotating field at a given point of space—the common centre of suitably arranged coils—may be produced by supplying the coils with polyphase currents. We now proceed to the study of the waves of magnetic flux which are produced in alternating-current machines.

Numerous types of alternating-current machinery consist essentially of two coaxial iron cylinders separated from each other by a narrow gap—the air-gap—as shown in Fig. 31. One of the cylindrical cores—generally the outer—is stationary, and forms the *stator* of the machine; while the other is maintained in rotation, and forms the *rotor*. The surfaces of the cores may be either continuous, as in Fig. 31, or more or less discontinuous. For the sake of simplicity, we shall assume that each surface is continuous. By means of suitable windings, embedded in the cores, a system of magnetomotive forces is made to act across the gap, producing in it a multipolar field, so that the direction of the flux is alternately from stator to rotor and rotor to stator. The distribution of this flux is generally more or less irregular, and depends on the particular arrangement of the windings, the discontinuities in the polar surfaces of the cores, etc. Now, just as in considering alternating e.m.f.s and currents we made no attempt to deal with the numerous wave-shapes which occur in practice, but confined our attention to the simplest possible wave—the pure sine wave—so in the present instance, instead of dealing with the more or less irregular distribution of the magnetic flux which occurs in practice, we shall select for special treatment the simplest case of all—that, namely, in which the magnetic flux is distributed in

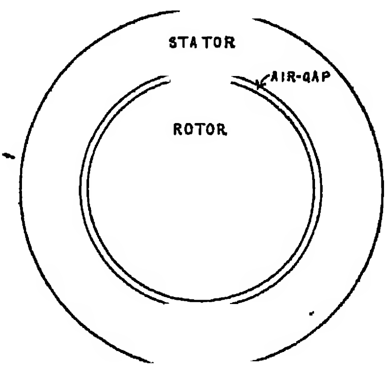


Fig. 31.—Type of Alternating-current Machine.

the air-gap according to the simple sine law. If the distance x be measured along the circumference of the rotor (Fig. 32), and if we select for our origin a point at which the magnetic field vanishes or

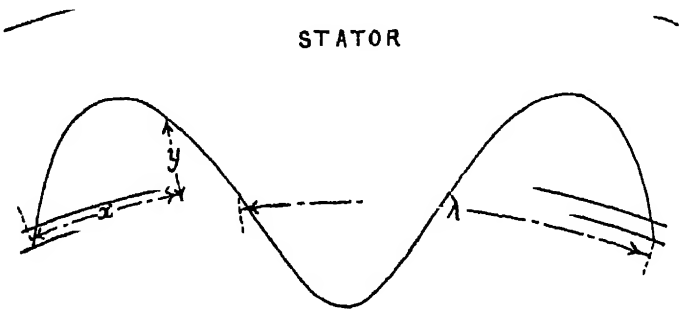


FIG. 32.—Simple Sine Wave of Flux.

changes sign, then according to the above supposition the value of the magnetic induction y at any point of the gap distant x from the origin may be represented by—

$$y = b \sin qx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The distribution of the flux may be graphically represented as in Fig. 32, where the values of y are laid off, to a convenient scale, along lines perpendicular to the rotor circumference.

We may speak of the distance λ which separates two corresponding points on the curve of induction, and within which are included all possible values of the induction, as the magnetic *wave-length*. The wave-length clearly corresponds to the distance between the middle points of two pole-pieces of the same name, or to twice the pole-pitch. Since the substitution of $x + \lambda$ for x leaves y unaltered as regards both magnitude and sign, we must have—

$$\sin qx = \sin (qx + q\lambda)$$

or—

$$q\lambda = 2\pi, \text{ i.e. } q = \frac{2\pi}{\lambda}$$

Equation (1) and the corresponding curve of Fig. 32 give the distribution of the flux at a given instant, with a given current circulating around the coils which produce the m.m.f.s.* Now, let us suppose that the current is alternating, so that the magnetic p.d. across the gap at any *given* point on the rotor circumference varies

* M.m.f. = magnetomotive force.

according to the simple harmonic law with *time*. Then b in equation (1) will no longer be a constant, but will be represented by—

$$b = B \sin \omega t$$

where $\omega = \frac{2\pi}{T}$, T being the period.

We now have—

$$y = B \sin \omega t . \sin qx$$

From this equation we see that while at every instant (*i.e.* for every value of t) the magnetic flux is distributed in space according to the sine law, and while the points of zero flux remain fixed, the

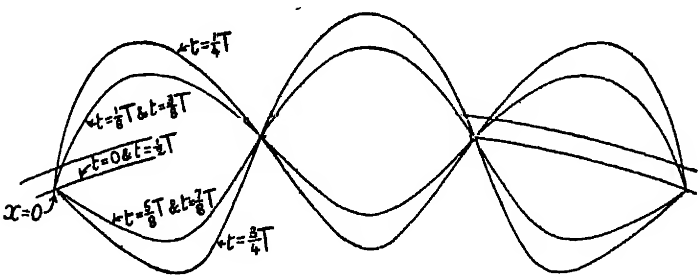


FIG. 33.—Oscillations of Simple Alternating Wave.

actual value of the flux at every point undergoes simple oscillations according to the sine law. This means that the ordinates of the curve in Fig. 32, oscillate about the base line. In Fig. 33 are indicated the successive positions of the curve corresponding to equal time intervals, each of which $= \frac{1}{8}$T. Thus our wave of magnetic flux is an oscillating wave whose zero points or *nodes* remain fixed; such a wave is frequently spoken of as a *stationary wave*. We may also speak of it as a *simple alternating wave* of magnetic flux.

§ 21. Analysis of Alternating into Two Rotating Waves, and vice versâ

The expression for y may be thrown into a slightly different form. We have—

$$\begin{aligned} y &= B \sin \omega t . \sin qx \\ &= \tfrac{1}{2}B . 2 \sin \omega t . \sin qx \\ &= \tfrac{1}{2}B\{\cos(\omega t - qx) - \cos(\omega t + qx)\} \\ &= \tfrac{1}{2}B \cos(\omega t - qx) - \tfrac{1}{2}B \cos(\omega t + qx) \end{aligned}$$

From this we see that y may be split up into—

$$y_1 = \tfrac{1}{2}\mathrm{B} \cos (\omega t - qx)$$

and—

$$y_2 = -\tfrac{1}{2}\mathrm{B} \cos (\omega t + qx)$$

Let us consider the meanings of y_1 and y_2. If we assign to t any definite value, then clearly both y_1 and y_2 will represent a magnetic flux distributed in the air-gap according to the simple sine (or cosine) law. Taking the particular instant $t = 0$, we find—

$$[y_1]_{t=0} = \tfrac{1}{2}\mathrm{B} \cos qx$$

and—

$$[y_2]_{t=0} = -\tfrac{1}{2}\mathrm{B} \cos qx$$

where the symbol $[y_1]_{t=0}$ denotes the value of y_1 at the instant $t = 0$. The values of $[y_1]_{t=0}$ and $[y_2]_{t=0}$ have been plotted in Fig. 34, and are shown by the full-line curves. When t has increased to $\tfrac{1}{8}$T, we find—

$$[y_1]_{t=\frac{1}{8}\mathrm{T}} = \tfrac{1}{2}\mathrm{B} \cos \left(\frac{\pi}{4} - qx\right)$$

and—

$$[y_2]_{t=\frac{1}{8}\mathrm{T}} = -\tfrac{1}{2}\mathrm{B} \cos \left(\frac{\pi}{4} + qx\right)$$

These new values of y_1 and y_2 are also shown in Fig. 34, the dotted curve representing $[y_1]_{t=\frac{1}{8}\mathrm{T}}$, and the chain-dotted curve $[y_2]_{t=\frac{1}{8}\mathrm{T}}$.

From this we see that during a time interval $\tfrac{1}{8}$T each curve has, without altering its size or shape, been displaced through a distance $= \tfrac{1}{8}\lambda$, and that the displacement for y_1 has been a *forward* one, and that for y_2 a *backward* one. Hence y_1 and y_2 represent two waves of magnetic flux *travelling* without change of size or shape in opposite directions around the rotor periphery. We speak of such a travelling or moving wave of magnetic flux as a rotating wave of flux or a *rotating magnetic field*, and the result just established shows that *a simple alternating or stationary wave is equivalent to, or may be replaced by, two rotating waves.*

The directions of rotation corresponding to y_1 and y_2 are indicated by the dotted and chain-dotted arrows respectively in Fig. 34. By adding the ordinates of $[y_1]_{t=\frac{1}{8}\mathrm{T}}$ and $[y_2]_{t=\frac{1}{8}\mathrm{T}}$ we, of course, obtain the ordinates of the curve corresponding to $[y]_{t=\frac{1}{8}\mathrm{T}}$ in Fig. 33.

It will be noticed that the amplitude of each of the two component rotating waves is equal to *half* the maximum amplitude of the resultant alternating or stationary wave.

Similarly, it may be shown that any rotating wave may be analyzed into two simple alternating waves whose amplitudes are the same as the amplitude of the rotating wave, and which are in

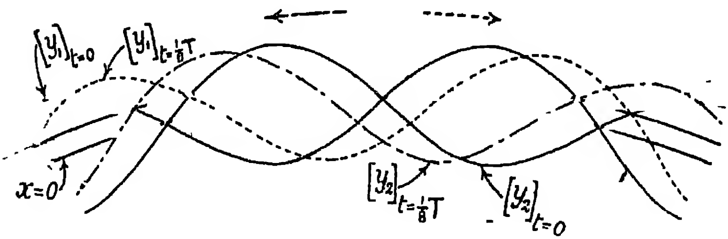


Fig. 34.—Rotating Waves of Flux.

quadrature with each other as regards both time and space. The proof of this proposition is extremely simple. For, taking the rotating wave—

$$u = \mathrm{U} \cos (\omega t - qx)$$

we may write this expression in the form—

$$u = \mathrm{U} \cos \omega t \cos qx + \mathrm{U} \sin \omega t \sin qx$$

But each of the terms on the right-hand side represents an alternating wave, and the maximum amplitude of each alternating wave is equal to the amplitude of the given rotating wave. Further, the waves are seen to be in quadrature as regards both time and space. Thus the proposition is established.

The substitution of two rotating waves for an alternating one, or *vice versâ*, is a device frequently employed in alternating current theory, as it in many cases leads to a considerable simplification in the treatment of certain classes of problems.

§ 22. Production of Rotating Waves of Magnetic Flux by means of Polyphase Currents

We shall now show that a rotating wave of magnetic flux may be produced by using suitably arranged windings supplied with polyphase currents.

Let us suppose that a two-phase system of mains is available. Imagine a winding embedded in the stator (or rotor) which, when traversed by a simple alternating current—obtained from one phase of the two-phase system of mains—generates a simple alternating wave of magnetic flux represented by—

$$y_1 = \mathrm{B} \sin \omega t \sin qx$$

where, as before, $\omega = \frac{2\pi}{\mathrm{T}}$ and $q = \frac{2\pi}{\lambda}$, T and λ denoting the period and the wave-length respectively of the wave of magnetic flux.

Let another winding, precisely similar to the first, but displaced relatively to it by $\frac{1}{4}\lambda$ (or half a pole-pitch), be arranged on the stator, and let a current be sent through it from the remaining phase of the two-phase system. This winding will generate a simple alternating magnetic flux wave precisely similar to that generated by the first winding, but displaced relatively to it as regards both time and space. For $\sin \omega t$ we must now, by reason of the phase difference of $\frac{\pi}{2}$ which exists between the two currents, write $\sin\left(\omega t + \frac{\pi}{2}\right)$, and—assuming that the origin from which x is measured remains unaltered, and that the second winding is displaced *backwards*, or in the negative direction of x, relatively to the first—for qx we must write $q\left(x + \frac{\lambda}{4}\right) = qx + \frac{\pi}{2}$. Hence, denoting by y_2 the value of the magnetic induction at any point x due to the current in the second winding, we have—

$$y_2 = \mathrm{B} \sin\left(\omega t + \frac{\pi}{2}\right) \sin\left(qx + \frac{\pi}{2}\right)$$

Each of these simple alternating waves may, as already explained, be replaced by two rotating waves, so that—

$$y_1 = \tfrac{1}{2}\mathrm{B}\{\cos(\omega t - qx) - \cos(\omega t + qx)\}$$

and—

$$\begin{aligned} y_2 &= \tfrac{1}{2}\mathrm{B}\{\cos(\omega t - qx) - \cos(\omega t + qx + \pi)\} \\ &= \tfrac{1}{2}\mathrm{B}\{\cos(\omega t - qx) + \cos(\omega t + qx)\} \end{aligned}$$

The resultant induction y at any point is given by—

$$y = y_1 + y_2 = \mathrm{B} \cos(\omega t - qx)$$

and we see that the result of the superposition of the two alternating waves is a pure rotating wave. Thus a two-phase system of currents may be made to give rise to a rotating wave of magnetic flux.

Similarly, a three-phase system of currents may be used for producing a rotating wave. Imagine three similar windings on the stator or rotor, displaced relatively to each other by amounts $\frac{1}{3}\lambda$, to be traversed by three-phase alternating currents. If y_1, y_2, and y_3 denote the three component alternating flux waves, we may write—

$$y_1 = \text{B} \sin \omega t \sin qx$$
$$y_2 = \text{B} \sin\left(\omega t + \frac{2\pi}{3}\right) \sin\left(qx + \frac{2\pi}{3}\right)$$

and
$$y_3 = \text{B} \sin\left(\omega t + \frac{4\pi}{3}\right) \sin\left(qx + \frac{4\pi}{3}\right)$$

or, using the simple transformation previously employed—

$$y_1 = \tfrac{1}{2}\text{B}\{\cos(\omega t - qx) - \cos(\omega t + qx)\}$$
$$y_2 = \tfrac{1}{2}\text{B}\left\{\cos(\omega t - qx) - \cos\left(\omega t + qx + \frac{4\pi}{3}\right)\right\}$$
$$y_3 = \tfrac{1}{2}\text{B}\left\{\cos(\omega t - qx) - \cos\left(\omega t + qx + \frac{8\pi}{3}\right)\right\}$$

Adding, we get for the resultant the rotating wave—

$$y = y_1 + y_2 + y_3 = \tfrac{3}{2}\text{B} \cos(\omega t - qx)$$

Special attention may be drawn to the following points. At the instant when any one of the component alternating waves reaches its maximum amplitude, it is coincident in position with the resultant rotating wave.*

Secondly, the amplitude of the rotating wave is, in the case of the two-phase system, equal to that of either component alternating wave, while in the case of the three-phase system the amplitude of the resultant is 1·5 times that of the component waves.

From the above it is at once evident that, while in the case of the two-phase system the e.m.f. induced in each phase by the resultant rotating wave is the same as that which would be induced by the alternating wave due to that phase if acting alone, the e.m.f. induced in each phase of the three-phase winding is increased 50 per cent. by the presence of the other two phases. This result may also be expressed by saying that there is no mutual inductance between the two circuits of a two-phase system, but that in a three-phase system the effect of mutual inductance is equivalent to a 50 per cent. increase in the self-inductance.†

* Take, *e.g.*, the component y_2. Its crest value is situated at the point defined by $qx + \frac{2\pi}{3} = \frac{\pi}{2}$, and this crest value rises to its maximum amplitude when $\omega t + \frac{2\pi}{3} = \frac{\pi}{2}$. But for these values of x and t we have $y = \frac{3}{2}\text{B}$, *i.e.* at the instant considered the crest of the resultant rotating wave becomes coincident with the crest of y_2. A similar result is easily established for y_1 and y_3.

† See Appendix I. for flux distributions corresponding to various types of windings, and Appendix II. for an account of the topographic method and the determination of

CHAPTER IV

§ 23 Theory of the wattmeter. Effect of self-inductance—§ 24 Effect of capacity—§ 25. Effect of eddy currents—§ 26. Effect of mutual inductance—§ 27. Correction for loss in wattmeter—§ 28. Power measurement in polyphase circuits—§ 29 Relations of line p.d.s and currents—§ 30. Case of balanced load. Tangent formula—§ 31. Power measurement by ammeters and voltmeters—§ 32. Three-point wattmeter method of measuring power factors and currents—§ 33. Measurement of energy.

§ 23. Theory of the Wattmeter. Effect of Self-inductance

One of the most important measurements in alternating-current circuits is the measurement of *power*. By power is here meant, as explained in § 7, the mean value of the power over a complete period. Although numerous methods, some very ingenious, have been devised for the measurement of power, it is nowadays generally admitted that by far the most satisfactory method is that involving the use of a *wattmeter*.

The oldest type of this instrument is the electrodynamometer wattmeter, which resembles in general construction the well-known Siemens electrodynamometer for the measurement of currents. It consists of two coils, a fixed and a movable one, arranged with their planes at right angles to each other, the movable coil being provided with a torsion head, by means of which it can always be restored to its original position when deflected by an electrodynamic couple. The fixed coil is placed in the main circuit, while the movable coil is connected in series with a high non-inductive resistance, and is joined across the two points between which power is being measured.

The connections are indicated in Fig. 35, where F denotes the fixed or main or "current" coil of the instrument, M the movable or shunt or "pressure" coil, r_1 the high non-inductive resistance in series with the pressure coil, and A the portion of the circuit in which power is being measured. The current coil represents the ammeter part of the instrument, and the pressure coil the voltmeter part. The dotted line indicates an alternative mode of connection for the pressure coil.

Consider in the first place the *ideal* wattmeter, in which the self-inductance of the pressure or fine-wire circuit is negligible in comparison with the resistance, and in which the fine-wire circuit is also utterly devoid of capacity. The current in this circuit will then be in phase with, and proportional to, the p.d. Hence the instantaneous couple acting on the instrument will be proportional to the product p.d. × current, *i.e.* to the power, and the mean couple to the mean power. The angle of torsion will therefore be proportional to the mean power.

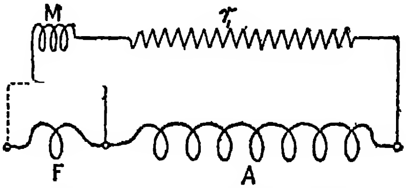


FIG. 35.—Connections of Wattmeter.

In the actual instrument, the self-inductance and capacity of the fine-wire circuit are made as small as possible by using a coil of the smallest possible number of turns, and winding the wire which forms the non-inductive resistance in sections on thin sheets of mica, the various sections being then suitably mounted and connected in series with each other.

It is more difficult in practice to reduce the self-inductance of the fine-wire circuit to a negligible amount than its capacity, and it is important to investigate what effect the presence of a small amount of self-inductance may have on the reading of the instrument.

Let R, L denote the resistance and self-inductance respectively of the load, and let r, l stand for the corresponding quantities in the fine-wire circuit of the wattmeter. If V = p.d., I = current in main circuit, then, denoting $\tan^{-1}\dfrac{L\omega}{R}$ by ϕ, and $\tan^{-1}\dfrac{l\omega}{r}$ by θ, we have—

$$\text{true power } w = \text{VI} \cos \phi \ (\S\ 7)$$

and if l were negligible the wattmeter reading would correspond to VI cos ϕ. The effect of l is a twofold one: it reduces the current in the fine-wire circuit in the ratio $\dfrac{r}{\sqrt{r^2 + \omega^2 l^2}}$, and it further causes the current to lag behind the p.d. by an angle θ. The first effect reduces the couple in the given ratio, while the second is obviously equivalent to a change in the power factor of the load from cos ϕ to cos $(\phi - \theta)$. Thus the reading of the wattmeter will be—

$$w' = \frac{\text{VI}r}{\sqrt{r^2 + \omega^2 l^2}} \cdot \cos (\phi - \theta) = \text{VI} \cos \theta \cos (\phi - \theta)$$

We thus have, dividing the former equation by the latter—

$$w = w' \frac{\cos \phi}{\cos \theta . \cos (\phi - \theta)}$$

If, therefore, θ is not negligible, then the only case in which the wattmeter reading will be correct is that corresponding to $\phi = \theta$. For values of $\phi < \theta$ the reading of the wattmeter will be too low, and for values of $\phi > \theta$ too high.

The multiplier $K = \frac{\cos \phi}{\cos \theta . \cos (\phi - \theta)}$, which converts the wattmeter reading into true power, is spoken of as the *correcting factor*. It may be thrown into various forms. Thus, since—

$$\begin{aligned}\cos \theta . \cos (\phi - \theta) &= \cos \theta(\cos \phi \cos \theta + \sin \phi \sin \theta) \\ &= \cos^2 \theta \cos \phi(1 + \tan \phi \tan \theta)\end{aligned}$$

we have—

$$K = \frac{\sec^2 \theta}{1 + \tan \phi \tan \theta} = \frac{1 + \tan^2 \theta}{1 + \tan \phi \tan \theta}$$

Now, since θ is always very small, we may write approximately—

$$K = \frac{1}{1 + \tan \phi \tan \theta} = 1 - \tan \phi \tan \theta$$

a form which shows very clearly the *rapid increase in the error with decreasing power factor* $\cos \phi$.

Another simple approximate expression for the true power, which may be used when the power factor is very low, has been suggested by Dr. Drysdale.* Using the last form of approximate value for K, we have, since the wattmeter reading is roughly correct, *i.e.* since roughly $w' = VI \cos \phi$, and also $\sin \phi$ = nearly 1 for small values of $\cos \phi$—

$$w = w' - w' \tan \phi \tan \theta = w' - VI \times \tan \theta$$

The above results may be exhibited very clearly by the aid of the simple vector diagram of Fig. 36, in which I is the vector of current in the main circuit, and V the p.d. vector. If there were no self-inductance in the fine-wire circuit, the current in this circuit would be proportional to V, and the reading to $IV \cos \phi = IV'$. By reason of self-inductance, the effective or resultant e.m.f. which maintains the current through the resistance of the fine-wire circuit is not V, but V_r, and hence the reading is proportional to IV'_r. The error is thus $I(V'_r - V')$. It is at once evident that: (1) the instrument reading is too low if $\phi < \theta$; (2) the instrument reading is correct if

* *The Electrician*, vol. xlvi. p. 774 (1901).

$\phi = \theta$; and (3) the instrument reading is too high if $\phi > \theta$, and the error increases with increase of ϕ.

It may be pointed out that it is easier to render the error arising from the self-inductance of the fine-wire coil negligible in wattmeters intended to be used on a high than in those for use on a low voltage circuit, since in the former case the non-inductive resistance may be made relatively higher, and so tan θ may be reduced.

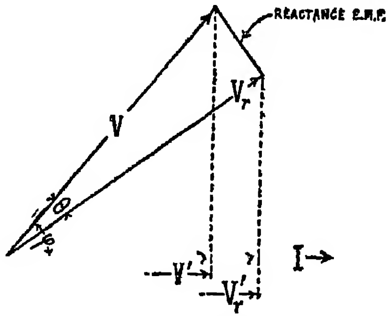
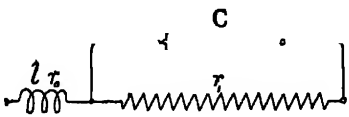


FIG. 36.—Effect of Reactance in Shunt Circuit of Wattmeter.

The main value of the formulæ obtained above lies, not so much, perhaps, in the fact that they enable us to apply a correction, as in showing under what conditions the correction may be safely neglected.

§ 24. Effect of Capacity

We may now consider the effect of capacity in the non-inductive resistance. Imagine this resistance not subdivided—as is generally the case—but wound double in a single coil. Suppose next, for a moment, that the middle point of the resistance, where the wire is doubled back on itself, is cut. Then clearly the two halves of the winding constitute a condenser, and if an alternating p.d. is maintained between them, a definite capacity current or condenser current will flow through the dielectric between them. Each half of the winding will at every instant be at a practically uniform potential. Let the cut ends be now joined. We then have, in addition to a capacit conduction current, and a c drop of potential along the wire.



Now, this drop of potential will obviously reduce the capacity current to *half* the value which it had when the resistance was cut, since the *mean* p.d. between the two halves of the resistance is now only half the p d. across its ends. The *equivalent* capacity is thus half the true capacity, and the effect due to it is clearly the same as if the resistance were devoid of capacity, but were shunted by a condenser of capacity C equal to half the true capacity of the resistance, as shown by the diagram of Fig. 37.

Applying to the branched portion of the circuit of Fig. 37 the rules established in § 9 for parallel arrangements of impedances,* we find for the joint resistance of the branched portion—

$$\frac{r_1}{1 + r_1^2C^2\omega^2}$$

and for the joint reactance—

$$-\frac{C\omega r_1^2}{1 + r_1^2C^2\omega^2}$$

From this we find the total resistance of the fine-wire circuit to be—

$$r_0 + \frac{r_1}{1 + r_1^2C^2\omega^2}$$

and its total reactance—

$$l\omega - \frac{C\omega r_1^2}{1 + r_1^2C^2\omega^2}$$

The current in the fine-wire circuit therefore lags behind the p.d. by an angle—

$$\tan^{-1}\frac{l\omega - \dfrac{C\omega r_1^2}{1 + r_1^2C^2\omega^2}}{r_0 + \dfrac{r_1}{1 + r_1^2C^2\omega^2}}$$

or, approximately—

$$\tan^{-1}\frac{l - r_1^2C}{r}\omega$$

where $r = r_0 + r_1$.

From this we see that the capacity of the non-inductive resistance, if not excessive, is beneficial, as it tends to neutralize the effect of the self-inductance of the fine-wire coil.

The formulae established in § 23 in connection with the effect of self-inductance alone are immediately available for the combined effect of self-inductance and capacity, provided that for θ we substitute $\tan^{-1}\frac{l - r_1^2C}{r}\omega$.

The capacity effect is greatly reduced by subdividing the non-inductive resistance into a number of sections. Let there be n sections, each wound non-inductively as before. If we imagine each section bisected, we obtain n condensers in series, each of capacity $\frac{1}{n}C$, where C is the capacity of the resistance when wound in a single

* See Note I. at end of chapter.

section. But since the joint capacity of n equal condensers arranged in series with each other is $\frac{1}{n}$th that of one of them, we see that by subdividing the resistance we have reduced the capacity to $\frac{1}{n^2}$ of its original amount. The subdivision of the resistance therefore affords a simple means of reducing the capacity to any desired extent.

In modern practice the various sections of the non-inductive resistance are not wound bifilarly (*i.e.* by doubling the wire on itself), but in the ordinary way on thin sheets of mica. This still further reduces the capacity (though very slightly increasing the self-inductance) and minimizes the risk of breakdown of the insulation.

§ 25. Effect of Eddy Currents

In most of the earlier wattmeters there was a source of error whose importance was not realized, and whose very existence, in fact, remained for a long time unknown. It is largely due to this source of error that a strong prejudice was established against the wattmeter, a prejudice which was only gradually overcome as the construction of the instrument underwent improvement. The error referred to is that due to *eddy currents* induced in any solid masses of metal used in the construction of the instrument—such as a metal case, metal supports for coils, or even a heavy winding for the main coil. The error due to eddy currents may be regarded as a twofold one. In the first place, the field due to the main coil is distorted by the action of the eddy currents, and so the torque acting on the movable coil is altered; and secondly, there will be an additional torque (which may correspond to either an increase or a decrease of torque, according to circumstances) acting on the movable coil, due to the presence in its neighbourhood of large conducting masses.

In order to avoid this source of error, the case of the dynamometer type of wattmeter is invariably constructed of some insulating material, such as ebonite, and the same material is also employed for the supports of the coils, etc. For large currents, the current coil should be carefully stranded.*

§ 26. Effect of Mutual Inductance

The errors arising from self-inductance, capacity, and eddy currents are the only ones present in the zero type of dynamometer wattmeter, in which the movable coil always occupies the same position when a reading is taken. But since this type of instrument requires a preliminary adjustment, it is not very convenient to use,

* Another solution of the difficulty is to use a suitably designed current transformer, so that the current coil may be wound with thin wire.

and is not much used commercially, most modern forms being deflectional or indicating instruments, in which the fine-wire coil is allowed to move, a suitable spring furnishing the controlling couple. Now, as soon as the fine-wire coil is displaced from its position at right angles to the main coil, there is, on account of the mutual inductance between the coils, an e.m.f. induced in it, and this e.m.f. is a variable one, depending on the relative positions of the two coils. Thus a further possible source of error arises in connection with indicating instruments.

In order to investigate this error, we may conveniently use a diagram similar to that of Fig. 36. With no mutual inductance between the coils, the reading would be proportional to V′ in Fig. 38. Owing to mutual inductance, the e.m.f. V_m due to it must be compounded with V, and the resultant treated as if it represented the p.d. impressed on the fine-wire circuit. It is thus evident that the reading will be proportional to V_m'. Now, in the perfect instrument the reading would be proportional to V′, so that the error when mutual inductance is taken into account is proportional to $V_m' - V'$. With no mutual inductance present, the error would be proportional to $V_r' - V'$, *i.e.* in the case shown in the diagram it would be greater. Thus mutual inductance, in the case shown, has the effect of *reducing* the error due to self-inductance.* It is easy to see that for negative values of ϕ, *i.e.* for leading currents, the error due to self-inductance would be *increased* by mutual inductance.

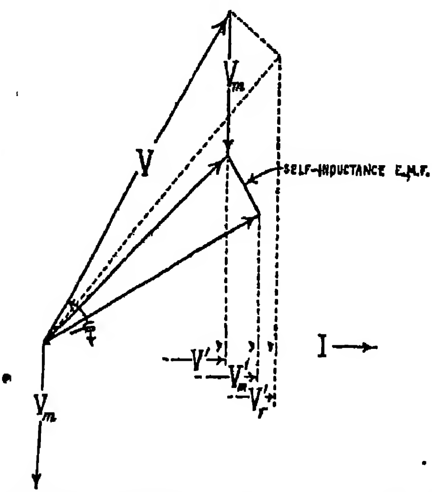


FIG. 38.—To illustrate Effect of Mutual Inductance.

If, however, the self-inductance and capacity effects are quite negligible ($V' = V_m' = V_r'$), then it is evident from the diagram that mutual inductance does not introduce any error.

We may summarize the above results by saying that mutual inductance tends to *correct* the errors of a wattmeter so long as the current is a *lagging* one, and to *exaggerate* the errors if the current is a *leading* one.

* Or capacity, or self-inductance and capacity combined.

§ 27. Correction for Loss in Wattmeter

Returning to the diagram of Fig. 35, which shows the connections of the wattmeter circuits, we notice that when the fine-wire or shunt circuit is arranged as shown by the full line, the power measured includes that wasted in the shunt circuit of the wattmeter, whereas with the dotted line connection the power measured includes that wasted in the current coil of the wattmeter. A correction is easily made in either case by calculating the power wasted from the known resistances of the circuits and the currents passing through them. The error may also be compensated for by providing an auxiliary fine-wire winding on the main coil, having a number of turns equal to that on the main coil, and arranged so as to oppose the effect of the main winding; the auxiliary winding being included in the fine-wire circuit. This arrangement suffers, however, from the disadvantage of increasing the self-inductance of the shunt-circuit.

§ 28. Power Measurement in Polyphase Circuits

The measurement of power in a single-phase circuit is readily effected, as explained in § 23, by the use of a wattmeter. A similar method, involving the use of *two* wattmeters, is applicable to a two-phase circuit, one wattmeter being included in each phase. If the phases are coupled, the wattmeter connections remain unaltered, and are as shown in Fig. 39, where C_1 and C_2 denote the current coils of the wattmeters, p_1 and p_2 their pressure coils, and r_1 and r_2 the high non-inductive resistances connected in series with the pressure coils. The sum of the two wattmeter readings gives the total power in the two-phase system. If only a single wattmeter is available, it may be used to obtain the two readings in succession; this, however, is not so satisfactory as a simultaneous reading of two wattmeters.

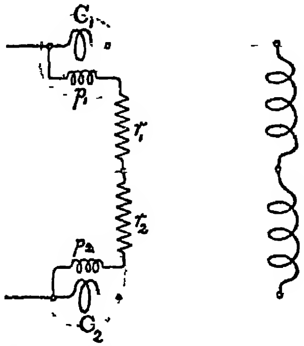


FIG. 39.—Power Measurement in Two-phase Circuit.

The measurement of power in a three-phase circuit is a much more complicated matter, and it is only in certain special cases that it approaches the simplicity of single-phase power measurement. We shall first consider the most general method of

finding the power in a three-phase circuit, a method which applies to every possible case, and which only involves the use of *two* wattmeters. For this reason it is known as the *two-wattmeter method.*

A large number of methods of measurement and calculation in connection with three-phase systems is based on various more or less arbitrary and frequently unjustifiable assumptions, such as sine waves of p.d. and current, perfect balance of the system, accessibility of the neutral point, or phase differences of exactly 120° between the three p.d.s or the three currents. Now, when the load is not a balanced one, not only are the three p.d.s in general unequal, but their phase differences are also unequal, and different from 120°.

The two-wattmeter method of power measurement is entirely free from any such arbitrary assumptions, as will be evident from a study of its theory. In Fig. 40, A, B, and C are the three line wires conveying power to the three terminals A′, B′, C′, to which the load is connected. The load may be connected either delta- or star-fashion. In either case, we may always arrange a star-connected load, as shown in Fig. 40, having a neutral point O, which is equivalent in every respect to the actual load. Let the instantaneous values of the currents along the three wires (reckoned positive when flowing from left to right) be i_1, i_2, and i_3, and let v_1, v_2, and v_3 be the instantaneous p.d.s between the neutral point O and the line wires (reckoned positive if directed towards O). The wattmeter connections are arranged as shown.

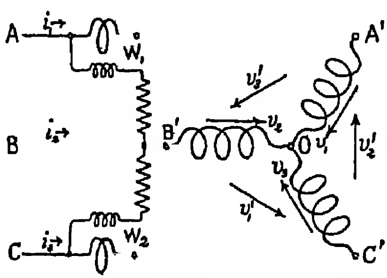


FIG. 40.—Power Measurement in Three-phase Circuit.

It is to be noted that the series coils of the wattmeters may be placed in *any* two of the line wires, provided the free ends of their shunt coils are in connection with the remaining line wire. If w = total instantaneous power, we evidently have—

$$w = v_1 i_1 + v_2 i_2 + v_3 i_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now, i_1 and i_3 are the currents traversing the series coils of the wattmeters, while i_2 does not flow through any wattmeter coil. It will, therefore, be convenient to eliminate i_2 from the expression for the instantaneous power. This is easily done by means of the equation—

$$i_1 + i_2 + i_3 = 0$$

which must obviously be true in every possible case. Putting $i_2 = -i_1 - i_3$ in (1), and rearranging the terms, we find—

$$w = (v_1 - v_2)\, i_1 + (v_3 - v_2)\, i_3 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

But since $v_1 - v_2$ is the instantaneous p.d. across the shunt circuit of the wattmeter W_1,* and $v_3 - v_2$ the instantaneous p.d. across the shunt circuit of W_2, it follows that the mean total power over a period (*i.e.* the mean value of w) is equal to the *algebraical sum of the wattmeter readings.*

§ 29. Relations of Line P.D.s and Currents

On account of the importance of the two-wattmeter method, we shall investigate the phase relations of the line p.d.s and line currents somewhat more closely in the general case of an unbalanced inductive load. In Fig. 41 is given a vector diagram of the line currents, the star p.d.s, and the line p.d.s.† The three vectors I_1, I_2, and I_3 represent the line currents. These lag behind their respective star p.d.s. V_1, V_2, and V_3 by the angles θ_1, θ_2, and θ_3, and for the sake of generality we have supposed these angles to be all different. The line p.d.s, whose instantaneous values are denoted by v_1', v_2', and v_3', in Fig. 40, and which are reckoned positive when they have a counter-clockwise direction around the mesh or triangle A′B′C′ in Fig. 40, are easily obtained by noticing that $v_1' = v_2 - v_3$, $v_2' = v_3 - v_1$, and $v_3' = v_1 - v_2$. Hence in the vector diagram of Fig. 41 these line p.d.s are represented by the sides of the triangle formed by joining the extremities of V_1, V_2, and V_3; their directions are indicated by the arrow-heads.

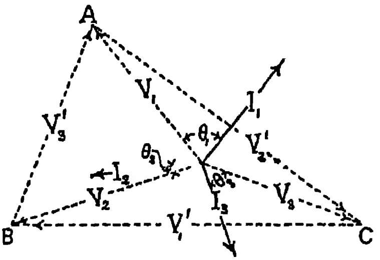


FIG. 41.—Vector Diagram of Three-phase Circuit.

For the sake of clearness in the diagram, it will be convenient to draw the triangle of line p.d.s as in Fig. 42, and from the vertices of this the current vectors I_1, I_2, and I_3.

We next notice that the connections of the two wattmeters W_1 and W_2 in Fig. 40 were supposed to be so arranged that a positive couple was obtained when the current and p.d. at any instant were both acting away from (or towards) the junction of the current and pressure coils. Now, from Fig. 40 it will be seen that the positive direction of v_3' is the same as that of v_1, and since the reading of W_1 is equal to the mean value of $+ v_3'i_1$, it is represented, in Fig. 42, by $V_3'I_1 \cos \phi_1 = V_3' \times AD$. On the other hand, considering the wattmeter W_2, we notice that, since in Fig. 40 the positive direction of

* Cf. § 17.

† This diagram should be compared with Fig. 40.

v_1' is opposed to the positive direction of v_3, the wattmeter reading will represent the mean value of $-v_1'i_3$, and hence will, in Fig. 42, correspond to $-V_1'I_3 \cos \phi_3 = V_1'I_3' \cos \phi_3' = V_1' \times CE$.

Now, it is evident that under certain conditions of load CE may be of opposite sign to CB or V_1', *i.e.* the reading of W_2 may become

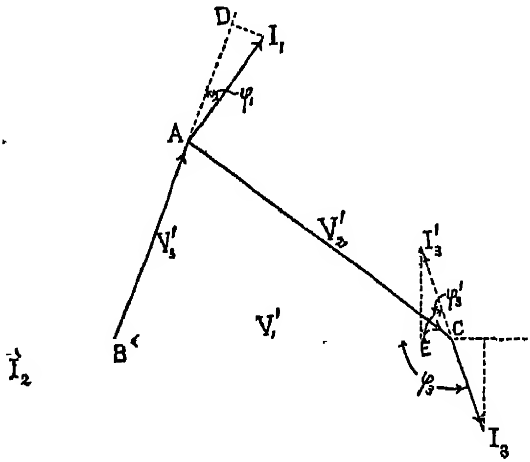


FIG. 42.—Vector Diagram of Three-phase Circuit.

negative. If the wattmeter is only capable of reading to one side of zero, this will necessitate a reversal of its shunt-coil connections, and the total power will then be given by the *arithmetical difference* of the readings of the two wattmeters. This case, although possible, is not of frequent occurrence, and, as may be seen by an examination of the vector diagrams, requires a very large angle of lag of I_3 behind V_3.

We have already (§ 7) defined the *power factor* of a single-phase circuit as the ratio of the true power to the volt-amperes, and the question now arises as to what meaning is to be attached to this term in the case of an unbalanced inductive three-phase load. When, as in Fig. 41, the currents all lag by different amounts behind their respective p.d.s, it is clear that the term "power factor" can have no definite physical meaning.

Having considered the most general case of power measurement in three-phase circuits, we may next examine certain special cases. The most interesting of these, from a practical point of view, are (1) that of a balanced circuit, and (2) that of an unbalanced non-inductive load. The first may be closely approached by either a symmetrical lamp load or a load of induction motors, and the second frequently arises in connection with a lamp load.

§ 30. Case of Balanced Load. Tangent Formula

When the circuit is balanced, the line p.d.s and line currents will, in the diagram of Fig. 41, be represented by lines of equal length. Hence ABC will form an equilateral triangle. Further, the vectors V_1, V_2, and V_3, representing the star p.d.s, will radiate towards the vertices of the triangle from its middle point, making angles of 30° with its sides, *i.e.* with the line p.d. vectors. The star of currents will consist of three equal arms, making angles of 120° with

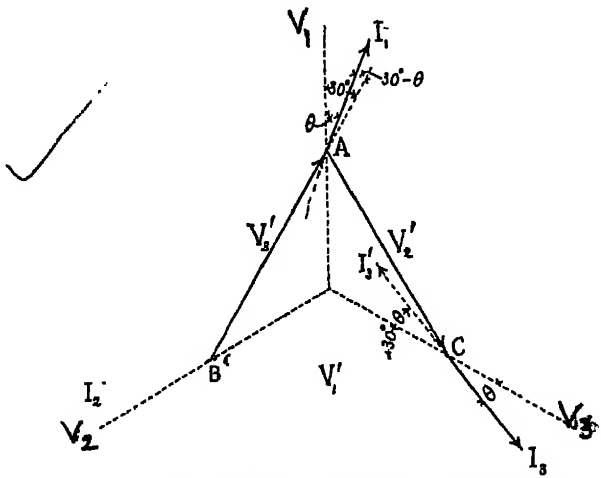


FIG. 43.—Vector Diagram of Balanced Three-phase Circuit.

each other, and displaced relatively to the p.d. star through an angle θ. The cosine of this angle, which by supposition is the same for each phase, is the *power factor of the balanced three-phase load.* The diagram of Fig. 42 now takes the shape shown in Fig. 43, and it will be seen that the reading of the wattmeter W_1 in Fig. 40 is $V_3'I_1 \cos (30° - \theta)$, while that of W_2 is $V_1'I_3' \cos (30° + \theta)$. Since numerically $V_3' = V_1' = V'$ say, and $I_1 = I_3 = I$ say, we have, using W_1 and W_2 to denote the two wattmeter readings—

$$W_1 = V'I \cos (30° - \theta)$$
$$W_2 = V'I \cos (30° + \theta)$$

Hence—

$$W_1 + W_2 = 2V'I \cos 30° \cos \theta$$
$$W_1 - W_2 = 2V'I \sin 30° \sin \theta$$

Dividing the second of the last two equations by the first, we find—

$$\tan 30^\circ \tan \theta = \frac{W_1 - W_2}{W_1 + W_2}$$

or—

$$\tan \theta = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}\ *$$

By using this formula, we are enabled to calculate the power factor $\cos \theta$ of the balanced three-phase system from the readings of the two wattmeters. The formula is frequently referred to by continental writers as the "tangent formula." It must be carefully noted, however, that the tangent formula is based on the assumption of *sine waves* of p.d. and current,† and cannot be relied upon when the wave-forms are greatly distorted.

If the power factor of a balanced three-phase system is unity, the load being entirely non-inductive, and $\theta = 0$, then it is evident, either from the vector diagram of Fig. 43, or from the tangent formula just established, that $W_1 = W_2$, *i.e.* the readings of the two wattmeters are equal. This is, in fact, the only case in which the two instruments give identical readings.

As the angle of lag θ increases, it is evident from Fig. 43 that, the currents being assumed to retain their original values, the reading of W_1 will increase, and that of W_2 decrease, until θ becomes equal to 30°. At this stage, the reading of W_1 reaches a maximum value for a given value of the line currents. Any further increase of θ will cause both readings to decrease (it is evident from the diagram that the reading of W_2 will decrease more rapidly than that of W_1). When θ reaches 60°, we have $30^\circ + 60^\circ = 90^\circ$, and the current I_3 is in quadrature with V_1', hence W_2 will give a zero reading. A still further increase in θ will cause the reading of W_2 to become *negative*.‡

A very special case of a balanced three-phase load is that in which the load is star-connected, with an accessible neutral point (a star connection is very common, but the neutral point is not generally accessible). A *single* wattmeter may then be used to measure the power. The current coil of the wattmeter is introduced into one of

* This may also be written in the form—

$$\tan \theta = \sqrt{3}\,\frac{\frac{W_1}{W_2} - 1}{\frac{W_1}{W_2} + 1}$$

† It may be shown that, with distorted wave-forms, the star p.d.s of a three-phase system cannot, in general, be represented by three co-planar vectors. Hence the ordinary graphical method fails (see, in this connection, a paper by E. Orlich in *Elektrotechnische Zeitschrift*, vol. xxiv. p. 59).

the line wires, while its pressure coil is connected between this line wire and the neutral point. The wattmeter will then evidently measure the power in one phase, and since the system is balanced, the total power is given by three times the reading of the instrument.

In this connection it may be mentioned that, with an accessible neutral point, the total power of an *unbalanced* system may be measured by means of three wattmeters, connected as just explained, each wattmeter measuring the power in one phase. The total power is the sum of the three wattmeter readings. This method, whose applicability is limited by the accessibility of the neutral point, is known as the *three-wattmeter method.**

§ 31. Power Measurement by Ammeters and Voltmeters

The case of an unbalanced non-inductive load is of considerable interest, not only on account of its frequent occurrence in practice, but also because in this case the power may be found from the readings of three voltmeters and three ammeters, connected so as to read the three line p.d.s and three line currents, without the use of any wattmeter.

A reference to the vector diagram of Fig. 41 shows that the directions of the current vectors must in this case coincide with those of the star p.d. vectors with which they are co-phasal. From this it is evident that, if the current vector diagram (showing the magnitudes and phase relations of the line currents) be given in the form of three vectors radiating from a point, it must be possible to so fit this diagram into the triangle ABC, Fig. 41, of the line p.d. vectors as to cause the current vectors, or those vectors produced, to pass through the angular points of the triangle.

Assuming, therefore, that the three line voltages and three line currents have been measured, we may proceed to find the power as follows. Construct the triangle ABC, Fig. 44 (*a*), of the line voltages, and the triangle of line currents, Fig. 44 (*b*). From this latter obtain the star of currents, Fig. 44 (*c*). Produce the rays of the star, if necessary, and, having made a tracing of it, fit it over the triangle of voltages. By means of a needle-point, mark the position of the centre O of the star inside the triangle. Remove the tracing, and from each of the vertices, A, B, and C of the triangle draw lines radiating from O; along these lines lay off lengths representing the line currents. Then the power is given by—

$$V_3' \times I_1 \cos\phi_1 + V_1' I_3 \cos\phi_2 = AB \times AD + CB \times CE$$

* See Note II at end of chapter.

since this would represent the sum of the two wattmeter readings if these instruments were employed. There is, however, another way of expressing the power. A reference to Fig. 41 shows that OA, OB,

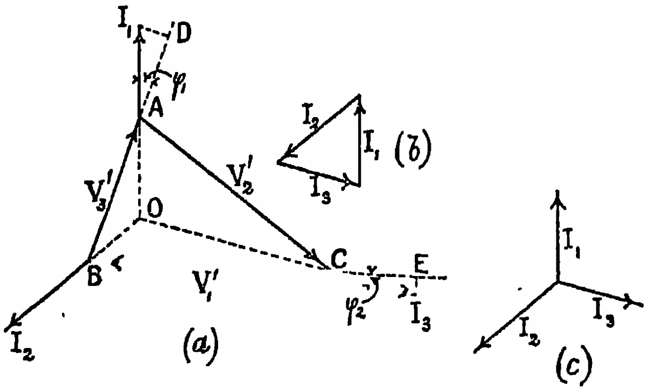


FIG. 44.—Vector Diagram of Unbalanced Non-inductive Three-phase Load.

and OC give the star voltages of the equivalent star-connected load. Hence, the load being non-inductive, and the currents in phase with the star voltages, we have for the total power—

$$OA \times I_1 + OB \times I_2 + OC \times I_3$$

§ 32. Three-point Wattmeter Method of Measuring Power Factors and Currents.

An extremely accurate method of determining the power factor of a given load is that known as the *three-point wattmeter* method. This term has been suggested for the method by Messrs. D. K. Morris and G. A. Lister.* As regards the origin of the method, there seems to be some doubt as to who was the first to suggest it †; but the most complete exposition of it hitherto published occurs in the paper by Messrs. Morris and Lister referred to above.

The main feature of the method is the use of a polyphase system of mains for determining the power factor of a single-phase load. Either two- or three-phase mains may be used for this purpose.

If a two-phase system is to be used, then, supposing A and B to denote the supply conductors of the two phases, the load whose power factor is required is connected in series with the current coil

* *Journal of the Institution of Electrical Engineers*, vol. xxxvii. p. 264 (1906).

† See correspondence on "Phase Measurement" in *The Electrician*, vol. lvi. pp. 896, 933, 977, and 1020 (1906).

of a wattmeter, and then across the conductors of, say, phase A. The shunt circuit of the wattmeter is first connected across phase A, and a reading w_1 is taken. If V_A = p.d. across phase A, I = current taken by load, and ϕ = angle of lag of current behind p.d., then (§ 7)

$$w_1 = V_A I \cos\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The shunt circuit of the wattmeter is next transferred to phase B; let us suppose the connections to be made so that, considered with reference to the current taken by the load, the p.d. across phase B leads with respect to that across phase A (the lead may be converted into a lag by reversing the connections of the pressure circuit). If V_B = p.d. across phase B, and w_2 = new reading of wattmeter—

$$w_2 = V_B I \cos\left(\phi + \frac{\pi}{2}\right)$$
$$= V_B I \sin\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Dividing (2) by (1), we obtain—

$$\tan\phi = \frac{V_A}{V_B} \cdot \frac{w_2}{w_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If $V_A = V_B$, this reduces to the simple form $\tan\phi = \dfrac{w_2}{w_1}$.

Knowing ϕ, V_A, and V_B, we can find I by means of either (1) or (2). Thus a wattmeter and a voltmeter are sufficient to enable us to find both the power factor and the current.

In connection with the determination of ϕ by (3), it will be noticed that since the ratio merely of two readings is involved, any error in the instruments will not affect the final result, provided the scale divisions are relatively correct—*i.e.* provided the percentage error is the same throughout the scale.

When three-phase mains are used, the load is connected in series with the current coil of a wattmeter and across any pair of mains, such as B, C in Fig. 45. The three line p.d.s across BC, CA, and AB respectively are measured by means of a voltmeter, and from these the triangle of voltages shown in Fig. 46 is constructed. The angles θ_2 and θ_3 thus become known. The phase relations of the voltages relatively to the current I are shown in Fig. 47. Let the wattmeter pressure circuit terminals be denoted by 1 and 2, and let the movable coil connections be such that when 1 is connected to B, and 2 to C, in Fig. 45, the wattmeter gives a positive reading w_1, which is given by—

$$w_1 = V_1 I \cos\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$\cos\phi$ being the required power factor of the load.

Let next the pressure circuit be transferred to the mains C, A. It is to be carefully noted that if 1 were placed on C, and 2 on A,

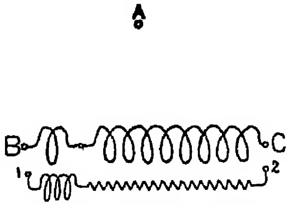


FIG. 45.—Three-point wattmeter method.

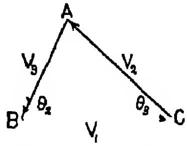


FIG. 46.—Triangle of voltages.

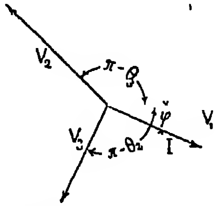


FIG. 47.—Phase relations of voltages and current.

the phase relation of the wattmeter shunt current and I would be the same as that of V_2 and I, as shown in Fig. 47. It is evident from Fig. 47 that the wattmeter reading would then be negative. In order to render it positive, we reverse the pressure circuit terminals—*i.e.* we leave 2 on C, and transfer 1 from B to A. The effect on the phase relation is the same as that obtained by reversing V_2 in Fig. 47, and it is evident that now the phase difference between the currents in the wattmeter circuits is $\theta_3 - \phi$, so that the new wattmeter reading w_2 is given by—

$$\begin{aligned} w_2 &= V_2 I \cos(\theta_3 - \phi) \\ &= V_2 I (\cos\theta_3 \cos\phi + \sin\theta_3 \sin\phi) \quad . \quad . \quad . \quad . \quad (5) \end{aligned}$$

Similarly, if we place 1 on B, and 2 on A, we obtain a reading * w_3, given by—

$$\begin{aligned} w_3 &= V_3 I \cos(\theta_2 + \phi) \\ &= V_3 I (\cos\theta_2 \cos\phi + \sin\theta_2 \sin\phi) \quad . \quad . \quad . \quad . \quad (6) \end{aligned}$$

Using the value of I given by (4) in (5) and (6), we find—

$$\tan\phi = \frac{V_1}{V_2} \cdot \frac{w_2}{w_1 \sin\theta_3} - \cot\theta_3 \quad . \quad . \quad . \quad . \quad (7)$$

and

$$\tan\phi = \cot\theta_2 - \frac{V_2}{V_3} \cdot \frac{w_3}{w_1 \sin\theta_2} \quad . \quad . \quad . \quad . \quad (8)$$

Either of these equations may be used for determining ϕ. Equation (4) then gives the current I.

* This may be either positive or negative, as is evident from Fig. 47. If the wattmeter reads in one direction only, and its reading is negative when 1 is on B and 2 on A, the connections must be reversed, 1 being put on A and 2 on B, but the wattmeter reading must, in that case, be used with the *negative* sign in the formula.

The above method is, of course, immediately applicable to the determination of the power factor of a balanced three-phase load. In this case, $V_1 = V_2 = V_3$, and $\theta_2 = \theta_3 = 60°$, so that $\sin \theta_3 = \frac{\sqrt{3}}{2}$, and $\cot \theta_2 = \cot \theta_3 = \frac{1}{\sqrt{3}}$. The formulæ (7) and (8) now become—

$$\tan \phi = \frac{2w_2 - w_1}{w_1\sqrt{3}}$$
$$= \frac{w_1 - 2w_3}{w_1\sqrt{3}}$$

§ 33. Measurement of Energy

For the measurement of the *energy* supplied to a three-phase circuit, two single-phase energy meters of suitable construction — such as the Aron clock meter or the Elihu Thomson meter—may be employed, the total energy being given by the sum of the readings of the two instruments. The connections of the current and pressure coils of these instruments are precisely similar to those used in the two-wattmeter method of power measurement shown in Fig. 40. For a balanced three-phase circuit, a single instrument will suffice if, by the creation of an artificial neutral point, this point is made accessible; the instrument then has its current coil in series with one of the mains, and its pressure coil forms one of the arms of an artificial star (all the arms being of equal resistance), the connections being as shown in Fig. 48.* Again, two meters connected as in the two-wattmeter method of power measurement may be combined into a single instrument, the rotating parts being mounted on a common axis; this has the advantage of compactness as compared with two ordinary single-phase meters.

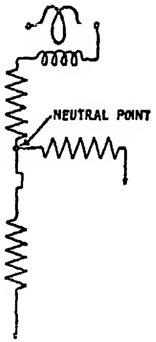


FIG. 48.—Connections of Energy Meter for Balanced Load.

* If the meter is arranged to give correct readings on a single-phase circuit, its readings when transferred to a three-phase circuit and connected as above give one-third of the total energy.

Note I.

The result given at the top of p. 52 is obtained as follows:—

Considering the branched portion of the circuit in Fig. 37, we have for the admittances of the two branches and the corresponding phase angles,

$$\frac{1}{r_1}, \text{ with phase angle zero,}$$

$$\text{and } C\omega, \quad " \quad " \quad " \quad \frac{\pi}{2}$$

Compounding the two admittances, we find for the joint admittance,

$$\sqrt{\frac{1}{r_1^2} + C^2\omega^2} = \frac{\sqrt{1 + C^2\omega^2 r_1^2}}{r_1}, \text{ with joint phase angle} - \tan^{-1} C\omega r_1.$$

If we denote the joint phase angle by a, then—

$$\sin a = \frac{-C\omega}{\sqrt{\frac{1}{r_1^2} + C^2\omega^2}} = \frac{-C\omega r_1}{\sqrt{1 + C^2\omega^2 r_1^2}}$$

$$\text{and } \cos a = \frac{1}{r_1\sqrt{\frac{1}{r_1^2} + C^2\omega^2}} = \frac{1}{\sqrt{1 + C^2\omega^2 r_1^2}}$$

The joint impedance is given by the reciprocal of the joint admittance, *i.e.* by—

$$\frac{r_1}{\sqrt{1 + C^2\omega^2 r_1^2}}$$

Hence—

equivalent resistance of branched portion = impedance × $\cos a = \frac{r_1}{1 + C^2\omega^2 r_1^2}$

and " reactance " " = impedance × ~~sine~~ (sin α) $= \frac{-C\omega r_1^2}{1 + C^2\omega^2 r_1^2}$

as given on p. 52.

Note II.

Expression for Power in Balanced Three-phase Circuit.

Consider a star-connected balanced three-phase load having a power factor $\cos\phi$. If I = current, and V_s = star p.d., then total power = $3V_sI\cos\phi$. Let V = line p.d.; then (§ 17) $V_s = \frac{V}{\sqrt{3}}$. Hence total power $= \frac{3}{\sqrt{3}}VI\cos\phi = \sqrt{3}VI\cos\phi$, or *the total power in a balanced three-phase circuit is equal to $\sqrt{3}$ times the product of the line p.d. into the line current into the power factor of the load.* This expression, it will be noted, involves the sine wave assumption.

CHAPTER V

§ 34. General conditions to be satisfied by alternate-current instruments—§ 35. Hot-wire instruments—§ 36. Electrostatic instruments for low voltages—§ 37. Methods of extending range of low-reading electrostatic voltmeters—§ 38. High-voltage electrostatic instruments—§ 39 Wattmeters—§ 40. Iron-cored measuring instruments—§ 41. Power factor indicators—§ 42. Oscillographs—§ 42(a). Prices of measuring instruments.

§ 34. General Conditions to be satisfied by Alternate-current Instruments

SOME of the instruments employed in connection with continuous currents are also available, without any modification, for use on alternate-current circuits, while others would give totally different readings on the two circuits. Permanent magnet instruments, which for accuracy and convenience of use are unsurpassed so long as we are dealing with continuous currents, are, of course, out of the question for alternating-current measurements.

The values of the p.d. and current with which we are concerned are their r.m.s. values. It is, therefore, evident that if a measuring instrument is to read correctly on both continuous and alternating-current circuits—*i.e.* if the readings are to be independent of frequency—the particular effect on which the reading of the instrument depends must be proportional to the *square* of the current or p.d. to be measured.

Considering instruments intended to measure currents, we have at our disposal several arrangements in which the force or couple, as the case may be, acting on the movable part of the instrument is determined by the square of the current.* The most important of these arrangements are: (1) A very thin wire traversed by the current, and fitted with suitable magnifying gear for indicating the amount of elongation due to rise of temperature. The elongation depends solely on rise of temperature, and the latter is completely determined if the mean value of the *square* of the current be given, it being quite immaterial as to whether the current is alternating or continuous. (2) An arrangement of two coils, with their planes inclined at any suitable angle, connected in series with each other

* And is independent of the *kind* of current used, *i.e* whether alternating or continuous.

and traversed by the current to be measured. The stress between the two coils is proportional to the product of their currents, *i.e.*, the currents being the same, to the square of the current. (3) A coil acting on a *feebly* magnetized core. So long as the magnetization is kept sufficiently low, the permeability may be regarded as constant, and hysteresis negligible. Thus for given relative positions of coil and core the stress between them will be proportional to the mean square of the current.

Passing now to the measurement of p.d.s, all the above arrangements may be employed, provided the instrument circuit is arranged so that the current in it is proportional to the p.d., and *independent of the frequency* of the p.d. This latter condition involves negligible reactance in comparison with the resistance of the instrument circuit. There is, however, a further arrangement available for the measurement of p.d.s, which is by far the most satisfactory. It consists of a fixed and a movable system of plates, the two systems being in connection with the points across which it is desired to measure the p.d. For a given relative position of the two systems, the electric field intensity will at any instant be in simple proportion to the p.d. But since the stress between the plates varies in proportion to the square of the electric field intensity, it follows that the mean value of this stress will be proportional to the mean square of the p.d.

We shall now describe a few important types of alternating current instruments whose action is based on the above principles.

§ 35. Hot-wire Instruments

The prototype of all hot-wire instruments is the at one time well known, but now entirely obsolete, Cardew voltmeter.

A much more compact construction was introduced by Messrs. Hartmann and Braun, and instruments of their design are now widely used.

In Fig. 49 are shown the working parts of a modern hot-wire voltmeter. The "hot wire" consists of a platinum-iridium wire maintained in a state of tension between two supports, one of which is fixed, while the other is adjustable, enabling the pointer of the instrument to be set to zero. When a current is sent through the "hot wire," it expands, and the sag increases. The extra sag is taken up by the pull exerted on the "hot wire" by a phosphor-bronze wire (not traversed by the current), one end of which is attached to a point near the middle of the "hot wire," while the other end is fixed. The phosphor-bronze wire is in its turn maintained in a state of tension by a silk fibre which exerts a side pull on it and which passes round the larger of two small pulleys mounted on

the axle carrying the pointer. The smaller pulley has another silk fibre attached to it, which is kept tight by a flat steel spring.

It is a well-known fact that a very slight amount of elongation in a suspended wire produces a relatively enormous increase of sag. Now, in the instrument under consideration we have a double magnifying arrangement of this description, the elongation of the hot wire being magnified by the phosphor-bronze wire, and the sag of this latter giving a still further magnification. A comparatively small elongation of the hot wire thus results in a large angular displacement of the pointer.

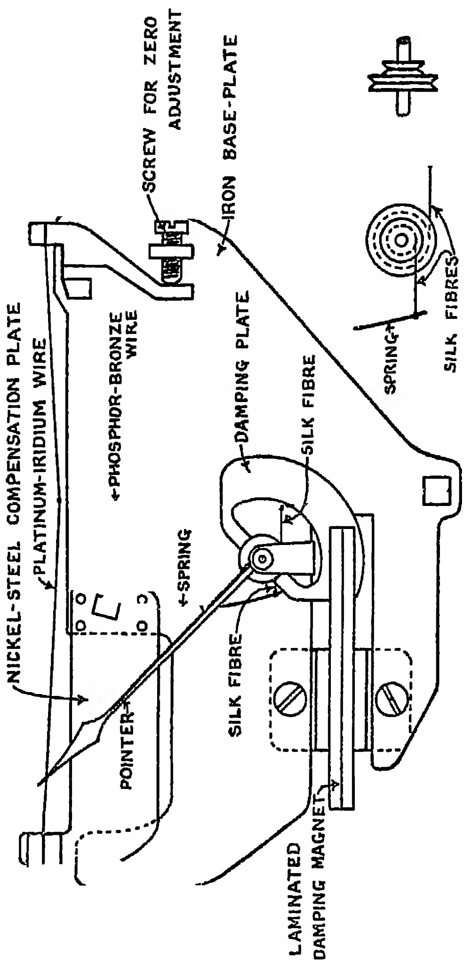


Fig. 49.—Hot-wire Instrument.

In order to prevent the instrument from acting as a thermometer, owing to the unequal expansions of the wire and the base-plate, a compensation for temperature changes becomes necessary. This is obtained by using an iron base-plate and attaching to it an auxiliary plate of nickel steel, as shown in Fig. 49. The relative lengths of the plates in the direction of the wire are such that the total expansion of the plates between the points of support of the wire is the same as the expansion of the wire itself for the same temperature rise.*

* The diameter of the wire is 0·03 mm., and the current corresponding to the maximum reading is 0·15 ampere. With this current, the wire reaches a temperature exceeding 300° C The length of the wire is about 20 cm., and its maximum

The compensating arrangement in the instrument described is obviously only intended to deal with comparatively slow changes of temperature; for if the temperature of the surroundings be suddenly changed, as by bringing the instrument from a cold room into an engine-room, or by exposing it suddenly to a cold draught, the hot wire, owing to its small mass, will acquire the new temperature of its surroundings much more quickly than the plate supporting it. With a platinum-iridium hot wire, however, the temperature rise corresponding to maximum deflection is so high compared with ordinary temperature changes that no serious error occurs even with comparatively sudden temperature changes. Such, however, is not the case with the bulk of the hot-wire instruments now in use, which are of an older design, having a platinum-silver hot wire. The maximum temperature reached by such a wire is only about 100° C., and as a result of this, and of the much higher coefficient of expansion (which is about twice that of platinum-iridium), very serious errors in the zero of the instrument, due to sudden temperature changes, may occur under ordinary conditions. Another advantage of platinum-iridium over platinum-silver is that owing to its greater tensile strength a thinner wire may be used, which renders the instrument less sluggish in its action.

An aluminium damping vane and a damping magnet are provided as shown in Fig. 49, to check any rapid vibrations of the pointer such as might arise from air currents impinging against the hot wire, or mechanical vibrations.

Among the most important advantages of hot-wire instruments may be mentioned their entire independence of frequency, wave form and external magnetic fields, combined with a reasonable price.

In the case of a hot-wire voltmeter, the hot wire is connected in series with a high non-inductive resistance mounted inside the case of the instrument. In an ammeter (unless for exceptionally small currents) the hot wire forms a shunt across a resistance, which in instruments for currents up to 200 amperes is contained in the instrument case, and in those for currents above this value is separate from the instrument proper, special connecting leads being provided for connecting the instrument across the shunt. In order to reduce the drop of potential in ammeters for large currents, it is usual to divide the hot wire into two or more sections connected in parallel, by means of very thin strips of silver foil connected to suitable points in the wire.

A serious disadvantage of the hot-wire type of instrument is its liability to be fused by a momentary excess of current.

expansion about 0·2 mm. This produces a change of sag in the phosphor-bronze wire amounting to 6 mm., which causes a rotation of the pointer through about 90°. *Elektrotechnische Zeitschrift*, vol. xxi p. 269 (1910).

Instruments constructed on the principles considered under (2) and (3) in § 34 are used to a considerable extent. As, however, details of their construction are fully described in treatises on continuous currents, we shall not refer to them further here. As examples of such instruments, we may mention Lord Kelvin's balances; the Weston standard voltmeter for alternating p.d.s; and the numerous forms of "soft-iron" instruments, whose great advantage is their cheapness.

§ 36. Electrostatic Instruments for Low Voltages

On account of the importance of the electrostatic type of instrument, we shall devote some space to a description of several examples of this type. One of the best known is Lord Kelvin's multicellular voltmeter, the construction of the working parts of which will be understood by a study of Figs. 50 and 51. In Fig. 50 is shown the movable system of vanes, or "needles," which are threaded on a rod carrying a damping vane at its lower end, and suspended by a fine wire from a coach-spring attached to a torsion-head provided with a tangent screw for adjusting the zero of the instrument. The object of using a coach-spring between the torsion-head and the suspension is to prevent the latter from being broken by a sudden accidental jerk or blow. If, in fact, the vanes are jerked downwards, the coach-spring yields sufficiently to allow the safety-sleeve to come into contact with the guide-stop before the suspending wire is over-strained By means of a clamping-spring, which may be raised by a screw provided with a milled head which projects through the bottom part of the

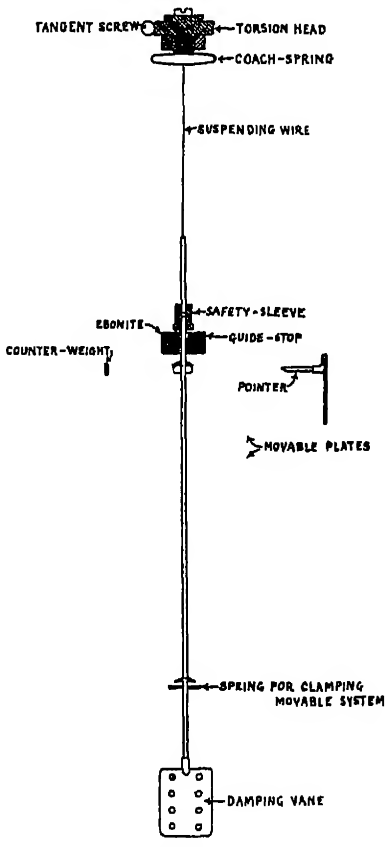


Fig. 50.—Multicellular Voltmeter.

case, the movable system may be raised until the conical portion shown above the pointer comes against the guide-stop. The suspension is then quite slack, and the instrument is capable of standing a considerable amount of ill usage without damage. The perforated damping-vane fits into a glass vessel filled with oil, the oil being at a sufficiently high level completely to cover the vane.

The movable vanes are in connection with one terminal of the instrument. The other terminal is connected to the two sets of fixed plates, one set on each side of the movable plates, as shown in Fig. 51. The fixed plates are, for the sake of simplicity, not shown in Fig. 50. Each set of fixed plates contains one more plate than the movable system, there being a fixed plate above each half of the uppermost moving plate, and a fixed plate below each half of the lowermost moving plate.

In the switchboard type of instrument the end of the pointer, bent down as shown in Fig. 50, is arranged to move over a cylindrical scale whose axis coincides with that of the moving system.

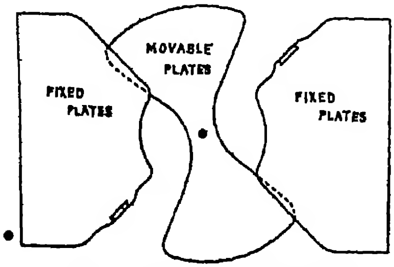


FIG. 51.—Multicellular Voltmeter.

Multicellular voltmeters of this type are constructed to read up to 600 volts. For higher voltages a modified form of the instrument, known as the "dial pattern," and illustrated in Fig. 52, is used. In this, there are only two movable plates, the axis of rotation is horizontal, and knife-edge supports take the place of the suspension used in the former or "vertical scale" pattern. The controlling couple is furnished by gravity, and oil damping is used. Instruments of this kind are made to read up to 8500 volts.*

A somewhat different type of instrument is the Ayrton and Mather electrostatic voltmeter, the working parts of which are shown in Fig. 53. The plates, both fixed and movable, are cylindrical, and electromagnetic damping is employed. When a p.d. is established between the movable and fixed portions, the former are sucked into the spaces between the latter. The controlling couple is furnished by gravity, but in instruments for low voltages spring control is used. The main feature of the instrument is the cylindrical shape of the

* In instruments of the multicellular type, the moving vanes are, in their zero position, displaced from the position of symmetry; otherwise they would be in a position of instability, and the direction of displacement would be indefinite, being determined by the accidental displacement, at the instant when the p.d. is applied, of the movable vanes from the exact position of symmetry.

plates, as a result of which the stress at every point of the movable plate acts at the same radius, so that for a given area of plate a much

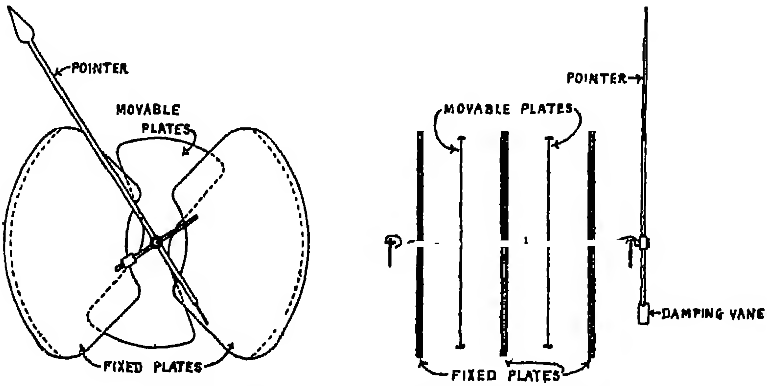


FIG. 52 —Dial Pattern of Multicellular Voltmeter.

larger deflecting couple is obtained than with a flat-plate instrument, in which the stress acts at varying radii, and so gives rise to a much smaller couple.

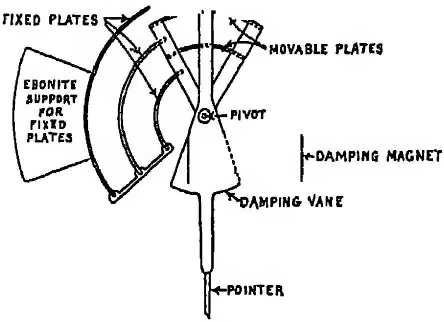


FIG. 53.—Ayrton-Mather Voltmeter.

§ 37. Methods of extending Range of Low-reading Electrostatic Voltmeter

It is frequently convenient to be able to extend the comparatively narrow range of an electrostatic voltmeter, and various methods of doing this are available. The most satisfactory of these is the

sub-division of the unknown p.d. by means of a resistance. Such resistances, known as "multipliers," are supplied by the makers of electrostatic voltmeters, and by means of them the range of an instrument may be greatly extended.

This extension may be carried on indefinitely, as it is simply a matter of providing suitable resistances. But for very high voltages the cost of the resistance becomes prohibitive, and the resistance itself becomes bulky. Other methods of adapting a low-reading voltmeter to the measurement of high voltages have therefore been used. By means of a transformer, the unknown p.d. may be transformed down to a fraction of its original value, and this fraction may be calculated with fair accuracy (provided the transformer is of suitable design) from the number of turns in the primary and secondary coils (§ 57). This method is also limited by the fact that a suitably designed transformer for this purpose is both bulky and costly when the voltages to be dealt with are very high. Another method, originally suggested by Ayrton, then by Peukert, and subsequently reinvented by Marchant and Worrall, consists in subdividing the p.d. by means of condensers. This method has been used to some extent, and is cheaper than the other two. It is, however, by no means free from difficulties, chief among which are leakage and variation of capacity with p.d. or frequency, or variation of capacity due to accidental external circumstances capable of affecting the capacity.

Any one of the methods described may be employed satisfactorily so long as the voltage is not excessive. But for very high voltages it is better to use specially constructed instruments. A good deal of attention has been devoted to the construction of such instruments, and in what follows we shall briefly describe a few typical forms.

§ 38. High-voltage Electrostatic Instruments

In Fig. 54 is shown Lord Kelvin's *Volt Balance*. This instrument is portable, and is suitable for pressures up to 30,000 volts (r.m.s.). The fixed plate is insulated by a thick disc constructed of mica. Above it is suspended an aluminium pan, which forms the movable system. The vibrations are damped by a thin aluminium vane arranged to swing between the poles of a damping magnet as shown in the figure. The bulk of the dielectric between the fixed and movable portions of the instrument is air.

In Fig. 55 is shown a voltmeter designed by Kintner and

manufactured by the Westinghouse Co.* The working parts are completely immersed in oil—an arrangement which provides the necessary electric strength to prevent sparking. The movable system consists of two hollow cylinders with hemispherical caps connected by a vertical plate, and suspended, by means of a corrugated insulating rod, from the spindle carrying the pointer, in the manner shown in Fig. 55 (*b*). Spring control is used. The movable system is subject

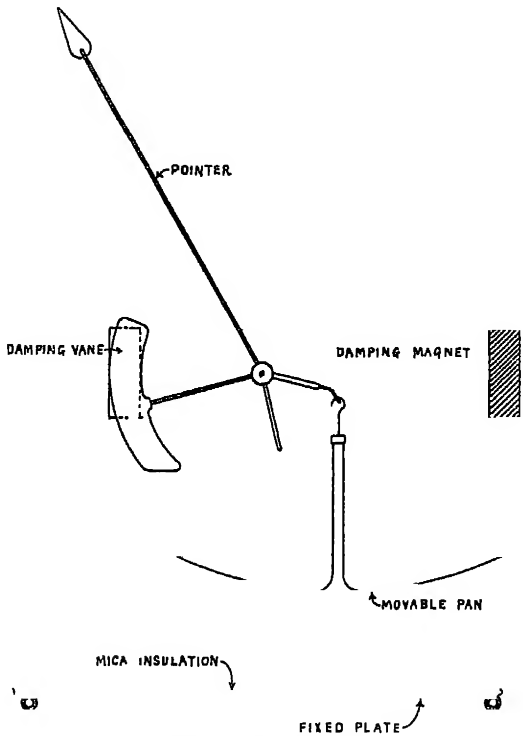
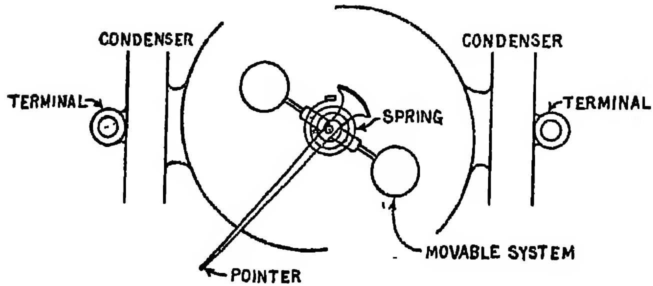


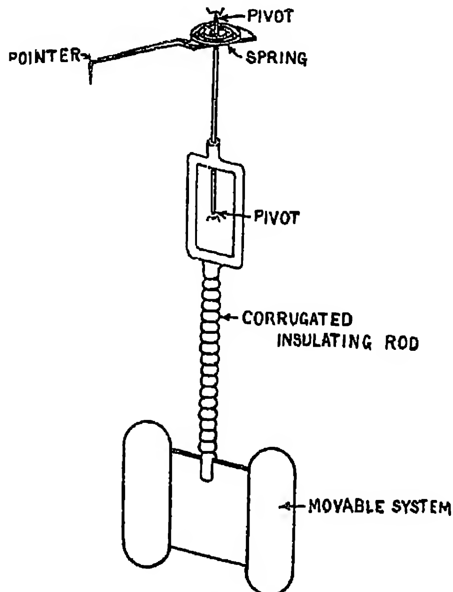
FIG. 54.—Volt Balance.

to the inductive action of two fixed curved plates, shaped so that as the movable system is rotated the distance between the cylinders and the fixed plates decreases. These plates are in metallic connection with one set of plates of a pair of plate condensers, whose other plates are connected to the terminals of the instrument. Owing to the buoyancy of the movable system due to the oil immersion, the frictional error is said to be very small. Instruments of this type may be constructed to read up to 200,000 volts.

* *Electrical World and Engineer*, vol. xlv p 1176 (1905), also vol. xlvii, p. 124 (1906).



(a)



(b)

FIG 55.—Westinghouse Electrostatic Voltmeter

§ 39. Wattmeters

Owing to the inconvenience of having to make an adjustment before taking a reading, the zero type of dynamometer-wattmeter has practically disappeared (except for laboratory purposes), all commercial types of wattmeter being direct-reading instruments. The

usual arrangement of fixed and movable coils is shown in Fig. 56. The current is led into and out of the movable coil by means of two hair-springs of phosphor-bronze, which also furnish the controlling couple.

A wattmeter of the simple form shown in Fig. 56 is subject to the error due to stray magnetic fields, as there is no arrangement for compensating the couple due to an external field. In order to remedy this defect, Lord Kelvin introduced the *astatic* type of wattmeter shown in Fig. 57. The movable system consists of two coils

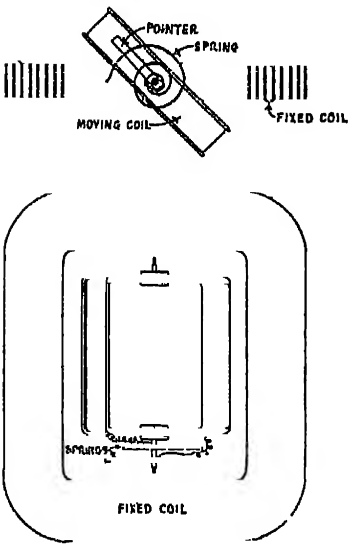


Fig. 56.—Indicating Wattmeter.

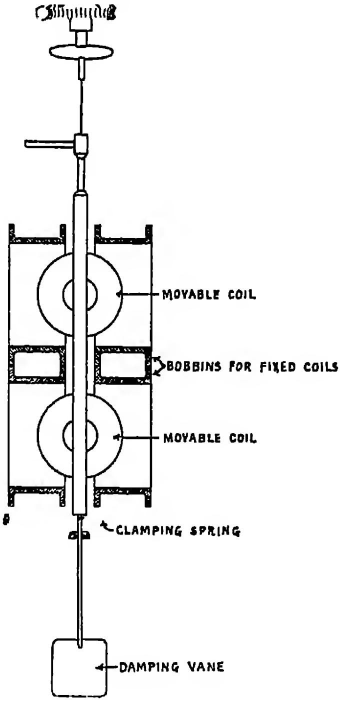


Fig. 57.—Kelvin Astatic Wattmeter

similar in every respect, but connected up so that the current circulates round them in opposite directions. Thus the couples exerted by a uniform external field on the movable coils are in opposite directions, and balance each other. Corresponding to each movable coil there is a fixed coil (wound in two sections, one on each side of the axis of rotation). Oscillations are prevented by an oil-damping arrangement precisely similar to that employed in the

multicellular type of voltmeter, and the method of suspension is also identical with that used in this latter type of instrument.

An astatic arrangement of movable coils, though rendering the reading of the instrument independent of a *uniform* external field, would still be liable to disturbance by a field of varying intensity. In general, however, it is possible to place the instrument so as to render the error due to non-uniformity of field very small.

§ 40. Iron-cored Measuring Instruments

A type of alternate current instrument, in which *shunt* electromagnets are used in a manner similar to that of permanent magnets in moving-coil instruments for continuous currents, has been

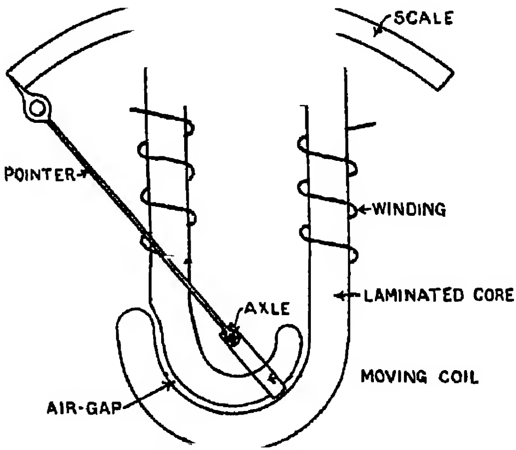


FIG. 58.—General arrangement of Iron-cored Instrument.

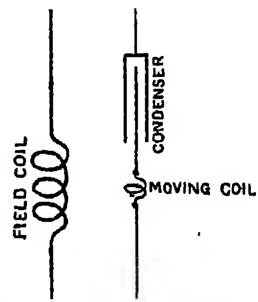


FIG. 59.—Connections of Iron-cored Voltmeter.

designed by Dr. W. E. Sumpner, and is manufactured by the General Electric Co.

The working parts of these instruments are represented diagrammatically in Fig. 58. They consist of an electromagnet provided with a laminated core of the shape shown, having a single annular air-gap, and of a moving coil pivoted so as to be free to swing in the gap, and controlled by the usual two phosphor-bronze hair springs, which, for the sake of simplicity, have been omitted from Fig. 58. The connections of the instrument depend on the particular use to which it is to be put.

In a *voltmeter*, the magnet winding is connected directly across

the two points between which the p.d. is to be measured, while the moving coil is joined in series with a condenser and then across the same two points, as shown in Fig. 59. Let the design and conditions of use of the instrument be such that the following assumptions are justifiable:—

(1) The resistance drop in the field-magnet winding is so small that the counter-e.m.f. may be taken to be equal to the p.d.

(2) The working range of the induction in the core of the electro-magnet is such that the Hopkinson leakage coefficient may be regarded as constant, and hence the field at any point of the gap as proportional to the total flux.

(3) The resistance drop in the moving coil is negligible, so that the current in this coil may be taken to be proportional to the p.d.

(4) The air-gap is of uniform radial depth, so that the gap field may be regarded as uniform over the working range.

On the basis of the above assumptions, it may be shown that the angular deflections of the instrument are proportional to the r.m.s. values of the p.d., and that the readings are *independent of wave-form and frequency.*

Let v = instantaneous p.d.; b = instantaneous gap induction. Then, by reason of (1), (2) and (4), $v \propto \dfrac{db}{dt}$. Again, on account of (3), the moving coil current is proportional to $\dfrac{dv}{dt}$. But since the instantaneous deflecting torque is proportional to the product of the instantaneous gap field and instantaneous moving coil current, we have: instantaneous torque $\propto b\dfrac{dv}{dt}$, and hence

$$\text{mean torque} \propto \int_0^{\mathrm{T}} b\frac{dv}{dt}dt,$$

where T is the period. Integrating by parts, we have

$$\int_0^{\mathrm{T}} b\frac{dv}{dt}dt = \Big[bv\Big]_0^{\mathrm{T}} - \int_0^{\mathrm{T}} \frac{db}{dt}v\,dt = -\int_0^{\mathrm{T}} \frac{db}{dt}v\,dt,$$

since bv has identical values at the upper and lower limits. Now, since $\dfrac{db}{dt} \propto v$, the last integral is proportional to $\displaystyle\int_0^{\mathrm{T}} v^2dt$ or to V^2, where V is the r.m.s. value of v. Thus

$$\text{mean torque} \propto \mathrm{V}^2,$$

and since no assumptions have been made regarding wave-form or frequency, the result is entirely independent of these.

In order that the assumption (2) made above may hold good, the

maximum value B of the induction in the electromagnet core must not exceed a certain limit. Now, since for a given wave-form $V \propto Bf$, or $B \propto V/f$, we see that the ratio $\frac{\text{p.d.}}{\text{frequency}}$ must not be allowed to exceed a certain value.

If the condenser connected in series with the moving coil is leaky, then in addition to the true capacity current there will be a leakage current which is in phase with the p.d. and proportional to it. The instantaneous value of this current is thus proportional to v, and the mean torque due to it proportional to

$$\int_0^T bv\,dt \propto \int_0^T b\frac{db}{dt}dt$$
$$\propto \left[\tfrac{1}{2}b^2\right]_0^T$$

which vanishes, b having identical values at the upper and lower limits. The leakage current is thus incapable of affecting the torque, and hence *the readings are not influenced by leakage in the condenser.*

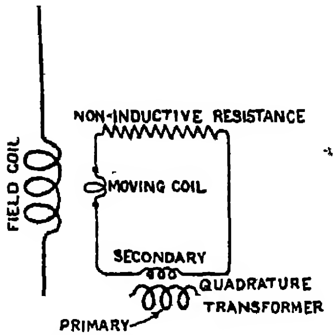


Fig. 60—Connections of Iron-cored Wattmeter.

If the instrument is to be used as a *wattmeter*, the connections are arranged as in Fig. 60. The electromagnet coil is, as before, connected directly across the mains, while the moving coil is joined in series with a non-inductive resistance and across the secondary of a small transformer of special construction known as a *quadrature transformer.*

The function of the quadrature transformer is to supply the moving coil circuit with a current which is proportional to the current in the main circuit, and in quadrature with it. The construction of this transformer is diagrammatically represented in Fig. 61. The main feature of the design is the use of a magnetic circuit with a relatively very long air-gap, so as to bring the magnetic flux into practical coincidence of phase with the exciting ampere-turns. Absolute coincidence of phase could be obtained by the use of an air-core transformer, but the iron core shown in the design illustrated serves two important purposes: (1) it acts as a *magnetic shield* to the windings, thereby preventing any influence of external magnetic fields on the current in the

moving coil circuit; (2) it reduces the magnetic reluctance of the circuit, and allows of a more compact form of construction for the transformer. The design of the windings is such that with the maximum current flowing in the secondary, the secondary ampere-turns are negligibly small in comparison with the primary ampere-turns. This condition must be fulfilled in order that the resultant ampere-turns may be practically equal to, and co-phasal with, the primary ampere-turns, and hence the magnetic flux in phase with the main current. The self-inductance of the moving-coil circuit being negligible in comparison with its resistance, the secondary current will be in phase with the secondary e.m.f., and hence in quadrature with the primary or main current.*

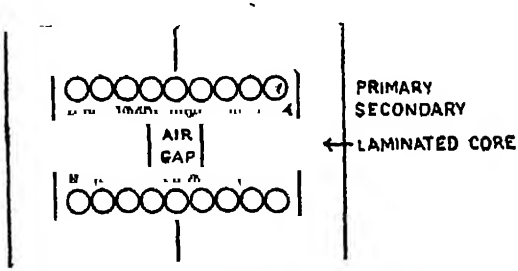


Fig. 61.—Quadrature Transformer.

Provided all the conditions stated above are fulfilled with sufficient accuracy, as well as conditions (1), (2) and (4) given above in connection with a voltmeter, it is easy to show that the deflection of the instrument is proportional to the mean power in the circuit, and is *unaffected by wave-form and frequency* within the working range of the instrument. For if i = main current, then the moving coil current $\propto \frac{di}{dt}$. Hence, if b as before denote the instantaneous gap induction in the shunt electromagnet of the wattmeter, we have

$$\text{instantaneous torque} \propto b\frac{di}{dt},$$

and

$$\begin{aligned}\text{mean torque} &\propto \int_0^T b\frac{di}{dt}dt \\ &\propto \Big[bi\Big]_0^T - \int_0^T \frac{db}{dt} i\,dt \\ &\propto \int_0^T vi\,dt \\ &\propto \text{mean power,}\end{aligned}$$

* The conditions are here entirely different from those prevailing in an ordinary transformer, in which at full load the primary and secondary ampere-turns nearly balance each other, and the magnetic flux is nearly in quadrature with either primary or secondary current (if the load be non-inductive).

a result not involving any assumptions as to wave-form or frequency.

The use of a wattmeter of this type is subject to the same restriction with regard to the ratio $\frac{\text{voltage}}{\text{frequency}}$ as that already pointed out in connection with the voltmeter.

The main advantages of these instruments are their independence of wave-form, frequency and external magnetic disturbances, and their comparatively substantial mechanical construction, rendered possible by the use of strong controlling springs, as a powerful deflecting torque is easily obtainable. On the other hand, their use is more restricted than that of instruments of the dynamometer type, as it is not permissible to extend the range of the instrument by connecting the shunt magnet in series with a non-inductive resistance.

An iron-cored wattmeter of the above type may be easily adapted to act as an *idle current ammeter*. All that is necessary is to substitute for the quadrature transformer a non-inductive resistance, the moving coil circuit of the instrument being joined across this resistance. If the main current be in phase with the p.d., the air-gap flux in the instrument and the moving coil current will be in quadrature with each other, and no deflection will take place. On the other hand, if the main current lag behind the p.d., the moving coil will experience a torque proportional to the idle or wattless component of the current.

§ 41. Power Factor Indicators

The power factor (§ 7) can always be calculated from the readings of a voltmeter, an ammeter, and a wattmeter. Instruments have, however, been devised for indicating directly the value of the power factor. Such instruments are known as *power factor indicators*. In Fig. 62 is illustrated the general arrangement of a type of power factor indicator devised by Punga, and manufactured by Messrs. Everett, Edgcumbe & Co. It is intended for use on a three-phase circuit, and consists essentially of two systems of coils, a fixed system of current coils and a movable system of pressure coils, the coils of each system forming angles of 120° with each other. Currents are led into and out of the movable system by means of very weak strips of phosphor-bronze, which exert no appreciable controlling couple. Each system gives rise to a rotating magnetic field when traversed by three-phase currents (§ 16). So long as the two fields coincide, no torque will be exerted on the movable system. But any change in the relative phase of the currents will

cause a displacement of the fields, giving rise to a torque which will rotate the movable system through an angle sufficient to restore coincidence of fields. Corresponding, therefore, to each value of the phase difference between the currents in the two systems, there is a perfectly definite position of equilibrium for the movable system. The value of the phase difference ϕ, or of the corresponding power factor $\cos \phi$, is read off on a scale over which moves a pointer attached to the movable system. The two systems of coils are connected up like the circuits of three wattmeters, the current coils being placed in series with the three line wires, and the three star-connected pressure coils being joined in series with three large non-inductive resistances, and then connected to the line wires. If the three-phase circuit on which the instrument is used is unbalanced, the reading corresponds to the mean value of the power factor for the three phases.*

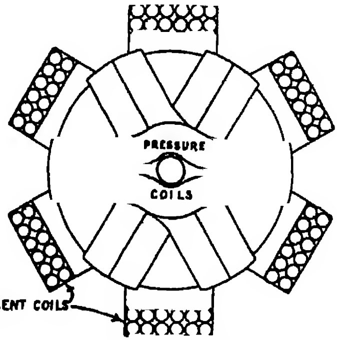


FIG 62.—Power Factor Indicator.

§ 42. Oscillographs

In many cases, it is important to know the exact shape of the p.d. or current wave. Numerous methods of determining the wave-shape are available, but by far the quickest and most convenient is that involving the use of a special instrument devised for this purpose

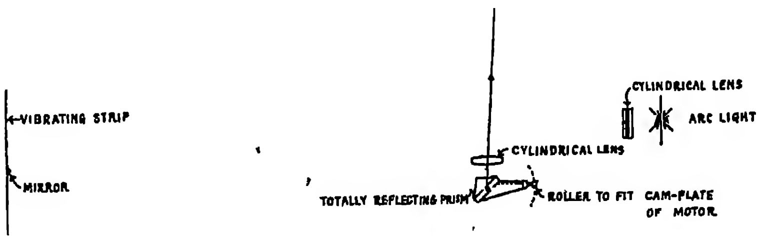


FIG 63.—Arrangement for recording Vibrations of Oscillograph.

and known as an oscillograph. We shall select for description a form of oscillograph due to Blondel.

* See § 196.

Every oscillograph is in reality a galvanometer whose moving system is capable of extremely rapid vibration, and which is fitted with a suitable arrangement for recording the vibrations. This arrangement is shown diagrammatically in Fig. 63, and is, except as regards details of construction, the same in all forms of oscillograph. Light coming from an arc passes through a cylindrical lens, and falls on the tiny mirror attached to the moving part of the oscillograph galvanometer. The vibrations of the mirror cause the reflected beam to sway in a horizontal plane. This reflected beam is received by a totally reflecting prism (or by a mirror), which is kept oscillating about a horizontal axis by means of a synchronous motor (§ 78) fitted with a suitable cam-plate, which engages the roller mounted on the tail-piece of the prism. The shape of the cam-plate is such that while the

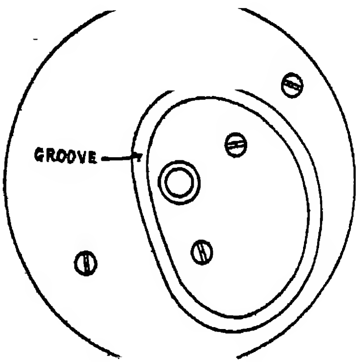


FIG. 64.—Cam-plate of Synchronous Motor.

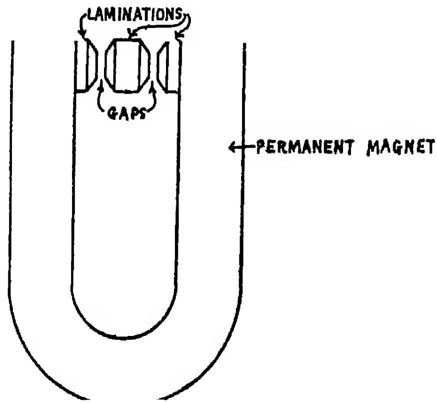


FIG. 65.—Magnet of Blondel Oscillograph.

prism is receiving the reflected beam its angular velocity is constant, so that the displacement of the reflected beam in a vertical plane is directly proportional to the time. When the prism reaches its maximum displacement, a sector mounted on the motor axle cuts off the light, and during this time the prism executes a quick-return motion, to resume its constant angular velocity in a forward direction as the sector passes out of the path of the beam.

On leaving the prism or mirror, the light is rendered convergent by a second cylindrical lens, and forms a tiny speck of light on the ground-glass screen which is provided to receive it. This speck is subject to two displacements at right angles to each other, one being due to the vibrations of the oscillograph galvanometer, and the other to the steady forward displacement of the prism (during the return

motion, the light is cut off as already explained). Thus the curve traced out on the screen shows the wave-shape at a glance. If it is desired to obtain a permanent record of the wave-shape, a photographic dark slide is substituted for the screen, and an exposure made in the ordinary way, by means of a shutter placed behind the first cylindrical lens.

Fig. 64 shows the form of cam-plate used in the Blondel oscillograph. The axle of the synchronous motor carries a gun-metal disc, and screwed to this are the two portions of the hardened steel cam-plate. It is important to use hardened steel for both the cam-plate and the roller at the end of the prism tail-piece in order to prevent a distortion of the curve due to wear of these parts.

The controlling field for the vibrating portion of the oscillograph galvanometer is provided by means of a powerful permanent magnet, shown in Fig. 65. This magnet is fitted with laminated tapering pole-pieces and core, which are separated from each other by two air-gaps, in which are placed the vibrating portions of the instrument. A *single* gap is quite sufficient if only one wave at a time is required. But frequently it is desirable to obtain simultaneously the records of *two* waves (such as a p.d. and a current wave), so as to determine their phase relation. Hence most oscillographs are made double.

In each gap is placed an extremely thin and narrow vertical band or strip of soft iron, with its width along the lines of force—a position which the band would naturally take up if free to rotate about a vertical axis. The tension of the band may be adjusted to the desired amount, and so its natural rate of torsional vibration about its axis controlled. This rate of vibration may correspond to a frequency as high as 40,000 per sec., so that the band is easily able to follow the comparatively slow vibrations due to the current whose wave-shape is required.

The powerful permanent magnet shown in Fig. 65 corresponds to the controlling magnet of an ordinary needle galvanometer, the soft-iron band corresponds to the needle or needles, and the deflecting coil is represented by two small coils (not shown in Fig. 65), one on each side of the gap, the common axis of the coils being normal to the field of the magnet (*i.e.* normal to the plane of the paper in Fig. 65). Each gap is, of course, fitted with its own coils. If a current is sent through either set of coils, the field of the corresponding gap is distorted, the band tends to take up a position with its width along the resultant field, and is thus twisted. The angle of twist is greatest at the middle point of the band, and it is to this point that the tiny mirror is attached.

Fig. 66 shows the details of construction of the frame which supports the vibrating band. This frame is made of German-silver in order to avoid excessive eddy currents. On each side of the vibrating

band is a tapering soft-iron pole-piece, an arrangement which gives a very intense field. The methods of fixing the ends of the strip and of adjusting its tension are clearly shown in the figure.

The entire lower portion of the frame supporting the band fits into a glass tube filled, or partially filled, with pure castor-oil. This provides the necessary damping, so that the band merely responds to the forced vibrations impressed upon it by the current, but is practically free from superposed natural vibrations (which would cause ripples in the curve). In order to prevent distortion of the luminous speck formed on the screen, the glass tube is fitted with a small lens, through which the light enters and leaves the tube. The tubes containing the bands slip into the spaces between the laminated pole-pieces and central core of the controlling magnet shown in Fig. 65.

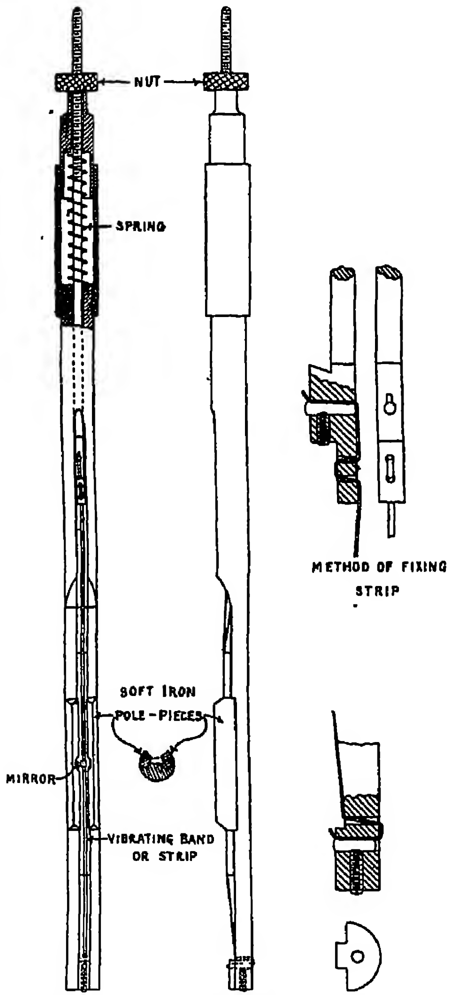


FIG. 66.—Details of Blondel Oscillograph.

Besides the oscillograph just described, Blondel devised another type, which has a moving-coil galvanometer. The development of this type in England we owe to Mr. Duddell, and the Duddell oscillograph is the best-known form of the instrument in this country. A powerful electromagnet is used, between whose pole-pieces are placed, in the narrow air-gap,* two parallel strips of

* Two air-gaps are provided in the double oscillograph

phosphor-bronze whose tension may be adjusted. The strips are stretched over bridge-pieces, the plane containing them being along the field. Across the two strips is fixed a small mirror. The current whose wave-form is required passes up one strip and down the other. As a result the strips move across the field in opposite directions, thereby deflecting the mirror. It is to be noted that here the strips vibrate bodily across the field, like the strings of a violin, while in the Blondel instrument which we have described there is no bodily vibration of the soft-iron strip, but simply a torsional vibration, the axis of the strip retaining a fixed position in space.

§ 42a. Prices of Measuring Instruments

Hot-wire instruments cost from about £4 to about £10, depending on the type and range.

Voltmeters of the electrodynamic type cost from £6 to £7.

Soft-iron instruments represent by far the cheapest class, the price ranging from £2 to £3 only.

Electrostatic instruments are expensive, and their price depends very much on the range required. For ranges comprised between 40 volts and 10,000 volts, instruments of this class would cost from £7 to £11. For a maximum reading of 30,000 volts, the price would be about £30; while for maxima of 100,000 and 200,000 volts, the prices would be about £50 and £100 respectively.

Iron-cored instruments of the Sumpner type cost from £8 to £10.

Oscillographs vary a good deal in price. The lowest price of a single oscillograph without any accessories is about £20, while a complete outfit may cost from £100 to £200.

CHAPTER VI

§ 43. Dynamo used as alternator—§ 44. Constructional differences between dynamos and alternators—§ 45. Types of alternators. Peripheral speed—§ 46. Standard type of low speed alternator—§ 47. Construction of turbo-alternators—§ 48 Fire risk in turbo-alternators—§ 49. Armature windings of alternators—§ 50. Methods of supporting coil ends—§ 51. Calculation of armature e.m f. Effect of varying length of polar arc—§ 52. Effect of varying number of slots—§ 53. General formula for e.m.f. of alternator—§ 54. Armature reaction in alternators—§ 55. Some technical data relating to alternators—§ 56. Dimensions, weights, and prices of alternators.

§ 43. Dynamo used as Alternator

ALTHOUGH the machines employed for the generation of alternating currents differ somewhat in details of construction from continuous-current dynamos, the principle of action is the same—the motion of a conductor across a magnetic field—and, in fact, any continuous-current generator may be easily modified so as to act as an alternator. The only addition necessary is the fitting of a suitable number of insulated contact-rings or "slip-rings" which are in *permanent* connection with certain points of the winding. By means of brushes making contact with the slip-rings, alternating currents may be supplied to an external circuit. If the commutator is retained, then the same machine may be made to supply at the same time both continuous and alternating currents, the continuous current being obtained as usual from the commutator, and the alternating currents from the slip-rings. Some very large generators of this kind were at one time in use in the United States, and were known as *double-current generators.*

The construction of the typical modern alternator is, however (for reasons which will be explained presently), different from that of the typical modern continuous-current dynamo. We shall commence our study of alternators by considering the action of a dynamo as an alternator when fitted with slip-rings as explained.

In the diagrams of Fig. 67 is shown a two-pole armature fitted with two slip-rings, diagram (*a*), connected to two diametrically opposite points, C and D, of the winding. For the sake of simplicity, the winding itself is indicated by a simple circle. Let the arrangement of the winding and the direction of rotation be such that e.m.f.s are induced in the two halves ACB and ADB of the armature having a direction from A to B inside the winding, as indicated by the arrows, and hence producing a p.d. from B to A across an external circuit connected to these two points. In the diagrams, the points B and A are supposed to be fixed *in space*, and to correspond to the

position of the brushes of a continuous-current machine. The two diametrically opposite points C and D are, on the other hand, points fixed *in the winding*, and are carried round in space, as shown in the consecutive diagrams, during the rotation of the armature. We shall suppose C and D to be in permanent connection with two slip-rings provided with brushes as shown in Fig. 67 (*a*),* by means of which permanent contact is maintained between the terminals of an external circuit and the points C and D in the winding. In the position of the armature corresponding to (*a*), it is clear that there will be zero p.d. between C and D, since the clockwise e.m.f. induced in the quadrant AC of the winding is exactly balanced by the counter-clockwise e.m.f. induced in the quadrant AD; similarly, the e.m.f.

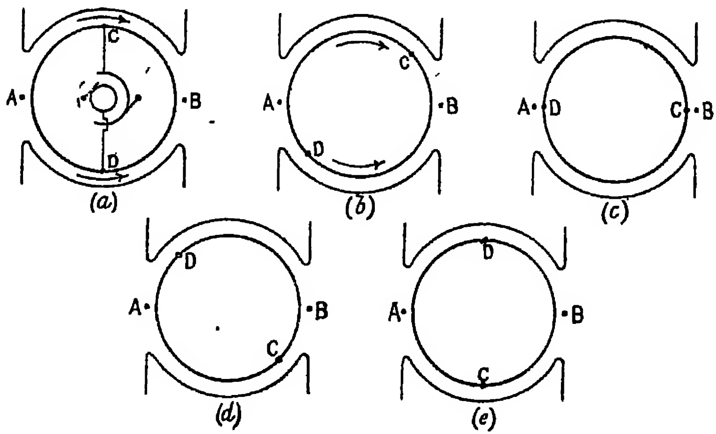


FIG. 67.—To illustrate Action of Continuous-current Armature when generating Single-phase Current.

in DB balances that in CB. When the armature reaches the position shown in diagram (*b*), the e.m.f.s in the sections AC and DB of the winding overpower those in the smaller sections AD and BC respectively, so that there is now a p.d. acting from C to D. Since the first two sections (AC and DB) steadily increase in length, while the last two (AD and BC) steadily decrease, the p.d. acting from C to D will increase until, as shown in diagram (*c*), the points C and D coincide with B and A respectively—when it reaches its maximum value. Beyond this position, the p.d. will steadily *decrease*, since, as is evident from diagram (*d*), the e.m.f.s in the sections AC and DB are now decreasing, while those in CB and AD are increasing.

* In order to simplify the figures as much as possible, the slip-rings are omitted from the diagrams (*b*) to (*e*).

The p.d. reaches a zero value in the position shown in diagram (*e*), the points C and D having now come halfway between A and B. It is evident that since (*a*) differs from (*e*) only in that C and D have now changed places, a similar series of changes to that just considered will take place in the p.d. across CD during the next half-revolution, the p.d. being now, however, directed from D to C instead of from C to D as during the first half-revolution. We thus get an *alternating p.d.* between the slip-rings connected to the diametrically opposite points C and D in the winding. Such an arrangement would, therefore, constitute a *single-phase generator*, or *single-phaser*.

It is clear that by selecting two pairs of diametrically opposite points, such as C, D and E, F in Fig. 68 (*a*), so arranged that the diameter CD is perpendicular to the diameter EF, we get, during the rotation of the armature, an alternating p.d. between C and D which differs 90° in phase from the alternating p.d. between E and F, the p.d.s being necessarily of the same frequency. By using *four* slip-rings connected to the points C, D and E, F, and connecting one external circuit to C, D, and another to E, F, we obtain a *two-phase system*. The machine now becomes a *two-phase generator*, or *two-phaser*.

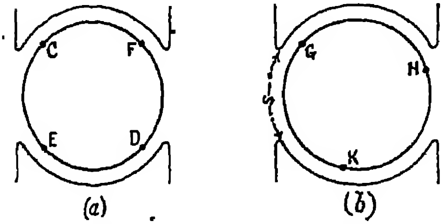


FIG 68.—Slip-ring Connections for Two- and Three-phase Currents

Lastly, by taking three points, G, H, and K, in the winding, 120° apart, as in Fig. 68 (*b*), and attaching them to three slip-rings, we get, between any pair of slip-rings, an alternating p.d. which differs by 120° in phase from the p.d.s between the two remaining pairs of rings. Three conductors connected to the slip-rings would thus form the mains of a *three-phase system*, the machine now acting as a *three-phase generator*, or *three-phaser*. The arrangement of the armature winding in this case would, it may be noted, correspond to a delta or mesh connection (§ 17).

It will now be evident that, by the simple addition of a suitable number of slip-rings, any ordinary continuous-current dynamo may be converted into an alternator. Such a mode of construction is used in practice when either (1) the same machine is required to generate both continuous and alternating current, the former being obtained from the commutator and the latter from the slip-rings; or (2) when a transformation from alternating to continuous current, or *vice versâ*, has to be effected. In the first case, the machine forms a *double-current generator*; in the second, a *rotary converter*.

§ 44. Constructional Differences between Dynamos and Alternators

If, however, we want a machine which is only required to generate single-phase current, the above arrangement admits of considerable improvement. A glance at the diagrams of Fig. 67 will show that with a uniformly distributed winding such as is used in continuous-current armatures, we have *opposition of e.m.f.s* in the different parts of the winding. Thus, in Fig. 67 (*b*) the e.m.f. in the section AC of the winding is opposed by the e.m.f. in the section AD; it is thus evident that in this particular position of the armature the conductors forming the section AD are not only useless, but positively harmful in (1) unnecessarily adding to the resistance and reactance of the armature, and (2) reducing its e.m.f. Such opposition of e.m.f.s in different parts of the winding is conveniently spoken of as *differential action.* In order to eliminate differential action, the winding must be arranged so that all the conductors forming a group of the winding between two contact-rings shall always be moving simultaneously across a field of the same polarity. In order to secure this result, the width of a band of conductors forming such a group must evidently not exceed the width of the interpolar space—*i.e.* the distance between the neighbouring polar horns of two consecutive pole-pieces (*s* in Fig. 68 (*b*)). Now, in continuous-current machines the interpolar arc is only about 30 per cent. of the pole-pitch (the pole-pitch is the distance between the middle points of two consecutive pole-pieces, which corresponds to half the armature circumference in a two-pole machine), so that if the same ratio of length of interpolar arc to pole-pitch were retained in alternators, we should, in a single-phase machine, utilize only 30 per cent. of the available winding space on the surface of the armature. In order to increase the width of the available winding space without at the same time introducing any appreciable amount of differential action, it is usual to make the lengths of polar and interpolar arcs equal, so that each of them, as well as the width of a group of the winding, is equal to half the pitch of the pole-pieces. In this reduction of the polar arc, and increase of interpolar space, we have one of the characteristic constructional differences between single-phase alternators and continuous-current dynamos.

Since with the arrangement indicated only 50 per cent. of the available winding-space is utilized with a *single-phase* armature, we have, as regards the armature winding, another important constructional difference between continuous-current machines and *single-phase* alternators; the winding in the latter, instead of being uniformly distributed as it is in the former, is broken up into groups separated by spaces devoid of conductors.

If, however, our alternator is to be a *two-phaser*, we may fill up

the vacant spaces on our armature with the conductors forming the second phase of the winding, thereby utilizing the entire available winding-space.

In a *three-phaser*, the pole-pitch is divided into three equal parts, and the width of each of these represents the width of a group of conductors belonging to the same phase. Here, again, the entire available winding-surface is utilized.

Since in a three-phase alternator the width of a group of conductors belonging to one phase does not exceed $\frac{1}{3}$rd of the pole-pitch, it follows that the pole-arc may be made equal to $\frac{2}{3}$rds of the pole-pitch without introducing any appreciable amount of differential action. This, accordingly, is the generally adopted value for the ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$ in *three-phase* alternators, and it is about the same as that commonly employed in continuous-current machines.

§ 45. Types of Alternators. Peripheral Speed

Until about the beginning of this century, the only types of alternators in general use were low and medium speed machines intended for direct coupling to engines of the reciprocating type. The peripheral speed in machines of this class generally ranges from about 5000 ft./min. to about 9000 ft./min. (about 25 metres/sec. to 45 m./sec.). Since the advent of the steam turbine, however, a new development has taken place in the construction of alternators, resulting in the introduction of a type of generator designed to run at much higher peripheral speeds—12,000 ft./min. to 20,000 ft./min. (about 60 m./sec. to 100 m./sec.).*

Turbo-generating sets present the following advantages when large outputs are in question:—(1) the cost of the generating plant is less than with reciprocating engines; (2) a very large saving in space is effected, with consequent large reduction in the capital expenditure on buildings; (3) in the case of large generating sets (above 1000 kw.) a turbine set has the additional advantage of higher efficiency.

§ 46. Standard Type of Low Speed Alternator

The standard modern type of low and medium speed alternator is one in which the armature is stationary and the field revolves. The general construction of this type will be understood from Fig. 69. The armature laminations are supported by a casting which forms the "yoke-ring," and which is generally divided into two sections, an

* The first turbo-alternators (75 kw.) were constructed in 1889 by Messrs. C. A. Parsons & Co. Their peripheral speed was about 9400 ft./min.

upper and a lower, bolted together.* The lower s
vided with two feet, by means of which it rests on
plate. In very large machines, however, the yoke-rin
four sections. This ring is provided with numerou

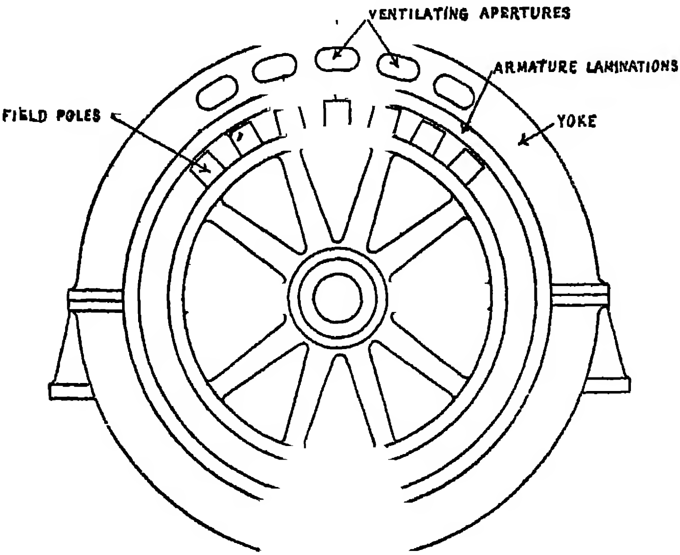


FIG. 69.—General Arrangement of Revolving Field Alternator.

apertures, only a few of which are shown in Fig. 69. The revolving field consists of a fly-wheel or spider, supporting a ring to which are bolted the field poles. In large machines the ring is a separate structure, bolted to the spider arms; or the boss of the spider is

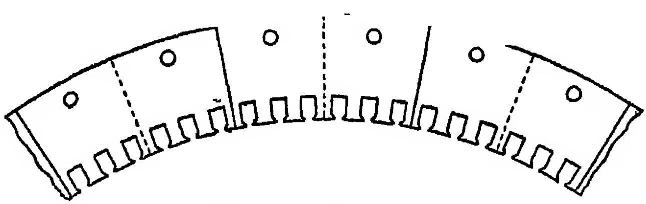


FIG. 70.—Method of building up Armature Core.

split, and after having been bored out to fit the shaft, is strengthened by two steel rings, which are shrunk on. Either of these expedients

* So long as its diameter does not exceed about 8 ft., the yoke ring may consist of a single casting

ensures freedom from any dangerous strains which may be set up in the arms of a large wheel during cooling when cast in one piece.

In order to render the armature ring more rigid, the laminations are arranged to overlap or break joint, as shown in Fig. 70, where the

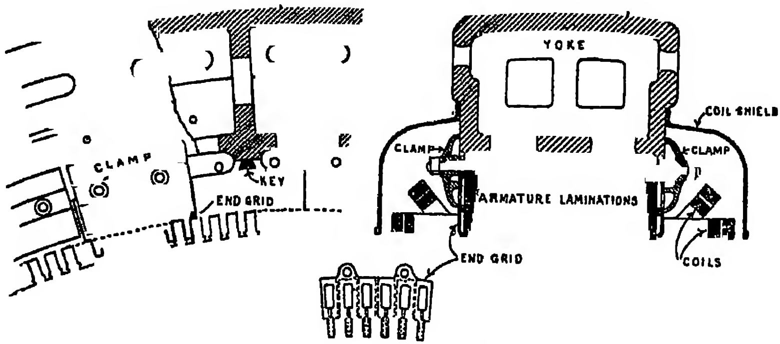


Fig. 71.—Method of supporting Armature Laminations.

full radial lines indicate the laminations of one layer, and the dotted lines those of the next. The thickness of the core-plates or laminations is from 0·014 inch to 0·018 inch.

There are various methods of supporting the laminations from the yoke-ring. One of these is illustrated in Fig. 71. As will be

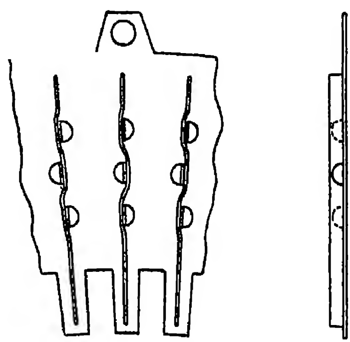


Fig. 72.—Ventilating Space-block.

seen, the laminations are held together by means of bolts between two sets of clamps, end grids being interposed between the clamps and plates. These end grids are provided with teeth, which fit over the teeth of the plates or laminations, and prevent them from spreading outwards. The core-plates are, in the design of Fig. 71, provided

with key-way projections, which fit over keys screwed to the yoke-ring. The end grids serve to ventilate the core, and additional ventilation is secured by providing gaps, or ventilating ducts (see Fig. 73), in the core, these gaps being formed by the interposition of ventilating space-blocks. One of the simplest forms of space-block is shown in Fig. 72. It consists of a steel plate, about $\frac{1}{32}$ inch thick,

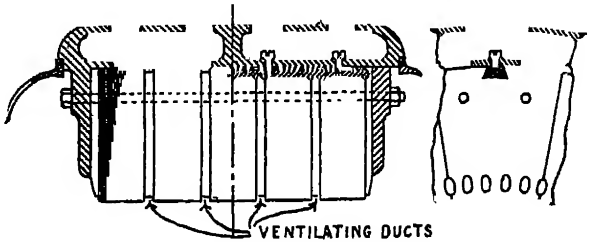


FIG. 73.—Armature Core Construction.

shaped like one of the core-plates, and having semicircular pieces cut and bent at right angles to the plane of the plate. These support bent radial steel strips as shown, which act as distance-pieces between the core plates. The width of ventilating duct varies from about $\frac{1}{3}$ inch to $\frac{3}{4}$ inch. It is usual to provide a ventilating duct to every 5 inches or 6 inches of core length.

Another mode of supporting the core-plates is shown in Fig. 73. Instead of having clamps on both sides, there is a ring cast on to the yoke, which acts as a clamping-ring. The core-plates are further secured by means of keys screwed to the yoke casting as shown.

In Fig. 74 is shown a similar construction for a very narrow core, in which case the keys may be dispensed with.

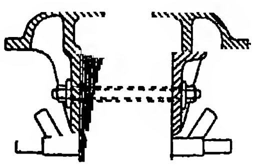


FIG 74.—Type of Construction for Narrow Armature.

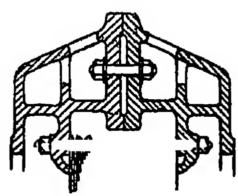


FIG. 75.—Yoke Construction.

A somewhat different design is illustrated in Fig. 75. Here the yoke-ring itself is divided by a plane normal to the shaft, the two halves of the ring forming clamps for the core-plates.

Three methods of securing the field-poles to the rim of the field-wheel are shown in Figs. 76, 77 and 78. In Fig. 76 each pole is

secured by two bolts passing through the rim, while in Fig. 77 additional safety is obtained by providing the rim with a clamping projection on one side and the field-pole with a similar projection on

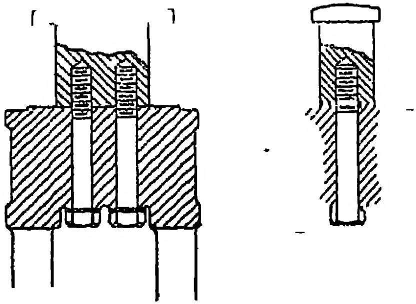
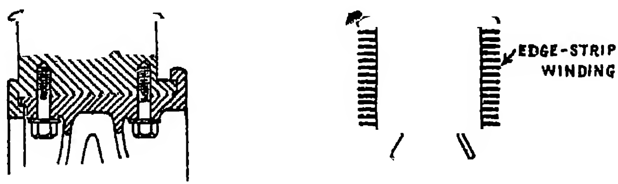


FIG. 76.—Field-pole Construction.

the other. For very high peripheral velocities, the field-poles may be dovetailed into the rim as in Fig. 78, and secured by means of wedges.

Owing to the high peripheral velocities common in alternators,



FIGS. 77 and 78.—Methods of supporting Field-poles.

ordinary wire winding is inadmissible for the field-poles, as the wires would roll over each other. For this reason, the usual form of field winding consists of thin strips of copper wound on edge in one or two

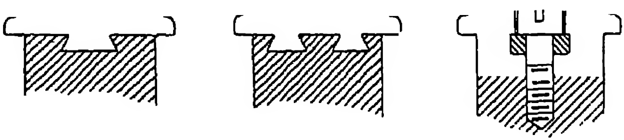


FIG 79.—Types of Laminated Pole-shoes.

layers. This kind of winding is known as *edge-strip winding*, and is shown diagrammatically in Fig. 78. Besides being mechanically strong, it possesses the further advantage of rapidly conducting the

heat to the outside of the coil, and so maintaining the coil at a uniform temperature, thereby preventing excessive rises of temperature in the interior of the coil.

The field winding of an alternator is generally designed for an excitation voltage not exceeding 240 volts.

In order to prevent excessive eddy-current loss in the ends of the

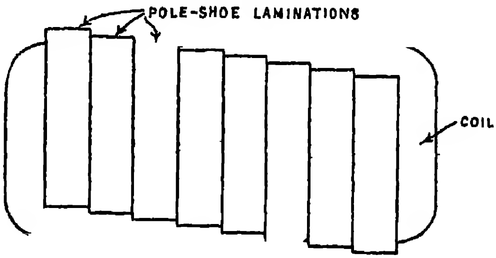


FIG. 80 —Showing construction of Skewed Pole-shoe

pole-pieces, these are frequently laminated. In Fig. 79 are shown various ways of securing the laminated pole-shoes to the field-cores.

For the purpose of securing a smooth wave-shape forming a close approximation to a sine wave, the Oerlikon Co. employs laminated pole-shoes, consisting of a number of sets of core-plates displaced relatively to each other, or "staggered," so as to form a series of steps, as shown in Fig. 80.

§ 47. Construction of Turbo-Alternators

The construction of the rotating field-magnets of turbo-alternators presents considerable difficulties, owing to the high centrifugal stresses which have to be dealt with, and the special arrangements which have to be provided for maintaining accurate balance and securing effective ventilation.

At the standard frequency of 50, the highest possible angular velocity—viz. that corresponding to a two-pole field—is 3000 revs. per min. The two-pole type of construction may be used for outputs up to about 20,000 kw. For larger machines * a four-pole field is used, the angular speed being 1500 r.p.m. A six-pole field—corresponding to 1000 r.p.m.—is only employed in machines of exceptionally large output.

In the early designs attempts were made to adapt the existing types of construction, which were primarily intended for low-speed machines, to the new conditions, with suitable modifications. Accordingly, in these early designs the rotors were provided with salient

* Up to about 40,000 kw.

poles, each radial pole being surrounded by a single field coil. Such rotors were found to present very serious mechanical difficulties and to run very noisily; further, they did not allow of the best utilization of the space available for the active materials. The *salient-pole* rotor has accordingly, in modern designs, been entirely displaced by the *cylindrical* rotor. In this, the winding is *distributed*, being contained in a suitable number of slots in the cylindrical rotor core. One example of this type of construction is shown in Fig. 81, which shows a two-pole rotor.

The core of this rotor consists of a cylindrical forging (the shaft forming part of the forging), with a number of grooves milled

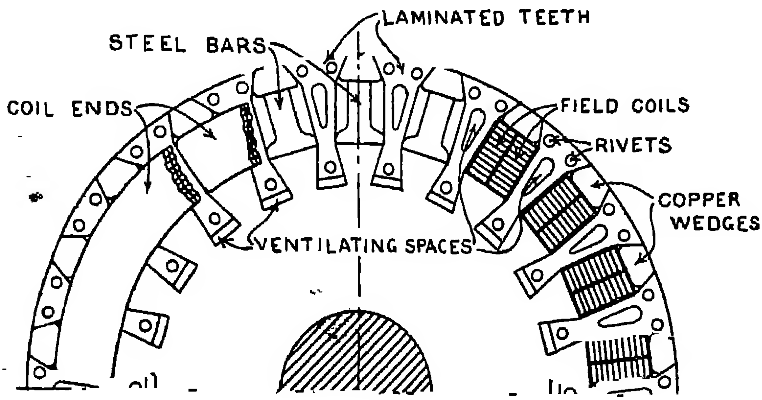


FIG. 81.—Cylindrical Two-pole Rotor.

out in its surface into which are dovetailed the laminated teeth which form the polar projections. The teeth are built up of stampings held together by rivets as shown. The axial length of each set of stampings is about 6 inches, and the rivet heads act as distance-pieces, so that there is a ventilating duct about every $6\frac{1}{2}$ inches along the core. In addition to these radial ducts, there are two sets of ducts in a direction parallel to the shaft which communicate with the radial ducts. One set consists of the spaces separating the bases of the teeth from the bottoms of the grooves into which they are slid; while the second set consists of central openings in the teeth themselves. The coils, after having been subjected to hydraulic pressure, are placed in position on the core, and the various sections of stampings are then slid into position. Copper wedges are then driven into the spaces above the coils, forcing the coils inwards and the teeth outwards. In the middle region of the polar surface are slots containing steel bars in place of coils. The coil ends are

supported by extensions of the core body, and are provided with a binding of wire; over this binding is fitted a ring of bronze to give additional strength, and also to serve as a connecting ring for the copper wedges, which are thus made to constitute a damping winding (§ 84).

In the most recent developments of the turbo-alternator there has been a steady tendency to raise the peripheral speed, which in some of the latest designs has as high a value as 25,000 ft. per min.

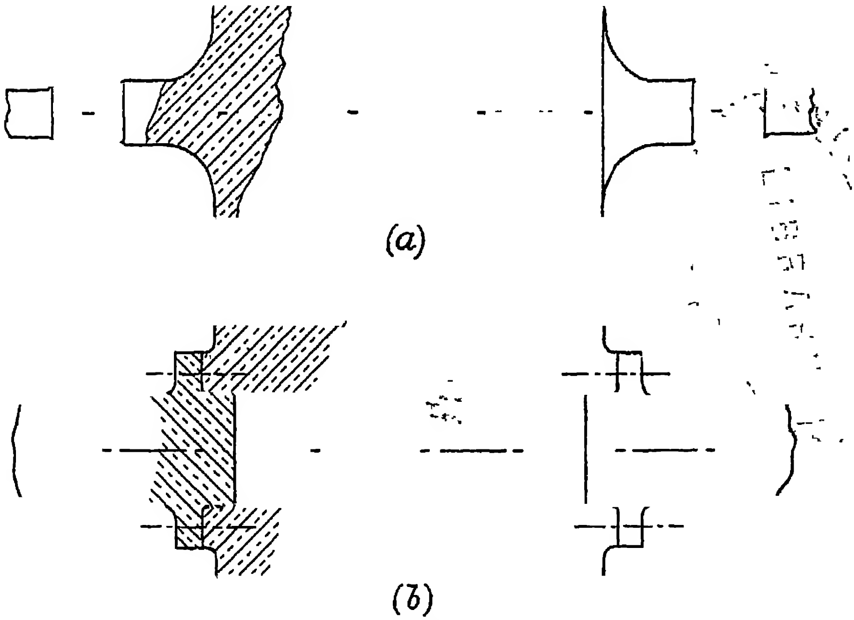


FIG. 82 —Typical forms of Turbo-Alternator Rotor.

(128m. per sec.), and to adopt the simplest possible type of construction for the rotor. In Fig. 82 are shown diagrammatically two typical forms of rotor construction. In the one (*a*), the rotor body consists of a single forging of high-grade carbon (·38 per cent.) steel, the shaft being formed by the reduced ends of the forging. This type of rotor would be used for the more usual outputs. For exceptionally large machines, the type shown in Fig. 82 (*b*) would be used. Here we have a hollow rotor core secured by bolts to end shafts which do not pass through the interior of the core.

In Fig. 83 (*a*) is shown a typical slot of a turbo alternator rotor. Beneath the portion of the slot which contains the conductors is

a ventilating channel through which air is blown. The conductors are secured in position by a strong wedge of bronze or steel.

Besides those already described, another form of rotor has been used. In this the core consists of *thick* sheets (about 2¼ ins.) without any central opening, rabbeted into each other and held together by bolts. The journals are forged solid with the end plates.

One of the most difficult mechanical problems in connection with the turbo-alternator rotor has been the provision of adequate support for the projecting coil-ends. For this purpose a structure known as an *end-bell* is used. In many cases the form of construction shown diagrammatically in Fig. 83 (*b*) is adopted. Here the end-bell takes the form of a ring centred on the rotor core at one end and on a disc mounted on the rotor shaft at the other. A more satisfactory form,

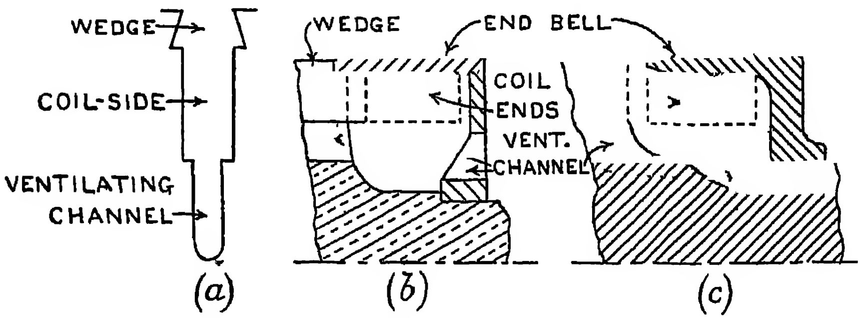


FIG. 83.—(*a*) Rotor Slot; (*b*) and (*c*) End-bell Designs.

advocated by Dr. S. F. Barclay,* and shown diagrammatically in Fig. 83 (*c*), is that in which the ring and disc are combined into a single cup-shaped piece mounted on the rotor shaft. End-bells are made of either steel or manganese bronze.

The general design of the stator of a turbo-alternator does not differ greatly from that of a low-speed alternator. In most cases, it becomes necessary to provide strong supports for the coil-ends, in order to prevent their being bent and damaged by the abnormal stresses which are called into play when an accidental short-circuit takes place. Such supports generally take the form of clamps or brackets bolted to the stator frame, the coil ends being bedded against the brackets (§ 50).

The ventilation of turbo-alternators is a difficult problem, as the cooling surface available per watt dissipated is much less than in the case of low-speed machines. The difficulty of effective ventilation increases rapidly with the output of the machine. In many cases

* *Journal of the Institution of Electrical Engineers*, vol lvi p. 486 (1918).

the cooling is effected by a strong blast of cool air (taken from outside the engine-room) which is maintained through the various ventilating ducts in the machine, the air after passing through the machine being conducted outside the engine-room by a special duct underneath the floor. As the outside air generally contains a considerable amount of dust which is liable to be deposited on the windings and to impair the insulation as well as interfere with the efficient cooling of the machine, it is usual to provide suitable filters for the incoming air. The cooling blast is in most cases maintained by fans mounted at the ends of the rotor. In order to simplify the cooling arrangements and do away with the necessity for air filters, the so-called *closed-circuit* system of air cooling has recently been introduced. This is shown diagrammatically in Fig. 84. The same air is used over and over again, so that no filtering or washing of the air is required. The cooling of the hot air as it passes out of the machine is effected by a series of spraying nozzles, and the mechanical carrying over of moisture into the machine is prevented by a series of baffle plates.

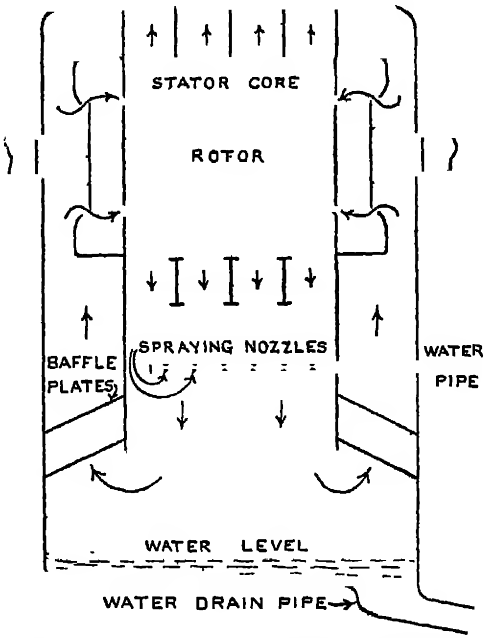


FIG. 84.—Closed Air Circuit System of Cooling Turbo-Alternators.

Attempts have been made to use water as the cooling medium instead of air. In some early designs, the stator frame was provided with a water jacket through which cooling water could be circulated. More recently machines have been constructed in which the rotor as well as the stator is water-cooled.*

* *Journal of the Institution of Electrical Engineers*, vol. lviii. p. 183 (1920).

§ 48. Fire Risk in Turbo-Alternators

Owing to the extremely high velocity with which the air is forced through the ventilating passages of a turbo-alternator, and the fact that the large kinetic energy stored up in the rotor (which generally carries the fan blades) enables it to run on for a very long time (½ hour to over 1 hour in large machines) and maintain a powerful fanning action after steam has been cut off, the occurrence of a fire in such a machine is very liable to prove extremely disastrous. The construction of the machine should therefore be carried out, as far as possible, with fire-proof materials, and provision should be made for extinguishing the fire in the shortest possible time. Arrangements are accordingly provided for shutting off the air supply and admitting some gas incapable of supporting combustion into the enclosing case of the alternator. Steam and carbon dioxide have been used for this purpose, and carbon tetrachloride has also been suggested.

§ 49. Armature Windings of Alternators

As regards the armature winding, by far the greatest number of alternators are provided with six slots per pole (though sometimes as many as twelve slots per pole are used). One advantage of choosing

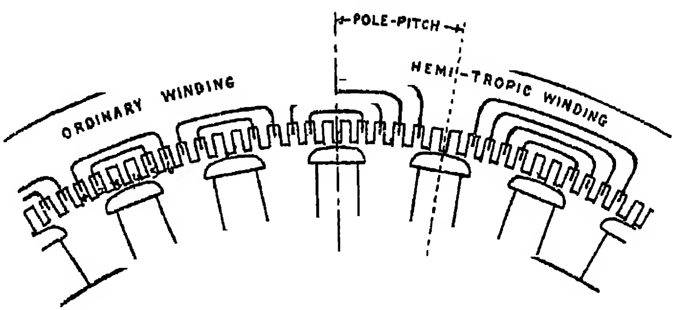


FIG 85.—Illustrating Two Types of Armature Winding.

this number lies in the possibility of standardizing the core-plates, so that the same core may be used for single-, two-, or three-phase machines. If the machine is to be a single-phaser, only three or four slots per pole would be utilized for the armature winding, the remaining slots being left empty. These vacant slots tend to improve the ventilation of the machine, and also offer the further advantage that a supplementary winding may, if necessary, be placed in them, thus

enabling the machine to supply a small two-phase load in addition to the main single-phase load.*

If we agree to use four out of the six slots per pole for a single-phase winding, then there are two ways of arranging the armature coils. These two methods are shown in Fig. 85. The first method, which we may describe as the ordinary method, consists in using *one coil per pole*, each coil being distributed over four slots. This method is illustrated by the left-hand side of Fig. 85. The second method, for which Prof. S. P. Thompson has suggested the term *hemi-tropic winding*, consists in using one coil per *pair* of poles. Each coil is now embedded in eight slots, as shown by the right-hand side of Fig. 85. If we consider only a single-phase winding, then it is immediately evident from Fig. 85 that the hemi-tropic winding, by reason of the greater mean length of turn, requires more copper and leads to a higher armature resistance than the ordinary winding. On the other hand, it offers a certain advantage in the case of three-phase machines, as will be seen presently.

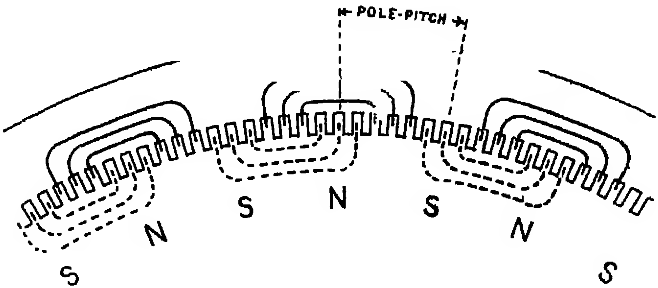


FIG. 86.—Two-phase Armature Winding.

Considering next a two-phase armature winding, and assuming as before six slots per pole, we have three slots available per pole per phase. The winding must therefore necessarily be hemi-tropic, as shown in Fig. 86, in which the winding of one phase is shown by the full lines, and that of the other by the dotted lines. The positions of the field-poles are indicated by the letters N and S. Owing to the overlapping of the coils, the coil-ends of one phase are brought out straight, and lie on a cylindrical surface (as in the typical modern

* If a single-phase generating station is required to supply current to a short tramway line, the transformation from single-phase to continuous current may be effected by means of a synchronous motor-generator set. A single-phase rotary converter, on account of its unsatisfactory performance, would not be used. But two-phase converters have been used for this purpose, the alternating currents corresponding to the second phase being obtained from the supplementary winding in one or more of the main generators, this winding being placed in some of the unused slots.

continuous-current armature with a barrel or "straight-out" winding), while those of the other are bent up so as to lie on a conical surface. The bent-up ends of the armature coils are shown in Fig. 88.

If the winding is to be a three-phase one, we have, assuming as before six slots per pole, two slots available per pole per phase. The winding might therefore be either of the ordinary or of the hemi-tropic type. The adoption of the ordinary winding (with one-half of a coil in a single slot) would, however, necessitate a somewhat complicated arrangement of coil-ends, as these would have to be arranged so as to lie on three different surfaces, a different surface corresponding to each phase. By adopting the hemi-tropic type of winding, shown in

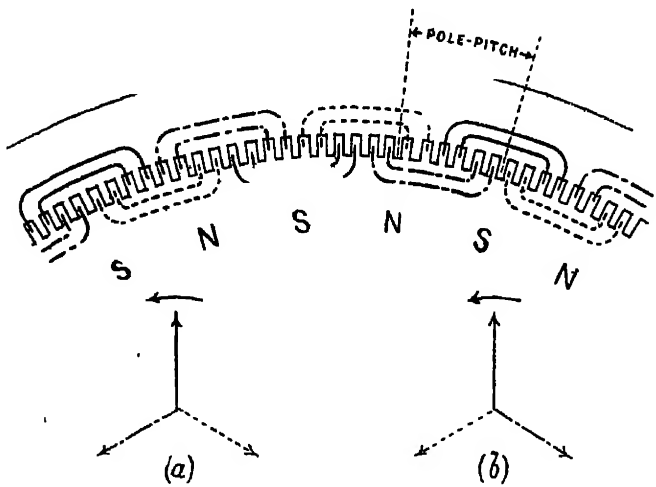


FIG. 87.—Three-phase Armature Windings.

Fig. 87, this difficulty is overcome, since the coil-ends may now be arranged to lie on two surfaces only (as with the two-phase winding), the coils being alternately straight and bent-up. The three phases in Fig. 87 are distinguished by full, dotted, and chain-dotted lines. It will be noticed that the coils belonging to any one phase are alternately straight and bent.

In the two-phase winding, the distance apart of two neighbouring coils belonging to different phases amounts to $\frac{1}{2}$ the pole-pitch (corresponding to a phase difference of $\frac{\pi}{2}$, or 90°), and in the three-phase winding to $\frac{1}{3}$ the pole-pitch. Now, a distance of $\frac{1}{3}$ the pole-pitch corresponds to a phase difference of only 60°. Thus,

if we consider three consecutive coils, A, B, and C, there will be a phase difference of 60° between the e.m.f.s in A and B, and B and C, and a phase difference of 120° between the e.m f.s in A and C. But by reversing the connections of B we alter the phase of its e.m.f. relatively to the other two by 180°, and so obtain three e.m.f.s differing 120° in phase, as required for a three-phase system.

A little consideration will show that if the direction of rotation of the field in Fig. 87 is counter-clockwise, or left-handed, the vector diagram of e.m f.s in the three phases is as shown at (*a*). If now the direction of rotation be reversed, the diagram assumes the form shown at (*b*), the dotted and chain-dotted vectors having changed places. Thus we see that a reversal of the direction of rotation of a three-phase generator is equivalent, so far as the external circuit is concerned, to interchanging the connections of two of the terminals.

From a manufacturing point of view, armature coils may be divided into two classes: former-wound and hand-wound. Former-wound coils require entirely open slots, whereas hand-wound coils may be placed in closed or semi-closed slots. Former-wound coils are cheaper, and are more easily replaced than hand-wound coils On the other hand, in fitting a former-wound coil into position there is some risk of damaging the insulation of the coil or slots. The use of open slots results in an irregular wave-shape (unless some such expedient as skewing the pole-pieces is adopted, see Fig. 80), and necessitates, as a rule, the use of laminated pole-shoes in order to prevent excessive eddy-current loss. Owing to the fact that thoroughly reliable insulation is more easily obtained with hand-wound coils (the insulating lining of the slot in this case consisting of a seamless tube), this type would appear to be preferable in the case of generators designed for very high voltages.

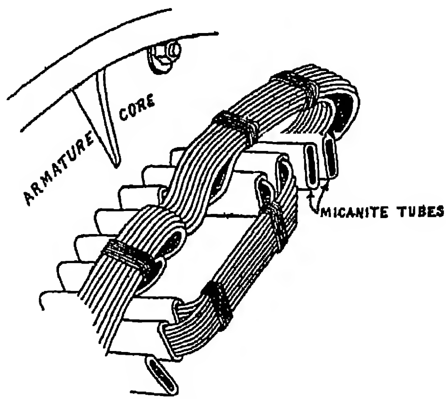


FIG. 88.—Showing Armature Coils in Pos

The insulating lining of the slots consists of some form of mica insulation, such as micanite or "reconstructed mica," and in the case of hand-wound coils seamless tubes of the insulating material are fitted into the slots, as shown in Fig. 88. In order to prevent disruptive discharge between the coils and the frame, these tubes are carried well beyond the core-ends, as shown in the illustration.

Since the end coils—*i.e.* those nearest the machine terminals—are subject to the greatest dielectric stress, it is advisable to provide these coils with additional insulation.

The actual thickness of the slot insulation depends, of course, on the voltage of the machine, and, according to Mr. H. M. Hobart,* the following table may be taken as representing good practice:—

R.m.s. voltage of alternator ...	500	1000	2000	4000	8000	12,000	16,000
Thickness of slot insulation, in mm.	1·30	1·75	2·47	3·35	4·6	5·67	6·67

The insulation thickness in the above table is measured from copper to iron, and includes the insulation of the conductors.

Since with increasing voltage a greater thickness of insulation becomes necessary, either the size of slot must be increased—and this will involve an increase in the diameter of the machine—or an equivalent increase must be made in the length of core. High-voltage generators are therefore, for a given power and speed, larger and more expensive than low-voltage ones. Nevertheless, the extra cost of a high-voltage generator is much less than that of the step-up transformers (§ 58) which would be necessary with a generator of low voltage.

§ 50. Methods of Supporting Coil Ends

The projecting coil ends of alternators of moderate size are generally arranged to be self-supporting—*i.e.*, there is no special mechanical support provided for them, and the stiffness of the coils themselves is depended upon to prevent their being bent by any mechanical forces which may be called into play. It must be remembered that the coil ends represent conductors conveying currents and placed in a magnetic field (that due to the neighbouring coil ends), and are hence subject to mechanical forces which increase as the *square* of the current (the magnetic field being proportional to the current). Although under normal working conditions such forces are practically insignificant, they may reach enormous values when certain abnormal conditions arise—such as the sudden short-circuit of a fully-excited alternator (§ 111). Experience with alternators of large output, and especially with turbo-alternators, has shown the absolute necessity of providing adequate mechanical support for the coil ends. One of the most satisfactory methods of doing this is shown in Fig. 89. The winding is of the "barrel"

* *Electrical Review*, vol. lvi. p. 681 (1905).

type, the projecting ends of the conductors being ar
each other in two layers on a conical surface. Two sup
one being external and the other internal, provide th

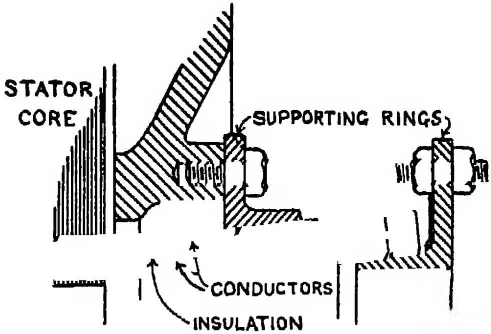


Fig. 89.—Arrangement of Coil Ends.

bracing for the coils. Fig. 90 shows an arrangement used by the Metropolitan-Vickers Co., Ltd. The coil ends are disposed in two

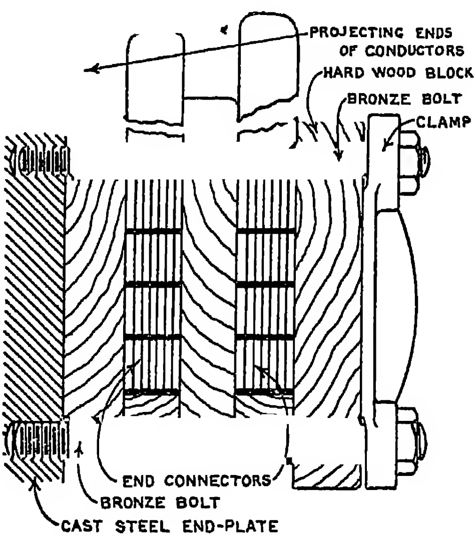


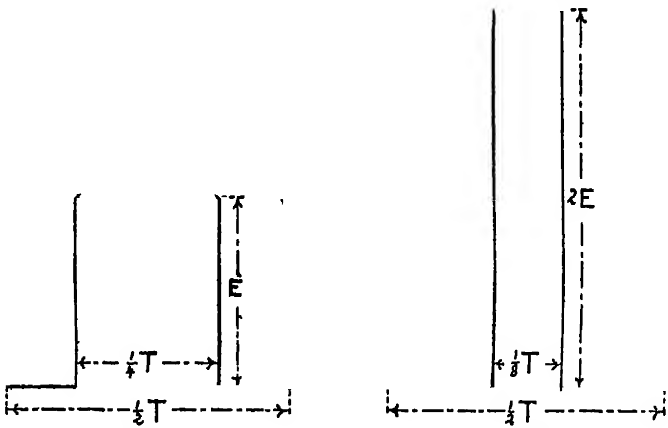
Fig. 90.—Method of securing Coil Ends.

ranges, and, by the aid of hard wood spacing-blocks, are securely clamped to a steel end-plate by means of bronze bolts. These coil ends or connectors form an independent set of conductors which,

after being secured in position, are joined to the straight conductors embedded in the stator core, the joints being carefully insulated.*

§ 51. Calculation of Armature e.m.f. Effect of Varying Length of Polar Arc

The calculation of the e.m.f. of an alternator is a more difficult and more uncertain matter than the calculation of the e.m.f. of a continuous-current dynamo. This arises from the fact that whereas in the latter case the e.m.f. (for a given speed) depends simply on the



(a) (b)

FIG. 91.—To illustrate Effect of Pole-width on E.M.F.

flux per pole and the number of conductors in series with each other between the brushes, in an alternator the r.m.s. value of the e.m.f. depends also on the ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$ and the number of slots per pole per phase. This will be evident from the following considerations.

Let the ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$ be $\frac{1}{2}$, and let us, for the sake of simplicity, suppose that the field is perfectly uniform under the pole-shoe, and ceases abruptly (without the formation of a magnetic "fringe") immediately outside it. It is then evident that the positive half-wave of e.m.f. induced in a single conductor is of the shape shown in Fig. 91 (*a*),

* For a very full account of the different methods of supporting the coil ends of large alternators, see a paper by Mr. Miles Walker, and discussion thereon, in the *Journal of the Institution of Electrical Engineers*, vol. xlv. p. 295 (1910).

the e.m.f. having a zero value during the first eighth of a period, a constant value, say E, during the next quarter of a period, and then suddenly falling to a zero value, which is maintained during the remainder of the half-period. It is easy to show that the r.m.s. value of the e.m.f. is $\frac{1}{\sqrt{2}}$E. Suppose next that the pole width is reduced to half its original value, the total flux remaining unaltered. On the same assumptions as before, the e.m.f. wave now takes the form shown in Fig. 91 (*b*). The r.m.s. value of the e.m.f. is now E instead of $\frac{1}{\sqrt{2}}$ E as in the first case, notwithstanding the fact that the total flux per pole has remained unaltered, and that the *arithmetic mean* of the e.m.f. in the conductor is the same as before. Thus for a given flux per pole, the r.m.s. value of the e.m.f. is increased by reducing the ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$. From this one might at first sight conclude that there is a great advantage in making the pole-pieces as narrow as possible. It must be remembered, however, that *for a given flux* a reduction in the polar area involves an increase of field intensity in the gap, and this means an increase in the field copper. Beyond a certain point, therefore, the increase of e.m.f. per conductor and consequent decrease of armature copper would be more than counterbalanced by the increase of field copper. The gap induction in modern alternators ranges from 6000 to 9000 C.G.S. lines per sq. cm.

§ 52. Effect of Varying Number of Slots

We have next to consider the effect of varying the number of slots in which the coil is embedded. If each side of a coil occupies only a single slot, the e.m.f.s of the various conductors may be practically taken to be in phase with each other, so that the e.m.f. per coil is equal to the e.m.f. per turn, multiplied by the number of turns in the coil. But by distributing the coil over a larger number of slots, we introduce phase differences between the e.m.f.s of conductors lying in different slots, and the addition of the e.m.f.s of the various groups of conductors must be carried out *vectorially* instead of arithmetically. The vectorial sum being, in general, numerically less than the arithmetical one, an increase in the number of slots per pole per phase results in a lowering of the e.m.f. (other things remaining the same). In spite of this disadvantage of a distributed as compared with a uni-slot or concentrated winding, distributed windings are invariably used nowadays, as they tend to reduce armature reaction and give rise to a smoother e.m.f. wave.

If the shape and size of the pole-shoe are such as to give rise to a distribution of the flux in the gap according to the simple sine law, the value of the e.m.f. in any given case is easily calculated. By way of example, we shall calculate the e.m.f. in one phase of a three-phase generator having six slots per pole, or two slots per pole per phase.

Let Φ = flux per pole, f = frequency, and Z = conductors per phase. Since each conductor cuts 2Φ lines per period, the arithmetic mean value of the e.m.f. induced in it is $2\Phi f$ (in C.G.S. units). The form factor of a sine wave being 1·11 (§ 3), the r.m.s. value, in volts, of the e.m.f. in each conductor is—

$$2{\cdot}22\Phi f \,.\, 10^{-8}$$

The conductors of each phase may be divided into two sets (corresponding to the two slots per pole per phase), such that the e.m.f.s of all the conductors in one set are in phase with each other. There being $\frac{1}{2}Z$ conductors in each set, the r.m.s. value per set is $1{\cdot}11Z\Phi f \,.\, 10^{-8}$. Now, since the distance apart of two neighbouring slots is $\frac{1}{6}$ of the pole-pitch, the phase difference between the e.m.f.s in the two sets of conductors is $\frac{1}{6} \,.\, 180°$, or 30°. Hence, compounding the e.m.f.s of the two sets vectorially, we find for the r.m.s. value of the e.m.f. per phase—

$$2 \,.\, 1{\cdot}11Z\Phi f \,.\, 10^{-8} \,.\, \cos 15° = 2{\cdot}14Z\Phi f \,.\, 10^{-8}\,*$$

In practice, however, the flux is not distributed according to the simple sine law, but is mainly concentrated under the polar arc, and in accordance with the explanation already given regarding the effect of narrowing the pole-piece, such a concentration of flux results in a *higher* value of the e.m.f. than that calculated on the assumption of a sine distribution.

§ 53. General Formula for e.m.f. of Alternator

The formula for the e.m.f. of each phase of an alternator may be written in the form—

$$E = kZ\Phi f \,.\, 10^{-8}$$

where k is a factor depending on the ratio $\dfrac{\text{pole-arc}}{\text{pole-pitch}}$, on the number of slots per pole per phase, and on the shape of the pole-shoe.

The earliest attempt to calculate the value of k for various cases is due to Kapp, who, in 1889, determined k for various kinds of windings and various values of the ratio $\dfrac{\text{pole-arc}}{\text{pole-pitch}}$, on the assumption

* The e.m.f. between two machine *terminals* is, for a star-connected winding, equal to 1·73 times the phase e.m.f. (§ 17).

that the field in the gap is uniform, and ceases abruptly at the edges of the pole-shoe. According to this assumption, the e.m.f. wave induced in each conductor is of the rectangular form shown in Fig. 98. The e.m.f. wave of the entire phase is obtained by superposing the waves corresponding to the various sets of conductors into which the winding of the phase may be divided; each set of conductors consisting of those conductors whose e.m.f.s are coincident in phase.

The assumption of a rectangular wave-shape for each conductor is, however, unjustifiable, on account of the formation of a magnetic fringe around the polar edges. Owing to the greater spreading of the flux due to the formation of this fringe, the actual e.m.f. is *less* than that calculated on the rectangular wave assumption.

More reliable results are obtained by taking into account the effect due to the fringe, as was first done by Mr. C. C. Hawkins in 1900.* The following table gives the values of k calculated by Mr. Hawkins, for the case of two slots per pole per phase, the slots being spaced $\frac{1}{6}$th of the pole-pitch apart:—

$\frac{\text{pole-arc}}{\text{pole-pitch}}$ =	0·30	0·50	0·70	0·90
k =	2·48	2 26	2·10	1·93

The above values are based on the assumptions of an air-gap length (single) equal to $\frac{1}{20}$th of the pole-pitch, of sharp rectangular polar edges, and of a width of pole-shoe equal to the width of pole or field-core.

The second of the above assumptions does not in general apply to practical cases, since the edges of the pole-shoe are always more or less rounded. Now if, retaining the same flux per pole and the same length of polar arc, we round off the edges of the pole-shoe, we thereby cause a greater concentration of the flux towards the middle portion of the pole-shoe, and a greater concentration of flux is, as we have seen (§ 50), accompanied by a rise of e.m.f. Thus for pole-shoes with rounded corners the value of k will be greater than that given by the above table.

A. Müller † gives the following values of k for two slots per pole per phase, the slots being $\frac{1}{6}$th of the pole-pitch apart:—

$\frac{\text{pole-arc}}{\text{pole-pitch}}$ =	0·4	0·5	0 6	0·7
k =	2·58	2·42	2·3	2·17

These values are based on the following assumptions: single air-gap length = $\frac{1}{50}$th of pole-pitch; radius of curvature of rounded pole-shoe corner = $\frac{1}{20}$th of pole-arc; height of pole-shoe = $\frac{1}{6}$th of pole-arc.

* *Electrical Review*, vol xlvii p 655 (1900).

† *Zeitschrift für Elektrotechnik* (Wien), vol. xxiii. p. 31 (1905).

§ 54. Armature Reaction in Alternators

When an alternator is loaded, the armature current reacts on the field, distorting and weakening (or, sometimes, strengthening) it. The nature of the armature reaction depends not only on the armature current, but also on its phase relation to the open-circuit or no-load e.m.f. The calculation of the exact effect produced by the armature current in any given case is an extremely laborious matter, and if accuracy is desired, must be carried out separately for each particular type of machine. But the general nature of the effects produced may be easily inferred from the following very simple considerations.

The currents in the armature windings of a polyphase generator may be assumed to give rise to a rotating wave of magnetic flux (§ 22). Since this wave travels around the stator periphery at the same speed as the rotating field-poles,* it will be stationary with respect to the poles. Its position relatively to them will clearly depend on the amount by which the armature current lags or leads, and the nature of the armature reaction will best be understood by considering the three cases of (1) zero phase difference, (2) lag of 90°, and (3) lead of 90°.

When the current is in phase with the e.m.f. in each phase of the winding, a reference to Figs. 86 or 87 shows that the crest of the rotating wave of flux falls exactly halfway between two pole-pieces, since at the instant when one of the component alternating waves reaches its maximum amplitude, it coincides in position with the resultant rotating wave (see concluding remarks of § 22). The relative position of the field-poles and the rotating wave of magnetic flux is shown in Fig. 92 (*a*); the field being supposed to rotate counter-clockwise as indicated by the arrow, and the radially outward direction of the flux being regarded as positive. It is evident at once that the effect in this case is a purely distorting or cross-magnetizing one (such as occurs in a continuous-current dynamo whose brushes have no lead), the field being weakened on the advancing side of each pole-piece, and strengthened on the other side. When the current lags by 90°, the rotating flux wave takes up the position shown in Fig. 92 (*b*), and it is at once evident that there is a powerful demagnetizing effect, whereas with a lead of 90°, Fig. 92 (*c*), there is an equally strong

* The speed of the rotating wave of magnetic flux is (§ 21) $\frac{\lambda}{T}$, λ being the wave-length and T the period. The wave-length, however, corresponds to twice the pole-pitch; hence $\frac{\lambda}{T}$ is also the speed of the field-poles, each field-pole advancing through a distance equal to twice the pole-pitch during a period.

magnetizing effect. For any intermediate value of the phase difference, the distorting and demagnetizing (or, for a leading current, magnetizing) effects are both present.

In the case of a single-phase alternator, the armature current gives

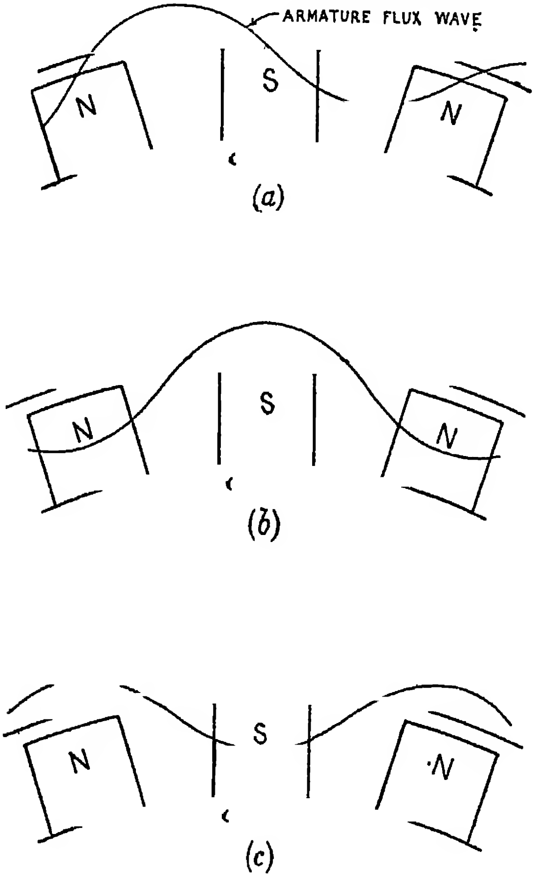


FIG. 92.—To illustrate Armature Reaction in Alternators (*a*) Current in Phase with e.m f., (*b*) Current lagging by 90°, (*c*) Current leading by 90°.

rise to a simple alternating flux wave (§ 20). Such a wave may, however, be resolved into two rotating waves (§ 21), travelling with equal speeds in opposite directions. One of these waves will be stationary with respect to the field-poles, and will produce the various effects (according to the phase difference of the current) already considered, while the other will move relatively to the field-poles with

a speed equal to twice that of the poles themselves. Since the effect of this second rotating wave alternates from a magnetizing to a demagnetizing one as it sweeps past the poles, its mean reaction is zero, and it merely reduces the efficiency of the machine by causing additional eddy-current losses.

§ 55. Some Technical Data relating to Alternators

In modern alternators, having a frequency of 50, the usual limits for the magnetic induction in the various parts of the magnetic circuit are as follows:—

Armature core	8,000 to 11,000
„ teeth	14,000 „ 18,000
Air-gap	8,000 „ 10,000
Field cores (cast steel)	14,000 „ 18,000
Yoke ring (cast steel)	8,000 „ 10,000
„ „ (cast iron)	3,000 „ 4,000

The Hopkinson leakage coefficient generally lies between 1·15 and 1·25.

The armature ampere-turns per pole per phase do not generally exceed 2000 in low-speed machines, but may be as high as 8000 in turbo-alternators.

The pole-pitch varies from about 10 to about 20 inches (roughly, 25 cm. to 50 cm.) in ordinary machines, and from 24 to 36 inches (roughly, 60 to 90 cm.) in turbo-alternators. The ratio $\frac{\text{width of slot}}{\text{pitch of slots}}$ is generally from 0·4 to 0·45, and the maximum slot depth does not exceed about 2 inches (5 cm.). The radial length of a field core (exclusive of pole-shoe) is from ·8 to ·9 of the pole-arc.

The air-gap generally absorbs about 75 per cent. of the total m.m.f. In a low-speed machine, the bulk of the remainder is absorbed by the field poles, and in a cylindrical rotor type turbo-alternator, by the rotor teeth.

The air-gap is so chosen as to make the ratio

$$\frac{\text{full-load field ampere-turns}}{\text{stator ampere-turns}}$$

equal to from 3 to 4, depending on the regulation desired. In low-speed alternators above about 500 k.v.a., the air-gap would not be less than about ·2 inch, while in large turbo-alternators with cylindrical rotors it would be of the order of ·8 to 1·5 inch.

The excitation voltage for alternators is of the order of 100 or 200 volts.

§ 56. Dimensions, Weights, and Prices of Alternators

A formula connecting the dimensions of an alternator with its speed and output was first proposed in 1891 by Esson. This formula is—

$$w = Kld^2m$$

where w = output in *watts* corresponding to a power-factor of *unity*; l = gross length of armature core; d = bore of the stator; m = revolutions per minute; and K = a coefficient depending on the type and output of the machine, and for a given type increasing with the output. We may term K the *output coefficient*, since the higher its value, the larger will be the output of the machine for given dimensions and speed.

If a large number of machines be examined, it will be found that the value of K shows large and irregular variations. Considering three-phase machines above 1000 kilovolt-amperes,* we find that K fluctuates between 0·03 and 0·06 if l and d be expressed in *inches* (if cms. are used, the corresponding limits for K are, roughly, 2×10^{-8} and 4×10^{-8} respectively). As a general rule, K has a smaller value for turbo-alternators than for low-speed machines. For a machine of given dimensions, a single-phase generator has an output which is only about 85 per cent. of that of a polyphase generator.

The weight of ordinary three-phase generators varies from about 1 cwt. per k.v.a. (kilovolt-ampere) for an output of 200 k.v.a. to about 40 lbs. per k.v.a. for an output of 10,000 k.v.a. The weight per k.v.a. in the case of turbo-alternators is, on the other hand, found to vary much less with speed and output, and averages, roughly, about 35 lbs. per k.v.a. Of this total weight, about 85 per cent. represents the weight of the iron or steel part of the machine, and 15 per cent. the weight of the copper.†

The cost of alternators varies from about £2 10*s.* per k.v.a. for an output of 100 k.v.a. to about £1 15*s.* per k.v.a. for very large outputs. The combined cost of steam turbines and turbo-alternators varies from about £10 per k.v.a. for an output of 100 k.v.a. down to as low a figure as £3 per k.v.a. for outputs of a few thousand k.v.a.

* It is usual to express the output of alternators in kilovolt-amperes, so as to avoid introducing the power-factor. The rated output of a machine in *watts* decreases in proportion to the power-factor.

† See A. G. Ellis, *Electrical Engineering*, vol. IV p. 838 (1908), and vol. V. p. 5 (1909).

CHAPTER VII

§ 57. Transformers. Ratio of transformation—§ 58 Constant potential transformer—§ 59. Examples of transformer construction—§ 60. Calculation of e.m f induced in transformer winding—§ 61 Losses in transformer. Core loss—§ 62 Copper losses. Best dimensions of core—§ 63. Heating of transformers Methods of cooling—§ 64. Δ, Y, V and T connections for three-phase transformers. Comparison of single-phase and polyphase transformers for polyphase circuits—§ 65 Auto-transformer or compensator—§ 66. Phase transformers—§ 67 Instrument transformers — § 68. Choking coils — § 69. Dimensions, weights, and prices of transformers.

§ 57. Transformers. Ratio of transformation

ONE of the most important advantages of alternating over continuous currents is the extreme ease with which the transformation from a low to a high voltage, or *vice versâ,* may be accomplished. Such transformations are effected by means of *transformers,* whose efficiency exceeds that of any other known apparatus.

A transformer consists essentially of a laminated iron core surrounded by two windings: a *primary* winding or primary, which is supplied with alternating currents, an alternating magnetic flux being thereby produced in the core; and a *secondary* winding or secondary, in which an alternating e.m.f. is induced by the alternating flux. By connecting the terminals of the secondary to an external circuit, a current may be obtained in this circuit.

In order to reduce the ampere-turns necessary to produce the required magnetization to as low a value as possible, the core of a transformer is arranged so as to form a *closed magnetic circuit, i.e.* the path of the magnetic flux lies entirely in iron, air-gaps being avoided.

By far the greater part of the alternating magnetic flux in a well-designed transformer will become linked with both primary and secondary. A certain small fraction of it, however, representing *magnetic leakage lines,* will become linked with the primary alone. As will be seen at a later stage, it is important in most cases to adopt a form of construction which will reduce the magnetic leakage to the smallest possible amount.

As an approximation, we may provisionally assume that magnetic leakage is negligible. From this it immediately follows that the e.m.f. induced by the alternating flux in each turn of the primary is equal to that induced in each turn of the secondary, so that the ratio of the total primary to the total secondary e.m.f. is simply equal to the ratio of the number of primary turns to the number of secondary turns. Again, in order to avoid excessive loss by heating of the coils, their resistances are always so chosen as to reduce the resistance drop (even at full load) to a very small fraction of the p.d. (in both primary and secondary), so that the p.d.s are nearly equal to the e.m.f.s, and we have, approximately—

$$\frac{\text{primary p.d.}}{\text{secondary p.d.}} = \frac{\text{primary turns}}{\text{secondary turns}}$$

This ratio is sometimes spoken of as the *ratio of transformation*. If, *e.g.*, a transformer is required to transform from 10,000 to 2000 volts, then the primary will have to be wound with five times as many turns as there are in the secondary, and the ratio of transformation will be 5 : 1.

§ 58. Constant Potential Transformer

Let us suppose that the p.d. across the primary is maintained constant. When the secondary circuit is open, the primary current adjusts itself to a value such that the e.m.f. due to the alternating flux just balances (neglecting the very small resistance drop) the primary p.d. Now, this balance of primary p.d. and primary (counter) e.m.f. must also—still neglecting the resistance drop—be maintained when the transformer is loaded, *i.e.* when the secondary circuit is closed. In order, therefore, to maintain the original value of the alternating flux corresponding to the given primary p.d., the resultant ampere-turns must be maintained constant. But this condition obviously implies that in the primary there must, in addition to the current which existed on open secondary circuit, be a further component of current—the *load* component—which has a value such that the ampere-turns due to it just suffice to wipe out the ampere-turns of the secondary, leaving a constant value for the resultant ampere-turns. We may speak of the inoperative primary ampere-turns (those balancing the secondary ampere-turns) as the *load ampere-turns*, and of the open-circuit ampere-turns as the magnetizing ampere-turns.* We then have the relation—

$$\text{prim. load ampere-turns} = \text{second. ampere-turns}$$

or—

$$\text{prim. load current} \times \text{prim. turns} = \text{second. current} \times \text{second. turns}$$

* The no-load ampere-turns of a transformer vary, according to size and frequency, from about 1 to about 5 per cent. of the full-load ampere-turns.

so that—

$$\frac{\text{primary load current}}{\text{secondary current}} = \frac{\text{secondary turns}}{\text{primary turns}}$$

Now, owing to the adoption of a closed magnetic circuit the open-circuit or magnetizing ampere-turns (or resultant ampere-turns of a loaded transformer) form a small fraction of the total primary ampere-turns at full load, so that the load ampere-turns corresponding to full load may be approximately taken to be equal to the total ampere-turns at full load. Hence, using the relation just established, we see that, approximately, *the primary and secondary currents are inversely as the turns in the windings.* For this reason the low-voltage winding will carry a heavier current than the high-voltage winding, and the cross-section of the conductor must be correspondingly greater. In practice the windings are designed so as to contain, roughly, equal amounts of copper.

Combining the result for the ratio of the p.d.s with that for the ratio of the currents, we see that the product p.d. × current has approximately the same value for primary and secondary. In the case of a non-inductive load, this conclusion is also otherwise obvious from the fact that if the losses be negligible, then the primary watts must equal the secondary watts.

A transformer whose low-voltage winding is the primary, is spoken of as a *step-up transformer;* and one whose high-voltage winding is the primary, as a *step-down transformer.*

§ 59. Examples of Transformer Construction

The most common form of construction for single-phase transformers is that illustrated in Fig. 93. The core consists of stampings arranged to form two upright cores connected by yokes at the top and bottom. The stampings are held together by insulated bolts, the positions of which are indicated by the small circles in the figure. In order to avoid butt joints (which introduce a short air-gap into the magnetic circuit and so increase its reluctance), the stampings may be made to overlap alternately at the corners, a layer consisting of two short core-plates and two long yoke-plates (as indicated by the full lines in the figure) being succeeded by another, in which there are two long core-plates and two short yoke-plates (dotted lines in figure). The butt joints in any two consecutive layers thus occur at different places, and the flux is free to pass from one layer to another through the lap joints (of large area, and therefore low reluctance) between different layers.* In order to economize copper as much as

* This principle of construction is not always adhered to It has the disadvantage that the dismantling of the transformer, and the subsequent building up of the core

possible, the coils are generally made circular, the core being given a cross-section which is stepped—as shown in the figure—so as to fit into the coils. In the figure the low-voltage coil is shown next the core, the high-voltage coil being outside. Sometimes, in order to reduce magnetic leakage, the high-voltage coil is wound in two sections, the low-voltage coil being sandwiched in between them. In other cases, each winding is subdivided into a number of very short coils, the sections of the high-voltage winding alternating with those of the low-voltage (the sectional coils of both windings

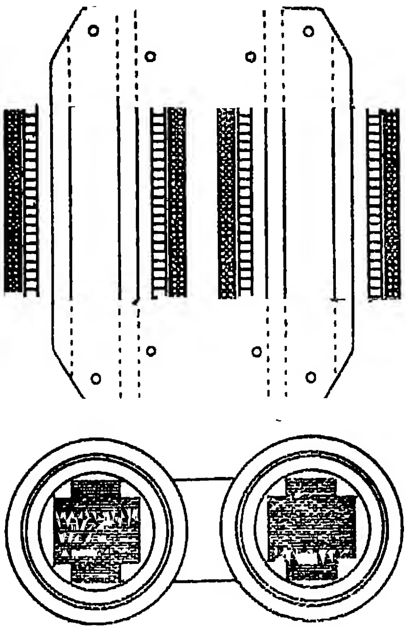


Fig 93.—Single-phase Transformer.

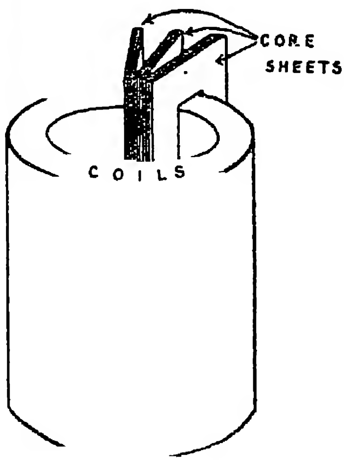


Fig. 94.—Berry Transformer.

being of the same size, and placed on top of each other). Even with the simple arrangement shown in the figure, it is advisable, in the case of very high voltages, to wind the high-voltage coil in a number of equal sections, separated from each other by barriers of insulating material.

Fig. 94 illustrates the construction of the Berry transformer. The circular coils having been wound, a number of bundles of core-sheets are

(in carrying out repairs), are rather slow processes. By adopting a butt-joint pure and simple between the core and yoke-plates, the transformer may be dismantled and built up again very quickly.

assembled around them, these bundles being arranged in radial planes. For the sake of clearness only three of these are shown in the figure.

In Fig. 95 is shown another type of construction. The coils are of oblong shape, the core-sheets being built up around them. The thick lines in the lower figure indicate insulating barriers between the sections of the high-voltage and those of the low-voltage windings. The core-plates are pressed together between two heavy end-plates connected by bolts passing outside the core-plates.*

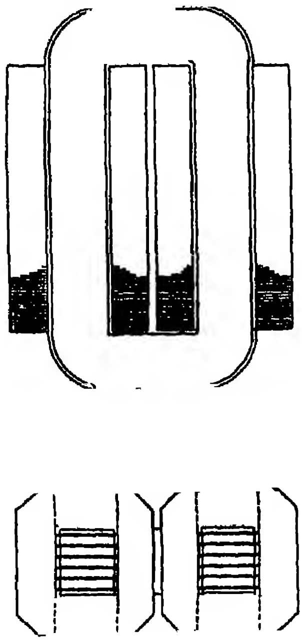


FIG. 95.—Type of Transformer Construction.

For three-phase work, three single-phase transformers may be used. A cheaper form of construction, however, results by combining the cores of the transformers so as to form a single three-phase core. One method of doing this is shown in Fig. 96.† The coils (not indicated in figure) are placed around three upright laminated cores, connected at the top and bottom by laminated yoke-rings. The cores are pressed against the yoke-rings by means of end-plates fitted with conical rims, which bear against the chamfered ends of the cores. In this, as in all other cases where butt joints are used, thin sheets of insulating material are interposed at the joints, in order to prevent the formation there of conducting grids (by the contacts between the two sets of plates), which would cause dissipation of energy by eddy currents. The algebraical sum of the magnetic fluxes in the three cores being zero, any two of the cores will, at a given instant, form a return magnetic circuit for the flux in the third core, so that the principle of a closed magnetic circuit will be realized.

In Fig. 97 is shown another method of combining the cores of the three transformers. The coils are shown in position in the lower part of the figure. This arrangement is not quite symmetrical, the flux passing through the middle core encountering a somewhat lower reluctance—on account of the shorter average length of path—than that passing through either end-core.

* The types of transformer shown in Figs. 93 and 95 are known as the *core-type* and *shell-type* respectively.

† This form of construction was a favourite one in the early days of three-phase currents, but is now being abandoned, mainly on account of its expense.

A form of construction applicable to either single-phase or poly-phase transformers, is shown in Fig. 98. The core of a single-

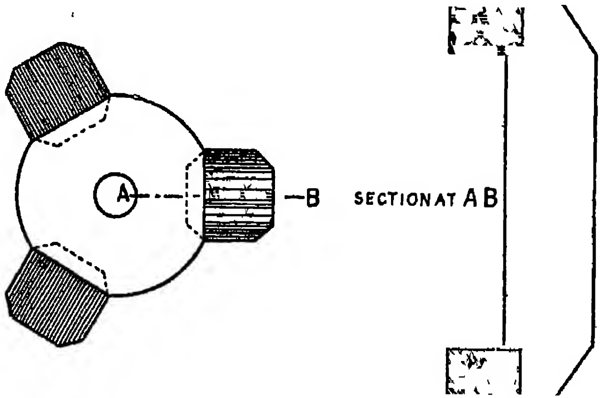


FIG. 96.—Core of Three-phase Transformer.

phase transformer would have the shape corresponding to Fig. 93.

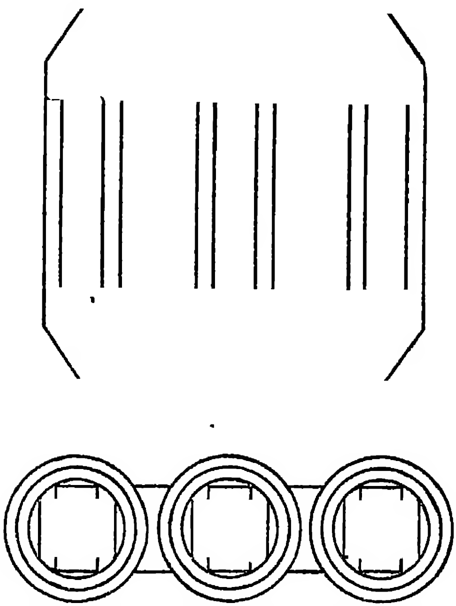


FIG. 97.—Type of Three-phase Transformer.

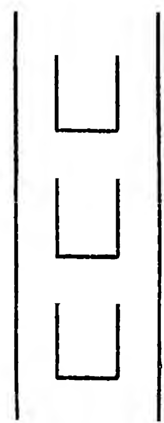


FIG. 98.—Transformer Core Construction.

In order to construct a three-phase transformer, three single-phase cores would be piled on top of each other as in Fig. 98, only the top core being provided with a top yoke.

The thickness of the core-sheets varies from 0·014 inch to 0·020 inch. The insulation (consisting of thin paper) between them occupies some 12 per cent. of the total thickness of the core.

So long as the voltage does not exceed about 10,000, there is no difficulty about leading the terminal conductors through the transformer containing case. For the lower voltages, up to about 3000, ferrules or bushings of prepared hard wood, ebonite, or porcelain, are used, and for the higher voltages porcelain alone should be employed. When, however, the voltage rises to 100,000 or 200,000 a plain porcelain bushing is no longer sufficient, and in order to prevent discharges to the case a much more elaborate form of terminal insulator becomes necessary. Two types of such insulators, suitable for voltages up to 200,000, are shown in Fig. 99. In the first type, the terminal is built up of several tubular porcelain component pieces; its lower portion is immersed in the oil contained in the tank (§ 63), while its upper portion projects a sufficient distance above the cover of the tank to prevent discharges over the external surface of the porcelain. The entire compound insulator is filled with oil, and for greater security a press-span cylinder is placed inside around that portion of the terminal conductor which is nearest the tank cover. The second type of insulator is built up around the terminal rod or conductor, and

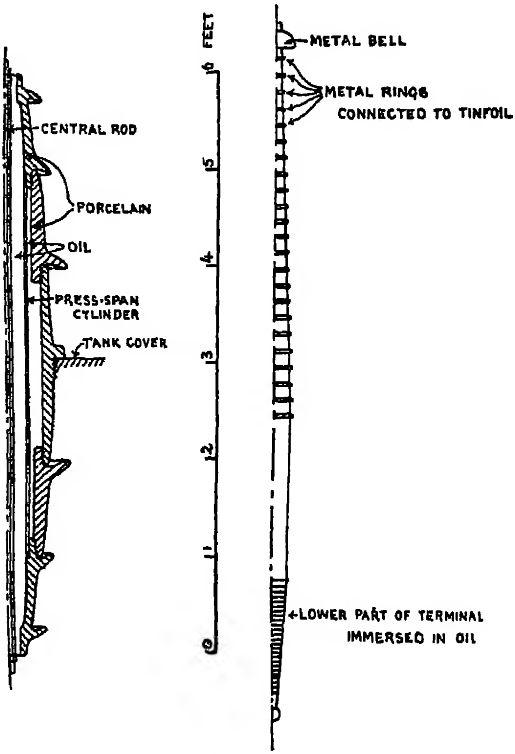


FIG. 99.—Types of 200,000-volt transformer terminals.

consists of alternate layers of insulation (impregnated paper or mica paper) and tinfoil, the tinfoil edges projecting outside the case being connected to smooth metal rings in order to prevent local brush discharges. The object of introducing the tinfoil layers is to equalize the electric stress throughout the thickness of the insulation. Complete equalization would be obtained if (the thickness of the insulating layers being constant) the lengths of the consecutive tinfoil layers were arranged so as to give equal capacities between neighbouring layers; the arrangement would then be equivalent to a number of condensers of equal capacity connected in series, and the p.d. across each condenser would be the same. The disadvantage of such a design would lie in the fact that the edges of the outermost layers of tinfoil would lie too close to each other, and there would be the danger of a surface discharge. In the actual insulator shown in Fig. 99, the length of the outermost layer of tinfoil is chosen so that along with the next layer it gives a capacity equal to that between the two innermost tinfoil layers, the lengths of the intermediate layers being arranged so as to give equal distances between their edges; the capacities decrease from the innermost layer outwards, pass through a minimum in some intermediate layer, and then increase again until the outermost layer is reached.

Another constructional difficulty which arises in connection with transformers intended for very high voltages is due to the phenomena which take place at the instant of connecting the high-voltage primary across the mains. Under normal working conditions, the p.d. between any two neighbouring turns, or equal groups of turns, is the same. But when the transformer is first switched into circuit, the whole of the winding is initially at zero potential, and one end of it is suddenly raised to a certain positive potential, the other to a certain negative potential. A current immediately begins to flow, but its establishment is retarded by (1) the radial component of the current, which forms the capacity current; and (2) by the self-inductance of the winding. Before a uniform potential gradient along the winding can be reached, each section of the winding must be raised to the corresponding potential by the charging or capacity current. As this process takes an appreciable though very short time, the end portions of the winding may, for a short fraction of a second, be subjected to a p.d. which is many times the maximum p.d. across them under normal conditions. It will be seen that such a momentary concentration of a high p.d. across the end turns may result in a breakdown of the insulation. To guard against this, two methods have been used. In one, the end turns of the winding are provided with extra heavy insulation, so as to enable them to stand the momentary application of a very high p.d.; while in the other, no special insulation is provided on the end turns, but heavily insulated

choking coils are included between the transformer terminals and the mains.

§ 60. Calculation of e.m.f. induced in Transformer Winding

The relation of the value of the maximum induction in the core to the e.m.f. induced in either coil at a given frequency is easily determined. Let B stand for the maximum value of the induction, a for the cross-sectional area of the core, in sq. cms., f for the frequency, and S for the number of turns in the coil. The maximum flux through each turn is $a\mathrm{B}$. This changes from $+a\mathrm{B}$ to $-a\mathrm{B}$ in $\frac{1}{2f}$ sec. The mean rate of change is thus $4fa\mathrm{B}$, the mean arithmetical value of the e.m.f., in volts, induced in each turn is $4fa\mathrm{B}\,.\,10^{-8}$, and the mean arithmetical value of the total e.m.f. $4fa\mathrm{SB}\,.\,10^{-8}$. Hence, if u denote the form factor of the e.m.f. wave (§ 1), we have for the r.m.s. value E of the induced e.m.f.—

$$\mathrm{E} = 4fau\mathrm{SB}\,.\,10^{-8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For a sine wave of e.m.f., $u = 1{\cdot}11$ (§ 3), so that—

$$\mathrm{E} = 4{\cdot}44fa\mathrm{SB}\,.\,10^{-8} \quad . \quad . \quad . \quad . \quad (2)$$

The usual value of B at a frequency of 50 is of the order of 10,000.

§ 61. Losses in Transformer. Core Loss

The losses taking place in a transformer may be divided into *core losses* and *copper losses*.

The core losses arise from hysteresis and eddy currents, and depend on the maximum value of the induction in the core. This maximum value (the amplitude of the induction wave) is frequently referred to as the induction simply.

We have seen that, owing to the smallness of the resistance drop, the e.m.f. induced in the primary is nearly equal to the primary p.d. Hence, if the latter be maintained constant, as is normally the case, the induced e.m.f. will also remain *nearly* constant. The constancy of the induced e.m.f., however, involves, in accordance with equation (1) above, the constancy of B. Thus the core losses, which depend on B, will remain nearly constant at all loads. As a matter of fact, there will be a *very slight* decrease with increasing load, owing to the slight decrease in the counter e.m.f. brought about by the increase in the resistance drop.

The hysteresis loss is found to increase considerably if the core-plates are subjected to great mechanical pressure. It is therefore advisable, in building up the core, to apply as little pressure as possible.

The core-sheets employed in some of the earlier transformers were found gradually to deteriorate, the hysteresis loss steadily increasing as time went on. This effect was found to be due to the maintenance of the core at a fairly high temperature by the losses occurring in it, and is known as the *ageing* of transformer iron. Manufacturers of transformer sheets are now, however, able to supply material which does not suffer from the "ageing" defect to any serious extent.

The power lost by hysteresis is, for a given maximum induction, proportional to the frequency, the loss per c.c. per cycle not being affected by changes in the frequency within the usual practical limits of frequency. If the frequency be maintained constant and the induction varied, the hysteresis loss is found, with modern transformer sheets, and at inductions in the neighbourhood of 10,000, to vary nearly as the 1·7th power of the induction (instead of Steinmetz's 1·6th power).

The eddy-current power loss is proportional to the square of the frequency and the square of the induction. This will be evident at once if we consider any eddy-current path of resistance r, the induced e.m.f. in which is E. In such a path, the power lost is given by $\frac{E^2}{r}$. But since E is proportional to both frequency and induction, the result stated above follows at once.

What is of interest to the designer and user of a transformer is the total core loss—hysteresis and eddy-current—which occurs in it. The simplest method of testing transformer sheets for the combined loss is to build up a magnetic square—as is done in Epstein's testing apparatus* the strips forming the sides of the square being 50 cm. long and 3 cm. wide, and the loss occurring in them being determined from the reading of a wattmeter connected to the four exciting coils surrounding the sides of the square.

The core loss may be greatly reduced by the use of a special silicon-iron alloy (containing about $3\frac{1}{2}$ per cent. of silicon) known as "stalloy." Experiment has shown that the addition of silicon in gradually increasing quantities at first increases the hysteresis loss,

* *Journal of the Institution of Electrical Engineers*, vol xxxviii p. 34 (1900). The magnetic circuit formed by the strips contains 4 butt joints, at each of which a layer of paper 0·2 mm. thick is interposed to prevent additional eddy current loss, the four sides of the square being pressed against each other by suitable clamps The weight of the core is 10 kg, and the standard of comparison is the loss at B = 10,000 and a frequency of 50

and then decreases it to a value much below that which it is possible to obtain with sheets of nearly pure iron. The following table gives the loss, in watts per lb., at various inductions in sheets 0·014″ thick at a frequency of 50 in the case of (*a*) "Lohys" iron and (*b*) "stalloy," both materials being manufactured by Messrs. J. Sankey & Sons.

LOSS IN WATTS PER LB. AT 50 ~.

B =	4000	5000	6000	7000	8000	9000	10,000	12,000
"Lohys" ...	0·31	0·43	0·57	0·73	0·91	1·11	1·35	–
"Stalloy" ...	0·19	0·27	0 34	0·43	0·53	0·63	0·73	0·97

If "stalloy" sheets were no more expensive than ordinary transformer sheets of nearly pure iron, they would be used exclusively Unfortunately, owing to their high cost, which is nearly double that of ordinary sheets, their use does not enable the manufacturer to build appreciably cheaper transformers. The main advantages attending the use of the more expensive material are a substantial reduction in size and weight and a higher efficiency.*

The hysteresis and eddy-current losses as calculated from tests on samples are always found to be less than the losses actually occurring in transformers. A little consideration will show that such a discrepancy might be expected to exist. For in calculating the losses we make the assumption that the induction is *uniformly distributed* over the cross-section of the core. Such, however, is by no means the case, as a single glance at Fig. 100, which represents a core stamping, will show. For the length of path corresponding to ABCD is very considerably shorter than that corresponding to EFGH, with the result that the flux will be crowded towards the inner surface of the core. Now, any departure from uniformity in flux distribution results in an increase of both the hysteresis and the eddy-current loss. An upper limit to these losses may be obtained by calculating them on the assumption that the value of B is that corresponding to its value close to the inner surface (ABCD) of the core.

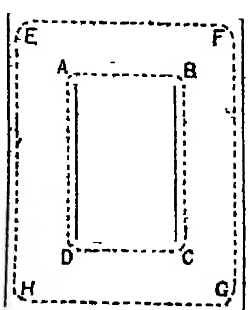


FIG. 100.—To explain Difference between Calculated and Observed Core Loss.

* See Appendix IV.

The following table gives the core losses of transformers of various outputs:—

Output, in k.v.a.	1	2	5	10	20	40	60	80	100
Core losses in watts	40	50	60	110	200	350	400	450	500

§ 62. Copper Losses. Best Dimensions of Core

The copper losses cannot be satisfactorily calculated from the continuous current resistances of the coils and their full-load currents, except in the case of coils wound with thin wire. Where conductors of large cross-section are used, the additional loss due to eddy-currents may bring about an apparent increase of resistance, amounting, in some cases, to as much as 20 per cent. For this reason, most makers prefer to avoid the use of such heavy windings, employing instead a number of coils wound with smaller wire and connected in parallel. The experimental method of determining the true copper losses is dealt with in § 101.

The total losses at full load depend on the output of the transformer. The following table gives the efficiencies which might reasonably be demanded in modern transformers:—

Output, in k.w.	1	5	10	20	50	100	150	250
Efficiency ...	94%	95%	95·5%	96%	96·5%	97%	97·5%	98%

Let us suppose that the average length of magnetic path and the total volume of iron are given, but that it is permissible to vary the relative cross-sections of the core-plates and yoke-plates. Any departure from a uniform cross-section for the entire magnetic circuit will increase the iron losses. On the other hand, however, a reduction in the core cross-section and an increase in the yoke cross-section will reduce the mean length of a turn in each winding, and so reduce the copper losses. Now, this reduction in the copper losses may more than counterbalance the increase in the iron losses. It has been found that in transformers of ordinary construction (Fig. 93) the total loss (iron and copper) reaches a minimum value when the yoke cross-section is about $2\frac{1}{2}$ times that of the core cross-section inside the coils.*

§ 63. Heating of Transformers. Methods of Cooling

The aim of the transformer manufacturer is to produce the cheapest transformer to comply with given conditions of regulation (*i.e.* voltage drop from no-load to full load), efficiency and temperature rise. As regards the first of these—regulation—this is largely a

* See A. Müller, *Zeitschrift für Elektrotechnik* (Wien), vol. xxii. p. 417 (1904); vol. xxiii p. 243 (1905).

matter of design in arranging a given amount of copper to the best advantage, and does not directly affect the amount of material to be used. On the other hand, considerations regarding the minimum efficiency and maximum temperature rise directly affect the size and weight of the transformer. It is found that with transformers of small output the heating difficulty does not arise, and the amount of active material required is dependent practically on the efficiency demanded. But as the size of the transformer increases, the problem of keeping down the temperature rise becomes more and more important, and in the case of large transformers it is the dominant factor in determining the minimum weight of active material, there being no difficulty with regard to the efficiency.

The maximum permissible temperature rise of the coils is 55° C.* Transformers are generally enclosed in suitable cases which may be either entirely of cast iron, or partly of sheet metal, plain or perforated. The transformer may be either exposed to ordinary air, or completely immersed in oil, in which latter case it is said to be *oil-insulated.* The use of oil as an insulating medium for transformers greatly simplifies the problem of reliable insulation, and for voltages above 30,000 oil insulation is essential. Oil is, however, freely used in the case of transformers working at very much lower voltages; its main function is then to act as a *cooling medium.* The maximum permissible temperature rise of the oil is 50° C.

The oil used in transformers is a pure mineral oil, which must be free from solid particles held in suspension, and from moisture, alkaline or acid impurities, and sulphur compounds. Its flash-point should not be below 175° C., and its burning point not below 200° C. Its density should lie between the limits 0·86 and 0·92. When intended for use in cold climates, it should not congeal at the lowest temperature to which the transformer is likely to be exposed under normal conditions. The oil should be free from volatile ingredients, and when 2 grammes of it are heated in a glass beaker 4″ high and $4\frac{1}{4}$″ in internal diameter placed in a hot-air oven maintained at 100° C., the loss by evaporation during 8 hours should not exceed 4 per cent.

The following methods of cooling transformers are in use:—

(1) *Natural air-draught method.*—When this method is employed, the case of the transformer is generally made of perforated sheet or expanded metal, so as to give the air free access to the core and windings, the natural circulation of the air being sufficient to keep the transformer cool.

(2) *Air-blast method.*—In this method, an auxiliary blower driven by a motor is employed, and a forced draught is maintained through ventilating channels in the core and between the sections of the windings.

* As measured by the increase of resistance method.

(3) *Natural oil-cooling method.*—The transformer is contained in the lower part of a large tank filled with oil, the natural circulation of the oil being depended on to keep the transformer cool. When a steady temperature is reached, the whole of the energy dissipated must be got rid of by the external surface of the containing tank, which is made corrugated for the purpose of increasing its area.

(4) *Air-blast oil-cooling method.*—In this method, a forced draught of air is maintained either through a number of cooling tubes passing through the oil tank, or else through the space between the oil tank and an external containing vessel, the arrangement forming an air-jacket.

(5) *Water-cooled oil insulation method.*—The heated oil is brought into contact with the surface of a coil of thin-walled tubing which

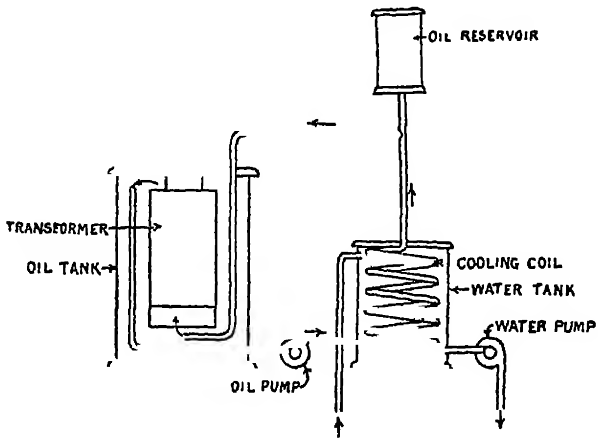


FIG. 101.—Method of Cooling Large Transformers.

is cooled by water. Two varieties of this method are in use. In the older and more common one, the oil tank is made much higher than the transformer, and in the upper part of it is fitted a coil of thin-walled brass piping through which cold water is circulated by a pump. In the second variety of this method, the oil tank is made only deep enough to take the transformer, with a sufficient clearance space above it, and the cooling of the oil is effected *outside* the tank, by forcing the oil through a worm immersed in cold water. The arrangement is shown in Fig. 101.* Two pumps are required, one to maintain sufficiently vigorous circulation of the oil, the other to maintain a flow of cold water through the cooling tank.

Comparing briefly the methods enumerated, we notice that while

* In the arrangement shown, the transformer is suspended from the cast-iron cover of the tank, in order to facilitate handling and inspection

some require no special auxiliary motor-driven apparatus, others do so. The provision of special cooling apparatus always increases the cost of the transformer, and this cost may become prohibitive for transformers below a certain output. In the case of such transformers, it is found cheaper to make the transformer somewhat larger and self-cooling, rather than of smaller size and requiring forced cooling. Accordingly we find that for transformers of small and moderate outputs methods (1) and (3) are in vogue, while for transformers above about 1000 k.v.a. output forced cooling is generally resorted to. Methods (2) and (4), and the first variety of (5) are in use up to outputs of about 5000 k.v.a. For still larger transformers, the second variety of (5) is generally employed.*

It may be noted that the use of air as an insulating medium has the advantage of keeping the surfaces of the core and coils clean, and thereby facilitates the carrying out of repairs. On the other hand, oil insulation, besides providing much greater electric strength than air, has the advantage that (owing to the larger thermal capacity per unit volume of oil) the transformer is able to stand a heavy overload for a much longer period than it could do with air insulation, as the presence of the oil prevents the rapid deterioration of the insulation which would inevitably take place in air at the same temperature. As illustrating the remarkable vitality of the oil-insulated type of transformer, it may be mentioned that a case is on record where such a transformer worked successfully at a temperature approaching 200° C.

The fire risk attending the use of oil-insulated transformers is now generally admitted to be small. It is, nevertheless, considered advisable to have such transformers installed in separate fire-proof compartments at some distance from the generating station, and to provide the necessary arrangements for draining off the oil rapidly.

§ 64. Δ, Y, V, and T Connections for Three-phase Transformers. Comparison of Single-phase and Polyphase Transformers for Polyphase Circuits

The coils of transformers intended for three-phase work may be connected either mesh- or star-fashion. The mesh connection is usually preferred, for if one of the coils should burn out, causing its fuses to blow, the supply of current to the corresponding phase would still be maintained, the remaining two coils taking up the load. But with a star connection the failure of one of the coils would evidently cut off the supply to *two* phases—one of the line wires becoming entirely disconnected. Thus a breakdown in the latter case would

* See Appendix V

have far more serious consequences than in the former. On the other hand, when dealing with very high voltages, the dielectric stress on each coil is reduced by adopting the star connection, since the phase p.d. then becomes only $\frac{1}{\sqrt{3}} = 0{\cdot}577$ of the line p.d. (§ 17).

In some cases, only *two* transformers may be available, and then the so-called "open delta" or **V**-connection is used. In this, the junction of the transformer windings is connected to one main, while the remaining two terminals are in connection with the other two mains, as shown in Fig. 102. It will be noticed that with this

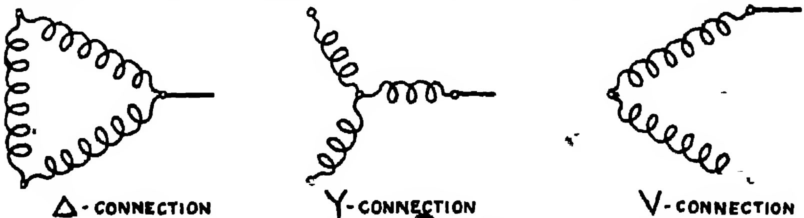


FIG. 102.—Connections for Three-phase Transformers.

arrangement the currents in two of the line wires are in phase with the transformer currents. Let us suppose the load to be a balanced three-phase non-inductive load, the line p.d. and current being V and I respectively. The total power in the three-phase circuit is then $\sqrt{3}$ VI,* and the load on each transformer is $\frac{\sqrt{3}}{2}$ VI = 0·866 VI. But either of the transformers when supplying a current I at a p.d. V to a *single-phase* non-inductive load would have an output VI. We thus see that when two **V**-connected transformers supply a three-phase non-inductive load, their outputs are about 13·4 per cent. less than when working on a single-phase non-inductive load; or two **V**-connected transformers supplying a balanced three-phase non-inductive load behave as if the power-factor of the load were 0·866 instead of unity.

A **V**-connection of transformers results if one out of three Δ-connected transformers fails and is switched out. Let us suppose that I, as before, represents the line current, the load being balanced and non-inductive. If one of the transformers is burnt out, a **V**-connection of the remaining two results, and the current through each rises from $\frac{I}{\sqrt{3}}$ to I, *i.e.* the current is increased in the ratio $\sqrt{3} : 1$, or by 73·2 per cent. If the Δ-connected transformers were loaded to their full output before the breakdown occurred, then

* See Note II. (p. 66) at end of Chapter IV.

owing to the 73·2 per cent. increase of current consequent on the failure of one of the transformers, those remaining in circuit would be capable of dealing with the load for a comparatively short period only.

Besides the V-connection, the so-called T-connection is sometimes used in the case of two transformers which are across three-phase mains. In this, one end of the winding of one of the transformers is joined to the middle point of the winding of the other (§ 66 and Fig. 105).

The relative advantages and disadvantages of a single three-phase transformer as compared with three independent single-phase transformers have frequently been discussed. For smaller sizes (up to about 200 k.w.) a single three-phase transformer is both lighter and cheaper, for a given output, efficiency, temperature rise, and regulation,* than three single-phase transformers. For very large transformers, on the other hand, the cost is lower in the case of three single-phase transformers (the weight is, as before, greater than that of a three-phase transformer). A three-phase transformer has the advantage over three single-phase transformers of maintaining better balance of the p.d. on the three phases, owing to the interlinking of the magnetic circuits.†

§ 65. Auto-transformer or Compensator

In addition to the ordinary transformers already considered, which have distinct primary and secondary windings, there is another type, known as the *auto-transformer*. In this the primary and secondary windings are represented by a single continuous winding, as shown diagrammatically in Fig. 103. The ends of the winding AB form the primary terminals, while the secondary terminals are represented by one of the primary terminals (A in Fig. 103), and terminal C, in connection with an intermediate point of the winding We may regard the portion AC of the winding as resulting from the fusion of the primary and secondary. Now, since the primary and secondary currents are nearly in phase opposition, it follows that such a fusion of the two circuits will give rise to a current in AC which is merely the difference of the primary and secondary

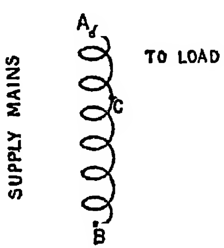


FIG 103 —Diagram of Auto-transformer.

* See Chapter X

† Eborall, "Howard Lectures on Polyphase Electric Working," p 40 (1902).

currents, and is thus less than the current in the secondary of an ordinary transformer. Hence the cross-section of the portion AC of the winding may be made less than would be necessary with a transformer of ordinary construction.

On account of this advantage, auto-transformers would be much more generally used than they are, were it not for the fact that in most cases it is absolutely necessary, from considerations of safety, to keep the two windings entirely distinct and heavily insulated from each other, so as to prevent all risk of the dangerous high voltage of the primary from reaching the secondary.

Auto-transformers are frequently used to supply single arc lamps from mains at a considerably higher p.d. than that taken by the lamp. They are then sometimes termed *economy coils*. Their main use is, however, in connection with the starting of various forms of alternating-current motors. When so used they are generally known as *compensators*. By means of a suitable switch, the motor is first connected across the portion AC of the winding (Fig. 103), and when it has run up to a certain speed, the switch is thrown over so as to connect it across AB. In the first or starting position, the voltage across the motor terminals is only a fraction of the normal voltage, while in the second position of the switch the motor is supplied at the full line voltage.*

§ 66. Phase Transformers

Cases sometimes arise in which it is required to pass from a two-phase to a three-phase system, or *vice versâ*. Such a transformation

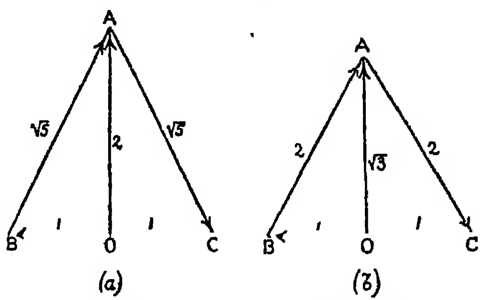


Fig. 104.—To illustrate Principle of Phase Transformation.

may be effected by a suitable combination of transformers, which are then termed *phase transformers*. Various solutions of the problem of phase transformation have been devised, but the best known is

* See § 72.

that due to C. F. Scott. The method of arranging the connections is shown in Fig. 105. The two-phase mains are connected to the primaries S_1 and S_2 of two transformers. If S_1 and S_2 have equal numbers of turns, the flux per turn will be the same in each transformer. The secondaries of the transformers are interconnected as shown, the end of the secondary OA of one transformer being joined to the middle point O of the secondary BC of the other. If we provisionally assume the secondaries to have equal numbers of turns, then the vector diagram of secondary voltages will be that shown in

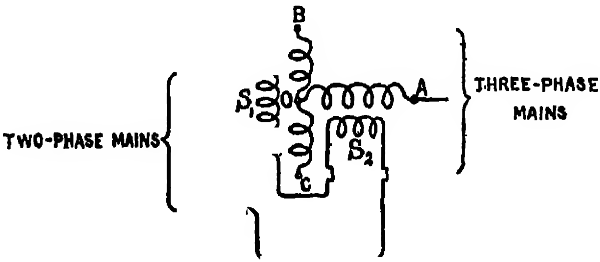


FIG. 105.—Scott's Method of Phase Transformation.

Fig. 104 (*a*), in which the vectors AC, CB, BA represent the voltages across the corresponding pairs of points in Fig. 105. It will be noticed that since the two vectors CB and OA in Fig. 104 (*a*) are equal in length, the length of either of the vectors BA and AC is $\frac{\sqrt{5}}{2}$ times that of CB. In order to equalize the lengths of the three vectors CB, BA, and AC, the length of OA must be reduced in the ratio $\frac{\sqrt{3}}{2}$,—a result which is easily accomplished by making the number of turns in the winding OA (Fig. 105) only $\frac{\sqrt{3}}{2}$ of that in CB, when the vector diagram assumes the form shown in Fig. 104 (*b*), the vectors CB, BA, AC now forming a balanced three-phase system.*

* See Appendix VI. for an account of phase-shifting transformers.

§ 67. Instrument Transformers

Instrument transformers are small transformers of special design used for the purpose of reducing either the p.d. or the current in a known ratio, so as to bring them within the range of measuring instruments of ordinary construction, and also for the purpose of insulating measuring instruments from high voltage circuits, so as to enable them to be handled with safety.

An instrument transformer employed for the purpose of reducing a high voltage in a known ratio is generally termed a *voltage, potential,* or *pressure-transformer*; while a transformer for reducing a large current in a known ratio is termed a *current-transformer*.

Instrument transformers are used in connection with voltmeters, ammeters, wattmeters, energy meters, and relays employed for actuating automatic switches. The conditions which must be satisfied by an instrument transformer depend to some extent on the kind of measuring instrument which is across its secondary circuit.

In the case of transformers connected to voltmeters, ammeters or relays, the only condition which the transformer has to satisfy is that of *constancy of the voltage or current ratio* over the working range. But if the transformer is to supply a wattmeter or energy meter, it must, in addition, also satisfy the condition of practically exact *phase opposition* of the primary and secondary currents. These conditions are fulfilled with a sufficient degree of accuracy by (1) constructing the transformer so as to reduce its leakage self-inductance to a very small value; (2) using a core material which has the least possible loss by hysteresis and eddy-currents, and a high permeability ("stalloy" is particularly suitable for this purpose).

The induction in modern instrument transformers does not generally exceed 1500, and is sometimes much less. The standard outputs for *potential* transformers are 15, 50, and 200 volt-amps. per phase; their standard secondary voltage is 110; the ratio error should not exceed 1 per cent. and the phase error should not exceed $\frac{1}{2}$ degree at unity power factor and any load not exceeding the normal output. As regards *current* transformers, the standard outputs are 15 and 40 volt-amps.; the standard full-load secondary current is 5 amps.; the ratio error should not exceed 1 per cent., and the phase error should not exceed 2 degrees, from full-load to $\frac{1}{5}$th of full-load current.*

* See § 194 for a more detailed account of instrument transformers

§ 68. Choking Coils

By a choking coil, impedance coil, or reactance coil, is meant a coil having a low resistance and high self-inductance. The primary of a transformer whose secondary is open forms a very powerful choking coil. In many cases, it is desirable to have some simple means of adjusting the reactance, and for this purpose various arrangements may be used. Thus, the coil may be provided with a sliding core, which may be clamped in any desired position relatively to the coil. Or the coil may be wound on the middle limb of a E-shaped core, and provided with a movable yoke, which may be placed at varying distances from the projecting ends of the core, thereby altering the length of the air-gaps in the magnetic circuit.

A choking coil affords a ready means of reducing the voltage of supply without any appreciable loss of power such as would occur if a resistance were used for the same purpose. This is due to the very low power factor of the coil, which enables it to take a large current at a high p.d. without the absorption of any large amount of power.

§ 69. Dimensions, Weights and Prices of Transformers

In the following table are given the approximate overall dimensions, total weights (including case) and prices of air-insulated self-cooling *single-phase* transformers for a frequency of 50 and voltages up to 5000.

Output in k.v.a.		1	2	5	10	20	50	100
Overall dimensions in inches.	Height	16	19	21	30	35	39	45
	Breadth	12	13	16	17	19	26	29
	Width	9	10	12	17	18	22	27
Total weight in lbs.		112	168	280	670	1000	2100	2900
Price, £		7	9	16	24	36	66	100

The corresponding data for air-insulated self-cooling *three-phase* transformers are as follows :—

Output in k.v.a.		1	2	5	10	20	50	100
Overall dimensions in inches.	Height	17	18	24	30	32	52	57
	Breadth	18	17	20	22	28	30	34
	Width	10	10	14	15	18	20	30
Total weight in lbs.		120	200	320	460	780	1700	3200
Price, £		5	8	12	23	34	60	90

Oil-insulated transformers, which are more suitable for voltages exceeding 5000, are lighter than air-cooled ones. In small sizes, up to about 10 or 20 k.v.a., the oil-insulated type is more expensive than the air-insulated one. Between about 20 and 50 k.v.a. there is little difference in price, while above 50 k.v.a. the oil-insulated type is the cheaper of the two.

It will be noticed that the price per k.v.a. steadily decreases, and at 100 k.v.a. amounts to about £1 per k.v.a. This decrease goes on with increasing output, until for outputs of about 500 k.v.a. and upwards the price reaches a value of about 10*s*. per k.v.a.

If we consider the weight of the active material only—viz. the core stampings and the windings—this is found to vary from about 60 lbs. per k.v.a. for an output of 2 k.v.a. to about 20 lbs. per k.v.a. for an output of 100 k.v.a. In the case of transformers of very large output (exceeding 1000 k.v.a.) the weight of the active material may be as low as 10 lb. per k.v.a.

The ratio of the weight of the steel in the core to that of the copper in the windings varies from about $3\frac{1}{2}$ to 3.

The output of *instrument transformers* varies from 15 to 200 volt-amperes, and their price from about £3 to about £30, the price being largely controlled by the primary voltage as well as by the output.

CHAPTER VIII

§ 70. Induction motors. Squirrel-cage rotor—§ 71. Rotor windings—§ 72. Starting resistances and protective devices for induction motors—§ 73. Lewis induction motor—§ 74. Example of induction motor. Variation of torque with position of rotor—§ 75. Asymmetry of hemi-tropic stator winding with odd number of pole-pairs. Method of obtaining symmetry—§ 76 General characteristics and some technical data of induction motors—§ 77. Dimensions, weights and prices of polyphase induction motors.

§ 70. Induction Motors. Squirrel-cage Rotor

IN connection with polyphase systems of distribution, a type of motor is employed which is known as the *induction motor*. The action of such a motor depends on the possibility (§ 22) of producing rotating waves of magnetic flux by means of polyphase currents.

Let a laminated stator be provided with polyphase windings (two- or three-phase) similar in every respect to the armature windings of a polyphase generator, and let the rotor consist of a solid cylinder of iron (*cf.* Fig. 31). When polyphase currents are sent through the stator windings, they give rise to rotating waves of flux—commonly spoken of as a rotating field—and as these waves sweep across the rotor they induce currents in it. The direction of these currents is, in accordance with Lenz's law, such as to oppose relative motion of the rotor and the rotating field. A driving torque is thereby exerted on the rotor, which (assuming that the resistances to its motion are not excessive) will run up to a speed only slightly below that of the rotating field. The difference between the speed of the rotating field and that of the rotor, on which depends the magnitude of the induced currents, is spoken of as the *slip*. Since with increasing load a larger driving torque, and hence larger induced currents, are required to keep the rotor running, it follows that the slip will increase with increase of load.

The rudimentary form of motor considered, in which the rotor is a solid cylinder of iron, although mechanically simple, would be unsatisfactory from the point of view of efficiency, as the loss in the rotor would be excessive.* In practice, therefore, the rotor takes the form of a laminated iron cylinder having copper conductors embedded in it.

* Such simple solid cylinders of iron are, however, actually employed in certain special cases, where the motor is used intermittently for very short periods only (as for starting up rotary converters), the efficiency being then of but little account.

The simplest form of rotor winding is that known as the *squirrel-cage* winding. It consists of a number of copper rods or bars arranged in holes or slots around the rotor periphery, and connected at each end to a ring of copper. One example of a squirrel-cage winding is given in Fig. 106. In this, the rotor is provided with a number of equidistant circular holes (open at the top) close to its periphery, and through these holes pass copper rods. The projecting ends of the rods are slotted, and into the slots, on each side, is fitted a copper ring, which is then soldered to the rods. In Fig. 111 is shown a slightly different arrangement. The conductors are, in this case, of rectangular cross-section, and fit into rectangular semi-closed slots in the rotor

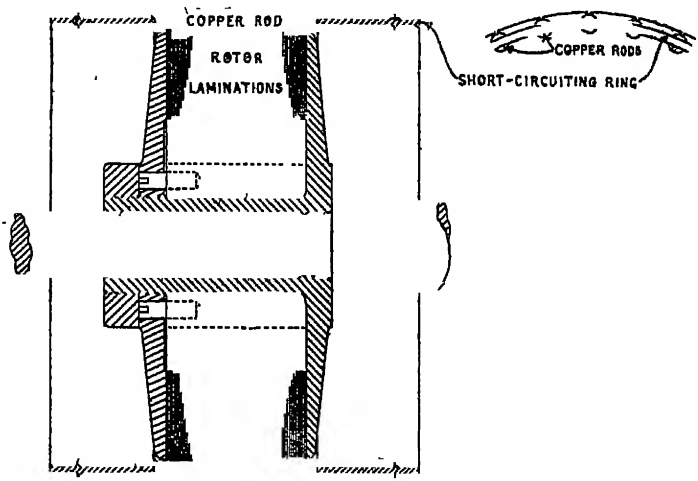


FIG 106.—Squirrel-cage Rotor.

core. The projecting ends of the copper bars are bolted to end-rings.

The squirrel-cage winding is at once the simplest and cheapest form of rotor winding, and is almost exclusively used for small motors—up to about 5 h p. In the United States, it is used for very large motors as well. It possesses, however, the disadvantages of taking a heavy starting current and exerting a comparatively feeble starting torque. It is thus incapable of starting against a heavy load, and must be run up to full speed on a loose pulley. For starting motors with squirrel-cage rotors, auto-transformers (§ 65) are employed.

In order that an induction motor may be capable of exerting a large torque *at starting*, its rotor circuits must have a comparatively

high resistance.* But high resistance in the rotor circuits would involve large heating loss and poor efficiency under normal running conditions. Thus, although a squirrel-cage rotor could easily be constructed to give a powerful starting torque (by the use of high-resistivity material for its conductors), its efficiency under normal running conditions would be extremely low. Large starting torque and high efficiency are, in fact, incompatible features, and the only way of securing both lies in the use of a rotor the resistance of whose windings at starting is high, and under normal running conditions, low. This variation of resistance is easily obtained by the use of an external rheostat, which may be connected in series with the rotor windings at starting, then gradually short-circuited as the motor gains speed, and finally cut out altogether. Now, a squirrel-cage winding does not readily adapt itself to the insertion of external resistances. It is for this reason that most of the larger European induction motors are provided with so-called *wound* rotors as distinguished from squirrel-cage or *short-circuited* rotors. The winding of these rotors is connected to slip-rings, which allow of the insertion of suitable starting resistances.

§ 71. Rotor Windings

Two types of three-phase windings, both of them star-connected, are commonly used in the case of rotors provided with starting resistances. One of these windings is similar to the form of armature winding for a three-phase generator already described (§ 48 and Fig. 86), while the other closely resembles the wave winding of a continuous-current armature, but differs from it in not being entirely symmetrical. This second kind of winding is frequently described as a *bar winding*, in order to distinguish it from the first type, which is spoken of as a *coil winding*. In the smaller sizes, coil windings are generally used, while the bar winding is commonly employed in large motors. In order to explain clearly how such a bar winding is carried out, we shall consider a particular case—that of a four-pole rotor having 96 conductors or bars arranged in 48 slots, so that there are 2 conductors per slot.

It will be sufficient to consider the winding of one phase, since all three phases are similarly wound. The number of conductors per phase is $\frac{96}{3} = 32$, and since the motor is a four-pole one, the number of conductors per pole per phase is $\frac{32}{4} = 8$. The winding of each phase must, therefore, clearly consist of four equally spaced groups of

* The reason for this will be understood later, when we come to study the theory of such motors in detail (Chapter XII.).

eight conductors, and these conductors have to be connected in series in such a manner that two conductors in immediate connection belong to two neighbouring groups.* Let us suppose all the conductors to be numbered, and let conductor 1 be connected at one end to the neutral point of the winding. The four groups of conductors forming the winding of the first phase are then as follows:—

Group I: 1, 2, 3, 4, 5, 6, 7, 8
„ II: 25, 26, 27, 28, 29, 30, 31, 32
„ III: 49, 50, 51, 52, 53, 54, 55, 56
„ IV: 73, 74, 75, 76, 77, 78, 79, 80

Between Groups I and II are the sixteen conductors belonging to the other two phases, and there are similar intervals between the remaining groups.

In arranging the connections, we select conductors which are as nearly as possible separated by a distance corresponding to the pole-pitch. Now, the pole-pitch is, in terms of the number of conductors comprised in it, equal to $\frac{96}{4} = 24$. But this value could not be adopted for the pitch of the winding, since with a double layer of conductors the pitch must correspond to an *odd* number.† Hence we select a *double* pitch for our winding, namely, 23 and 25, using these numbers alternately; the mean pitch of the winding is thereby made to correspond to the pole-pitch.

We must commence with a pitch of 25, since a pitch of 23 would lead us to conductor $1 + 23 = 24$, which does not belong to our phase at all. Hence our winding proceeds thus—

1—26—49—74

A difficulty occurs at this point. We have travelled once round the rotor periphery, and the next step (assuming that we go on using the pitches 25 and 23 alternately) would bring us to conductor $74 + 23 = 97$, which is conductor 1. But this would close the winding. Hence we select that odd conductor in Group I which is nearest to conductor 1. This is obviously conductor 3, and the step from 74 to 3 (or 99) corresponds to a pitch of 25. We thus find it necessary to break the alternate sequence of the two pitches at the end of the first round, and to use the same pitch twice in succession. The same difficulty occurs, and is similarly overcome, at the end of each revolution. Bearing this in mind, we obtain the following for the first half of the winding table, each horizontal line representing one complete revolution:—

* So that their e.m.f.s may be additive.

† Otherwise, starting from conductor No. 1, which is in the top layer, we should be using up odd conductors, all of which are in the top layer; this would not allow of the compact arrangement of end connections, which becomes possible when odd and even conductors (*i.e.* conductors in the top and bottom layer respectively) are connected alternately.

1—26—49—74
3—28—51—76
5—30—53—78
7—32—55—80

We have now used up half the conductors of the phase, and at this stage a fresh difficulty arises. All the odd conductors in Group I have been used up, so that we cannot proceed from conductor 80 (which is the last *even* conductor in Group IV) to an *odd* conductor in Group I. A pitch of 25 gives conductor $80 + 25 = 105$, or 9, which does not belong to our phase at all. The nearest odd conductor in Group I is 7, but this has already been made use of. If we try going back to Group III, we again find that all the odd conductors have been used up. Two possible courses are open—to step forward to Group I, using the even conductor 2 of this group, or to go back to Group III, using the nearest even conductor corresponding to a pitch of 24 (*i.e.* conductor $80 - 24 = 56$). In either case, the even conductor 80 will have to be connected to another even conductor. It will be found, however, that if we go forward to Group I, we shall involve ourselves in still further difficulties, necessitating further irregularities in the winding; whereas by going back to Group III, thereby reversing the original direction of travel around the rotor circumference, and continuing to step round in this reverse direction, we encounter no further difficulties, the winding proceeding exactly as before, the only difference being in the negative sign of the pitches. We thus complete the winding as follows:—

56—31—8—79
54—29—6—77
52—27—4—75
50—25—2—73—slip-ring

The peculiarities exhibited by the above example are characteristic of every rotor bar winding of the wave type in which the conductors are arranged in two layers. In each case, we have to (1) break the alternate sequence of the pitches after the completion of each revolution; and (2) take a *backward* step, represented by the mean pitch,—thereby reversing the direction of travel—after the completion of half of the winding, when the odd and even conductors in alternate groups respectively have been used up.

If, however, the conductors are arranged in a single layer, with evolute end connectors (instead of the "straight-out" or "barrel" type), there is no reason why an even pitch should not be used. In that case, after each revolution, the normal pitch must be increased by unity. Taking the example just considered, but assuming the

number of slots to be doubled, so as to bring all the conductors into the same layer, we have the following winding table :—

1—25—49—73
2—26—50—74
3—27—51—75
4—28—52—76
5—29—53—77
6—30—54—78
7—31—55—79
8—32—56—80

The voltage for which a rotor is wound is immaterial * so far as the behaviour of the motor is concerned, and is merely a matter of convenience from a manufacturing point of view. It generally lies between 100 and 400 volts across the slip-rings when the rotor is at rest and the slip-rings are open-circuited.†

§ 72. Starting Resistances and Protective Devices for Induction Motors

An induction motor, whose rotor is at rest, forms a short-circuited transformer having large magnetic leakage. If the full supply p.d. is applied to the stator, the initial value of the current, before the rotor has attained any appreciable speed, is very large, and may amount to as much as four times the normal full-load current. In order to limit this initial rush of current, with the consequent drop of voltage on the system, suitable starting devices must be employed in connection with all motors exceeding about 7 b.h.p. The nature of these starting devices depends on the type of rotor used.

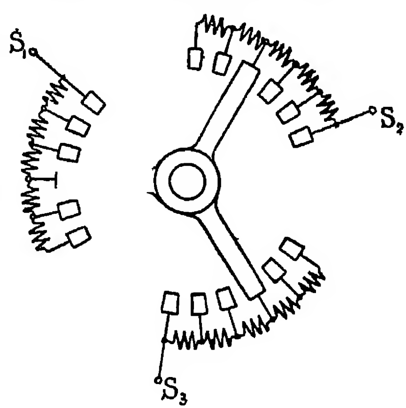


Fig. 107.—Starting Resistance for Wound Rotor.

In the case of motors provided with *wound rotors* connected to slip-rings, a three-phase starting resistance of the type shown in Fig. 107 is generally employed. It consists of a three-armed switch

* Assuming the same amount of copper to be used in each case.
† In large motors, the standstill rotor voltage may be as high as 1000.

moving over three sets of contacts connected to resistances. The terminals S_1, S_2 and S_3, are in connection with the slip-rings, the movable arms forming the neutral point of the starting resistances. In the case of large motors, it is desirable to use a large number of sections in the starting resistance—with a corresponding large number of contacts—in order to reduce the sudden changes in the current at each step. A large number of steps with a relatively small number of fixed contacts may be obtained by the use of the starting resistance designed by Kahlenberg, shown in Fig. 108. In this, it will be noticed, the neutral point of the resistance

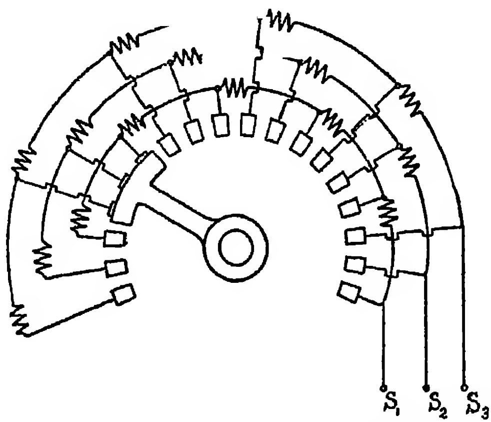


FIG. 108.—Kahlenberg's Starting Resistance.

is represented by a single movable arm, whose contact is of sufficient width to cover three of the fixed contacts. As the arm moves over these contacts, it cuts out a section of the resistance in one phase only at each step; this temporarily destroys the balance of the rotor windings, which, however, is of no great importance.

As regards the actual form of the starting resistance, this may consist of spirals of wire or metal grids, or of a liquid resistance. In the latter case, the liquid (a solution of either caustic or washing soda) is contained in a suitable tank, and a set of metal blades insulated from each other and connected to the rotor slip-rings is arranged so that it may be gradually lowered into the liquid, the contact area steadily increasing and the resistance decreasing, until finally the blades are short-circuited by metal contacts. The advantages of a liquid over a metallic resistance are perfect continuity of change in the resistance, and practical indestructibility, since the only effect of an overload is to cause the liquid to boil away.

Contact with the slip-rings is maintained by means of carbon brushes. In order to do away with the loss of power due to heating at the contact surfaces, it is usual to provide an internal short-circuiting arrangement which is brought into play when the motor has attained full speed. This device consists of three contact springs mounted on the shaft and connected directly to the ends of the rotor winding; the springs may be short-circuited by being brought into contact with a sliding sleeve actuated by a lever. The brushes may then be lifted off the slip-rings, as the winding is short-circuited independently of the brushes; special brush-lifting gear being frequently provided for this purpose.

It is desirable that the starting resistance should be combined with two protective devices, similar to those used in connection with continuous current motors—*viz.* an *overload* and a "*no-voltage*" release. In Fig. 109 is shown diagrammatically a form of starter,

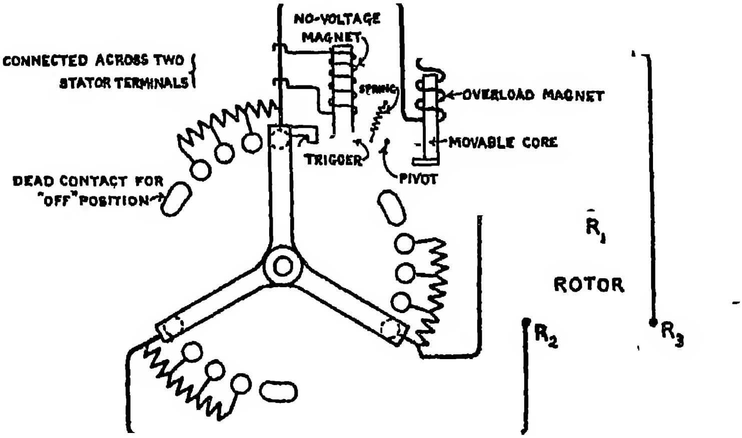


FIG 109 —Rotor Starting Resistance.

patented and manufactured by G. Ellison, which is provided with such protective devices. The points R_1, R_2 and R_3 are in connection with the slip-rings. The switch arm is fitted with a fly-back spring which tends to throw it into the "off" position. Normally, it is kept in the running position by a latch or trigger which forms part of a pivoted soft-iron armature acted on by the "no-voltage" electromagnet. This electromagnet is connected across two of the stator terminals or supply mains, and should an interruption of supply take place, the armature drops into a position where it is no longer able to engage the switch arm, which flies back into the "off" position. The overload magnet is connected in the rotor circuit as shown, and consists of

a solenoid acting on a movable core. When the rotor current exceeds the safe limit, the core is sucked in, and the latch knocked out of gear. It will be noticed that this form of protective device does not allow of the short-circuiting of the rotor independently of the brushes, and hence involves a slight sacrifice of efficiency.

The starting of induction motors provided with *short-circuited* or *squirrel-cage* rotors presents a somewhat different problem. Such motors take an abnormally large current at starting if supplied at the full voltage, although, as will be seen later (§ 122), they are incapable of exerting any large starting torque. In order to limit the starting current, the stator must be initially supplied at a lower voltage than the normal, this voltage being raised to the normal as the speed increases. The following four methods of starting squirrel-cage motors are in use :—(1) direct connection to the mains, without any attempt to reduce the starting current; (2) the star-delta change-over connection; (3) the auto-transformer, and (4) the starting resistance method. Of these, (1) is confined to motors of small output (up to 7 or 10 b.h.p.). For motors of medium size, method (2) is largely used. In this method, the stator windings are not permanently interconnected, the motor being provided with six terminals. The windings are designed for a Δ coupling under normal running conditions. By means of a change-over switch, they are connected in Y at starting, the p.d. across each phase of the winding being only $\frac{1}{\sqrt{3}}$ of the supply p.d. By throwing over the switch into the running position, the stator windings are Δ-coupled, the voltage across each stator phase being thereby raised to its normal value. In method (3) the starting p.d. is lowered by means of an auto-transformer (§ 65). In method (4), a three-phase starting resistance is used in the stator circuit, similar to the resistance employed in connection with wound rotors, except that it has no neutral point (Fig. 110).

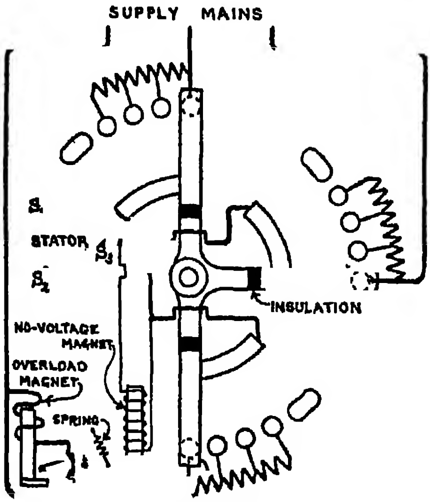


Fig. 110.—Stator Starting Resistance.

In all cases where the starting current taken by an induction

motor having a short-circuited rotor considerably exceeds the full-load current, a difficulty arises in connection with the fuses or circuit-breakers which protect the motor against an overload, as these are liable to act at the moment of starting the motor. A very common though not quite satisfactory method of meeting this difficulty consists in providing a double set of fuses connected to a throw-over switch, by means of which a much heavier set of fuses may be introduced at starting, the usual fuses being afterwards substituted by throwing over the switch. The danger of this arrangement lies in the fact that the throw-over switch may inadvertently be left in the position corresponding to the heavier fuses, which would not afford adequate protection to the motor. An ingenious method of overcoming the difficulty in question is used in Ellison's starter for squirrel-cage motors, shown in Fig. 110. The over-load magnet coil, it will be noticed, is not in circuit until the whole of the resistance has been cut out. This starter is also provided with a "no-voltage" release.

§ 73. Lewis Induction Motor

The use of wound rotors with slip-rings introduces numerous complications—an external starting resistance, and in some cases intricate internal short-circuiting and brush-lifting devices. Attempts have, therefore, been made to design motors which combine the advantage of simplicity possessed by a rotor having no slip-rings with the high starting torque and moderate starting current characteristic of the wound rotor designed for use with an external starting resistance. One of the most ingenious solutions of this problem is to be found in the motor patented by Mr. F. Lewis, and manufactured by the Electric Construction Co., of Wolverhampton. The construction of this motor is based on the fact that whereas a squirrel-cage rotor may be used in connection with any rotating field, quite independently of the number and sequence of the magnetic poles, a wave-wound rotor will only develop a torque in a field having the correct number and sequence of magnetic poles. In the Lewis motor, the rotor is provided with a double winding—a low-resistance wave winding, which is permanently short-circuited on itself instead of being connected to slip-rings, and a *high-resistance* squirrel-cage winding. At starting, the low-resistance winding is rendered inoperative by reversing the normal polarity of one half of the field. Thus, considering the case of an eight-pole motor, in which the normal sequence of poles is NSNSNSNS, at starting the sequence is arranged to be NSNSSNSN. The rotor, therefore, behaves as if it were provided with the *high-resistance* squirrel-cage winding alone, and starts with a powerful torque. If it were allowed to run on this

winding, not only would an excessive temperature rise take place, but the efficiency would also be poor. The normal polarity is therefore restored as soon as the speed has reached a certain limit, and the low-resistance wave winding now comes into play. The speed rises, the slip decreasing, and with it also the currents in the squirrel-cage. Although under normal running conditions the squirrel-cage conductors are still traversed by feeble currents and do part of the driving, yet the bulk of the torque is due to the wave winding, the presence of the squirrel-cage being simply equivalent to a slight increase of cross-section in the conductors of the wave winding. The squirrel-cage consists of conductors of small cross-section arranged in the upper parts of the slots containing the large conductors which form the wave winding.

§ 74. Example of Induction Motor. Variation of Torque with Position of Rotor

In Fig. 111 is shown a 5-h.p. three-phase induction motor, having a short-circuited rotor.* The stator is provided with a three-phase coil winding embedded in open slots, the coils being wound on formers and then fitted into position in the slots. The stator stampings or laminations are supported by a hollow cylindrical casting, which carries two end-shields containing the bearings. In order to improve the ventilation, each end-shield has four windows, closed by expanded metal gratings. The bearings are of the usual self-oiling type, each having two oiling rings, which ride loose on the shaft and dip into oil-wells. The rotor core-plates are mounted on a spider, to which they are secured by means of a key. The rotor conductors are bolted to the short-circuiting rings. It will be noticed that whereas there are forty-eight slots in the stator, the number of rotor slots is twenty-nine, so that the two numbers have no common factor. This arrangement is invariably adopted in the case of motors having squirrel-cage rotors, and its object is to prevent variations in the torque with varying position of the rotor; such variations occur to a marked extent when the number of stator and that of rotor slots have a common factor. These variations are due to the fact that in certain positions of the rotor the reluctance of the magnetic circuit is less than in others, and the rotor will always tend to pass from a position of higher to one of lower reluctance. If, *e.g.*, the stator and rotor have the same number of slots, then on exciting the stator the (open-circuited) rotor will tend to move so as to bring about coincidence of the teeth and slots in the two cores.

* The author is indebted to the Electric Construction Co., Ltd., of Wolverhampton for drawings from which Fig. 111 has been prepared.

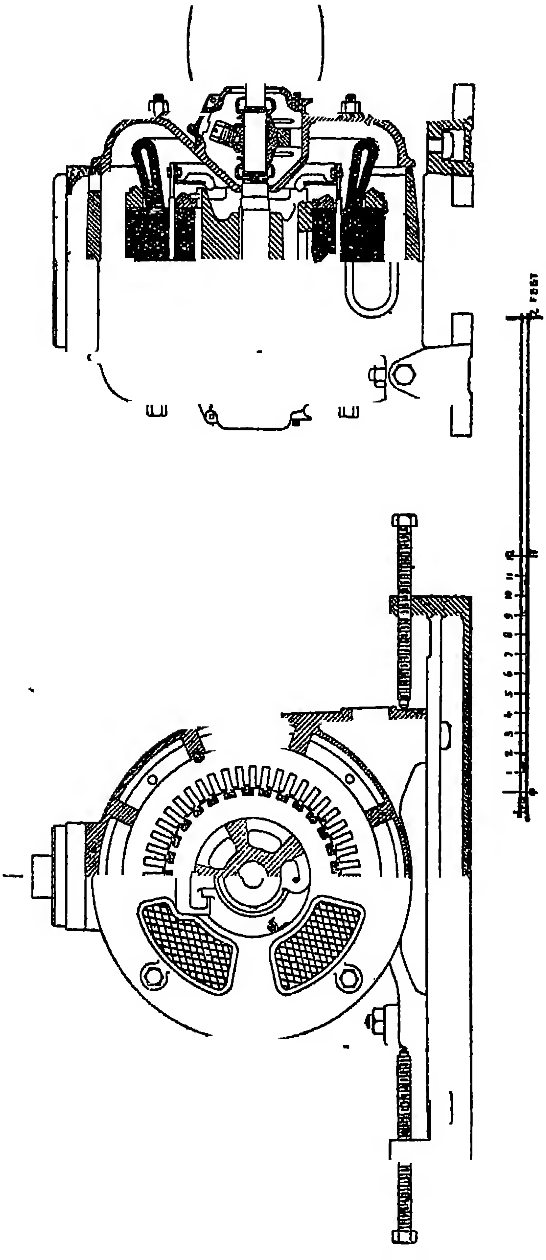


FIG 111.—Five Horse-power Induction Motor (The Electric Construction Co.)

With a *wound* three-phase motor, however, it is impossible to prevent the stator and rotor slots from having a common factor. For since there must be at least three slots per pole per phase (corresponding to the three phases), the stator and rotor slot numbers must both be multiples of six. It is found, accordingly, that the starting torque of such motors has a larger value in certain positions of the rotor than in others.

§ 75. Asymmetry of Hemi-tropic Stator Winding with Odd Number of Pole-pairs. Method of obtaining Symmetry

When the hemi-tropic type of winding (§ 48) with alternately straight and bent coils (Fig. 87) is adopted for the stator, a slight asymmetry is introduced into the arrangement of the coils in cases where there is an *odd* number of pairs of poles. This will be readily seen by considering a six-pole motor. With a hemi-tropic winding there will be three coils per phase (one coil per *pair* of poles), or a total of nine coils. If we proceed to arrange the coils on the stator core, making them alternately straight and bent, we are left with an odd coil, and in order to fit this in between a bent coil on one side

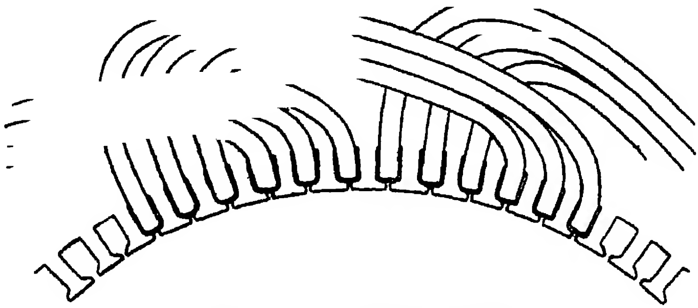


FIG. 112.—Type of Stator Winding.

and a straight coil on the other, one half of it must be "straight" and the other "bent." The shape of the projecting ends of the odd coil will thus be somewhat irregular, and will differ from that of the ends of the remaining two sets of coils. This asymmetry may, however, be entirely done away with by adopting the arrangement shown in Fig. 112. Instead of being made alternately straight and bent, the coil ends are here all of the same type, being bent obliquely so as to clear each other.

§ 76. General Characteristics and some Technical Data of Induction Motors

In its general characteristics the polyphase induction motor resembles the ordinary shunt-wound continuous-current motor. It is essentially a constant-speed motor, the drop of speed from no load to full load being very slight. One of the disadvantages of this type of motor is its somewhat low average power factor, on account of which the current drawn from the mains is larger than it need be for the amount of power developed by the motor. The power factor of an induction motor which is running light is very low, being generally less than 0·2. It increases, however, very rapidly with increase of load. The slip at full load, when expressed as a percentage of the speed of the stator field, varies from about 10 per cent. for very small motors to below 2 per cent. for very large ones. The following table may be taken as representing the average performance of modern induction motors :—

Horse-power	1	5	20	300	1000
Efficiency at full load	80	85	88	93	95
Full-load power factor	0·80	0·84	0·87	0·91	0·94
Slip as percentage of field speed	10	6	4	2	1·2

The power factor of an induction motor depends very largely on the length of air-gap, and it is important to keep the air-gap as small as mechanical considerations will allow. The extreme shortness of the air-gap of induction motors is an important constructional feature, which forms a striking contrast to the relatively very long air-gap of the continuous-current type of motor. Thus, taking the case of a railway motor, the air-gap of the continuous-current type will generally lie between 3 and 7 mm.; whereas an induction motor will have a gap ranging from 1 to 3 mm. Even a 250-h.p. modern induction motor may have a gap as small as 1·5 mm.*

Owing to the shortness of the air-gap, exact centering of the rotor relatively to the stator is a matter of great importance, as a slight amount of eccentricity may result in a strong side-pull, tending to bend the shaft. The bearings must be of very ample proportions, so that the wear is inappreciable. The effect of a slight amount of eccentricity in producing a side-pull has formed the subject of several

* According to K. Pichelmayer (*Dynamobau*, p. 524), the least permissible value of the air-gap length, assuming first-rate workmanship, may be taken as given by $\delta = 0{\cdot}03 + 0{\cdot}02\sqrt[3]{D}$, where δ is the gap length and D the rotor diameter, both expressed in cm.

investigations. The following very simple formula is given by J. K. Sumec:—*

$$P = \frac{B^2}{8\pi} \cdot S \frac{\varepsilon}{\delta} \cdot \frac{1}{\left\{1 - \left(\frac{\varepsilon}{\delta}\right)^2\right\}^{\frac{3}{2}}}$$

where P = total side-pull, in *dynes;* B^2 = mean square of magnetic induction around the rotor periphery, on the assumption that the rotor is exactly coaxial with the stator; S = total cylindrical surface of rotor, in sq. cms.; δ = (single) air-gap; ε = eccentricity (distance between axes of stator and rotor).

The depth of slot depends on the pole-pitch (*i.e.* the distance, measured along the rotor circumference, between the centre lines of two consecutive poles). It varies from about 1 in. (2·54 cm.) for a pole-pitch of 5 in. (12·5 cm.) to about $1\frac{3}{4}$ in. (4·45 cm.) for a pole-pitch of 20 in. (50·8 cm.). The number of slots per pole per phase is proportional to the pole-pitch, being at the rate of about 0·6 slot per pole per phase per inch of pole-pitch.†

Assuming a sine distribution of the magnetic flux in the air-gap, the maximum gap induction generally lies between the limits of 5000 and 6000.

The (r.m.s.) ampere-conductors per cm. length of rotor periphery vary from about 200 to about 300.

In order to avoid difficulties when starting and excessive noise arising from vibration while running, the rotor slots should be fewer than the stator slots, and should differ from them by P if P is even, and by 2P if P is odd, where P is the number of pole-pairs.‡

§ 77. Dimensions, Weights, and Prices of Polyphase Induction Motors

If we make use of the formula—

$$\text{brake horse-power} = kld^2m,$$

milar to that employed in § 56 in connection with alternators, l denoting the gross core-length (*i.e.* the length including ventilating ducts) and d the diameter of the rotor, and m the revolutions per minute of the motor, we find that k has a value ranging from about 16×10^{-6} for a motor of 5 b.h.p. to about 40×10^{-6} for a motor of 1000 b.h.p. if l and d be expressed in inches (the corresponding

* *Zeitschrift für Elektrotechnik* (Wien), vol. xxii. p. 727 (1904)

† Macfarlane and Burge, *Journal of the Institution of Electrical Engineers*, vol. xlii p. 249 (1908).

‡ W Stiel, *Zeitschrift Vereines Deutscher Ingenieure*, vol. lxv. p. 152 (1921).

values for the cm. as the unit of length are about 1×10^{-6} and $2{\cdot}5 \times 10^{-6}$).

The ratio l/d is dependent on the number of poles, the core-length l being in most cases about equal to the pole-pitch.

Since for a given size of motor frame the output will increase roughly in proportion to the speed, it is clear that both the weight and the cost of a motor of given output will depend not solely on the output, but on the ratio of the output to the speed. In the following table are given the approximate weights and prices of motors for various values of the ratio $\dfrac{\text{b.h.p.}}{\text{revs. per min.}}$.

$\dfrac{\text{b.h.p.}}{\text{revs. per min.}}$	0·001	0·002	0·005	0·01	0·02	0·05	0·1	0·2	0·3	0·5	0·75	1·0	1·2	1·4	1·6
Weight, in cwt.	2·2	2·7	3·7	5·2	8·0	16	30	54	71	96	124	152	165	180	194
Price, in £	15	22	31	39	53	85	133	220	284	372	450	515	547	570	590

The weights and prices given in the above table are for motors with wound rotors, mounted on foundation rails. Squirrel-cage motors are from 25 to 10 per cent. cheaper, the difference of price decreasing with increase of output.

If we consider merely the weight of the *active* materials, we find that the ratio of the weight of active iron to that of copper varies from about 3 to about 2, decreasing with increase of output.

CHAPTER IX

§ 78. Alternator used as motor. Synchronism—§ 79. Stability of synchronous motor—§ 80. Magnitude and phase of current for various conditions of load. Overload capacity—§ 81. V curves of synchronous motor—§ 82. Condenser action of over-excited synchronous motor. Use of synchronous motor as compensator—§ 83. Hunting of synchronous motor—§ 84. Prevention of hunting—§ 85. Starting of synchronous motor—§ 86. Paralleling or synchronization of alternators—§ 87. Everett-Edgcumbe rotary synchronizer—§ 88. Siemens and Halske three-phase synchronizer—§ 89. Parallel running of alternators. Starting of new machines.

§ 78. Alternator used as Motor. Synchronism

A SINGLE-PHASE alternator is, like a continuous-current dynamo, a *reversible* machine—*i.e.* it is capable of being *driven* as a motor when supplied with alternating currents. The possibility of using a single-phase alternator as a motor is immediately obvious. For, during the rotation, each armature conductor comes alternately under cover of poles of north and south polarity. If, then, we send current impulses through the armature winding so timed that they always give rise to a driving torque, a series of impulses will be communicated to the rotor, and the effect will be the same as that of a steady driving torque whose value is equal to the mean value of the fluctuating torque due to the current impulses. Since these current impulses must obviously alternate in direction, a reversal of current taking place in an armature conductor as it passes from a field of one polarity into a field of opposite polarity, it is evident that they will constitute an alternating current. We thus see the possibility of communicating a driving torque to the machine by sending an alternating current of suitable frequency through its armature. The frequency of this current must obviously be the same as that of the e.m.f. generated in the armature coils. Hence the frequency of the alternating current which drives the machine is equal to $P \times \frac{m}{60}$, where P = number of pairs of poles and m = revs. per min. The corresponding speed of the alternator is known as the *synchronous* speed, or speed of *synchronism*, and it is evident that an alternator is only capable of running as a motor at this particular speed.

A polyphase alternator may also be used as a motor. This is

at once evident from the fact that the polyphase alternating currents flowing in the armature windings give rise to a rotating field, and if we imagine the magnet wheel to be rotating at the same speed—that of *synchronism*—and to be suitably placed relatively to the field due to the armature currents, a driving torque will be exerted on it. As in the case of a single-phase machine, a polyphase synchronous motor is only capable of running at one particular speed—that of synchronism. For otherwise we should get an irregular succession of driving and retarding impulses, whose mean algebraic value is zero.

Another way of regarding such a polyphase motor is to consider each armature phase as acting independently, and giving rise to a fluctuating driving torque, such as we get in a single-phase machine. The fluctuating torques due to all the phases become fused into a single steady driving torque.

On account of the fact that alternators, whether single- or polyphase, when used as motors are only capable of running at one particular speed—that of synchronism—they are termed *synchronous motors.*

§ 79. Stability of Synchronous Motor

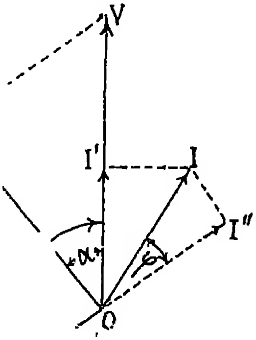
We have assumed the alternating current to be so adjusted as to give rise to the necessary value of the driving torque required to overcome all the resistances to the motion. We shall now show that when a motor is so running, with a definite p.d. across its terminals, it is in a condition of *stability*—*i.e.* any tendency on the part of the motor to run faster or slower, due to a decrease or increase of load, is *automatically* checked by a suitable change in the magnitude and phase of the current.

It is obvious that in order to provide the necessary driving torque, the succession of impulses contributed by the current in any one phase need not necessarily be of the same sign; we may, for example, have each large driving impulse succeeded by a smaller retarding impulse, so that there is, on the whole, a preponderance of driving impulses, and a resultant mean driving torque. Such a succession of alternate impulses will occur if the reversal of current does not take place at the precise moment when an armature conductor is passing from a field of one polarity into a field of opposite polarity; and corresponding to each wave of current there will be *four* impulses—two large driving ones and two small retarding ones.

Let us suppose that the p.d. across the armature of the synchronous motor and the exciting current supplied to its field are maintained constant. Let V = p.d., E = open-circuit e.m.f. corresponding to

given exciting current, and I = armature current per phase. In what follows, we shall consider the action of one phase only, so that the reasoning will apply to both single- and poly-phase motors.

Let, in Fig. 113, OV = V denote the p.d., and OE = E the e.m.f. induced in the armature winding. The vector resultant of these two, OR, gives the e.m.f. available for overcoming the armature impedance. The current is represented by OI = I, and lags behind OR by an angle whose tangent $= \frac{\text{armature reactance}}{\text{armature resistance}}$.* If OI′ = projection of OI on OV, and OI″ = projection of OI on OE produced backwards, then V × OI′ (= VI cos /IOV) represents the total electrical power supplied to the motor (excitation not included, of course), while E × OI″ (= EI cos ϕ) represents that portion of the total power which undergoes conversion into mechanical power.



otor Diagram of ous Motor.

If the load is suddenly decreased, acceleration begins to take place, and the vector OE swings forward (*i.e.* in a counter-clockwise direction) gradually gaining in phase on OV. The effect of this is to reduce OR, and so to reduce the current OI in the same ratio. The driving power † E × OI″ is thereby decreased, and this decrease will go on until the driving power becomes equal to that required to deal with the decreased load.

The opposite effect takes place with a sudden increase of load. Thus the current taken by the motor automatically adjusts itself, both as regards magnitude and phase, to the exact value required; hence the motor is running under stable conditions.

* It is here assumed that the armature possesses a definite constant self-inductance —an assumption which is by no means legitimate. It must, therefore, be understood clearly that the theory given is only *approximate*.

† By the "driving-power" is here meant the total power which actually undergoes conversion into mechanical power, the "driving power" therefore includes the power lost in overcoming frictional resistances.

§ 80. Magnitude and Phase of Current for Various Conditions of Load. Overload Capacity

By assuming various positions for OE relatively to OV, we can determine the load with which the motor is capable of dealing for each position of OE, and the magnitudes and phase relations of OR and OI relatively to OV. This might be done graphically, but more accurate results may be obtained by calculation, as follows:—

If θ = angle by which the motor e.m.f. is in advance of the p.d., r = resistance, and ωL = reactance of armature, then we have—

$$\mathrm{OR} = (V^2 + E^2 + 2VE \cos \theta)^{\frac{1}{2}} \quad \text{. . . . . (1)}$$

$$I = \frac{\mathrm{OR}}{\sqrt{r^2 + \omega^2 L^2}} \quad \text{. . . . . (2)}$$

$$\sin \alpha = \frac{E}{\mathrm{OR}} \sin \theta \quad \text{. . . . . (3)}$$

$$\phi = \pi - \left(\theta + \tan^{-1}\frac{L\omega}{r} - \alpha\right) = \pi - \tan^{-1}\frac{L\omega}{r} - (\theta - \alpha) \quad . \text{(4)}$$

$$\text{driving power} = EI \cos \phi \quad \text{. . . . . (5)}$$

The above five equations enable us to find the values of I and the driving power for various values of θ; V and E being maintained constant.

For given values of V and E, the relations connecting θ and I, and θ and driving power, may be graphically exhibited by means of curves. Such curves have been plotted in Figs. 114 and 115, for the case of a motor whose armature has a resistance of 0·2 ohm, and a reactance of 2 ohms, the p.d. being maintained constant at 1000 volts throughout. The different curves relate to different values of the motor e.m.f., *i.e.* to different excitations. The ascending portions (shown dotted) of the power curves in Fig. 115 correspond to a condition of instability. For, let us suppose that the motor is running under conditions corresponding to a point on the dotted branch of one of the power curves, and let there be a slight increase of load. This will cause a retardation of the rotor, *i.e.* a decrease of θ—the angle by which the motor e.m.f. is in advance of the p.d. Now, along the dotted branch of a curve a decrease of θ will be accompanied by a *decrease* of driving power, so that the retardation will go on, and the motor will drop out of step. On the other hand, if we suppose a slight decrease of load to take place, acceleration will result, θ increasing, and with it also the driving power, so that the acceleration will go on until the top of the curve is passed, and some point

on the stable descending branch is reached. Along any descending branch, the condition of stability is satisfied, a momentary retardation

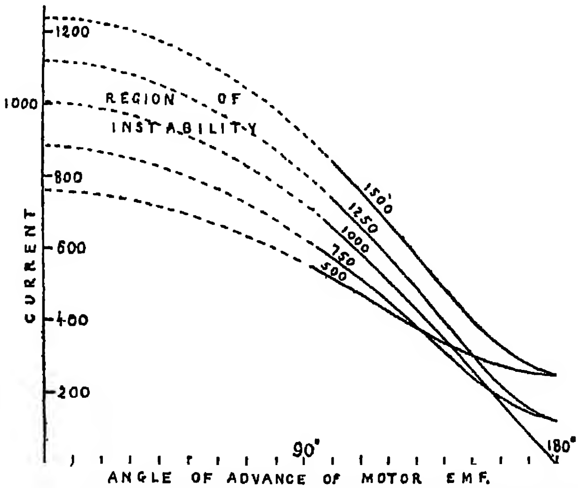


FIG. 114.—Variation of Current with Angle of Advance of Motor e.m.f.

resulting in an increase of driving power, and a momentary acceleration in a decrease. The conditions represented by the dotted branches

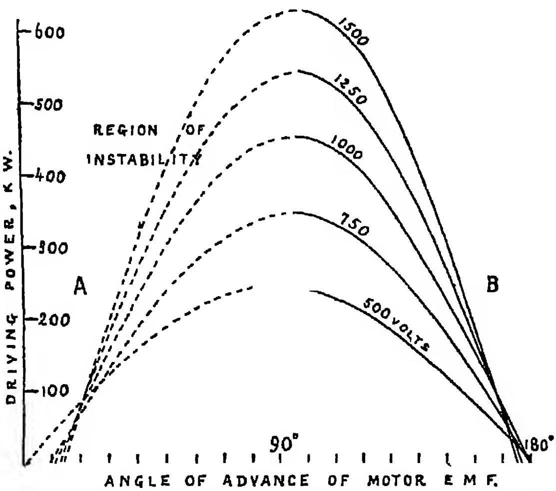


FIG. 115.—Relation connecting Driving Power with Angle of Advance of Motor e.m.f.

of the power curves could therefore exist only momentarily—as, *e.g.*,

when, owing to an extremely heavy overload, the motor is caused to drop out of step. The values of the armature current corresponding to unstable conditions of running are also shown dotted in the current curves of Fig. 114.

The power curves of Fig. 115 show very clearly that corresponding to each excitation (or each value of the motor e.m.f.) there is a certain maximum load—represented by the top of the curve, where it becomes

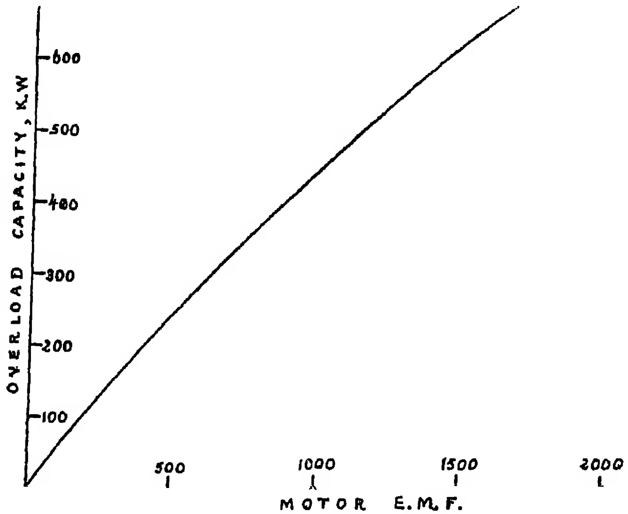


FIG. 116.—Connection between Excitation and Overload Capacity.

horizontal, and where the stable descending branch passes into the unstable ascending one—beyond which the motor will refuse to run. This maximum load is seen to increase with the excitation, and the relation connecting the maximum load with the motor e.m.f. is represented graphically in Fig. 116. From this it will be seen that if a large overload capacity is desired, the motor e.m.f. should have a relatively large value.*

* If the excitation is increased indefinitely, then beyond a certain value the overload capacity *decreases* with further increase of excitation. The value of the excitation corresponding to this is, however, so great as to be practically unrealizable under ordinary conditions of working.

§ 81. V-Curves of Synchronous Motor

By assuming any constant value for the load, and drawing a straight line such as AB in Fig. 115 corresponding to this load, we can determine, from the intersections of this line with the consecutive power curves, the values of θ corresponding to various motor e.m.f.s. By then referring to the curves of Fig. 114, we can find the values of the armature current corresponding to the different values of θ, and so obtain the relation connecting the armature current with the e.m.f. when the load is maintained constant. One such curve, corresponding to a load of 250 k.w., is shown in Fig. 117. It will be noticed that with this load the motor will not run at all unless its excitation

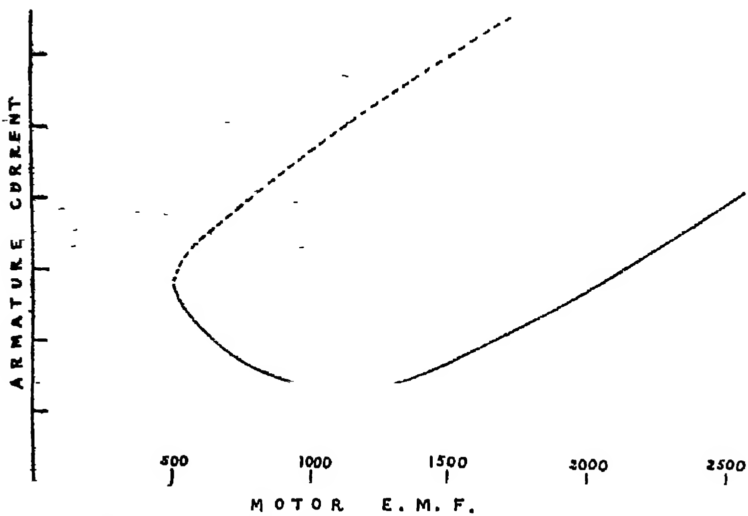


Fig. 117.—Relation connecting Excitation and Armature Current.

is such that the e.m.f. exceeds about 500 volts. As the exciting current increases, the armature current decreases to a minimum value, and for still higher excitations again increases. This variation of the armature current with varying excitation is an important characteristic of the synchronous motor, first pointed out by Mr. Mordey.* The variations in the armature current for a given range of variation in the excitation are greater at lighter loads, and the curve obtained when the motor is running light is more nearly V-shaped than that

* *Journal of the Institution of Electrical Engineers*, vol. xxii. p 128.

shown in Fig. 117. On account of their shape, these curves of a synchronous motor are frequently spoken of as its V-curves. The dotted branch represents conditions of instability.

§ 82. Condenser Action of Over-excited Synchronous Motor. Use of Synchronous Motor as Compensator

A reference to the vector diagram of Fig. 113 shows at once that when θ and ϕ are known, we can find the angle VOI $= \psi$, since $\psi = \pi - (\theta + \phi)$. Now, ψ is the angle by which the armature current lags behind the p.d., so that cos ψ gives us the power factor. By once more making use of the power curve of Fig. 115, and equations (1), (3) and (4) of § 80, we can determine the relation connecting motor e.m.f. with the angle ψ for a constant value of the load. We then find that for small values of the e.m.f., ψ is positive—*i e.* the current lags behind the p.d.; as the excitation is increased, ψ decreases, reaching a zero value (corresponding to a power factor of unity) when the armature current is at its minimum, and beyond this point it assumes a negative value, corresponding to a *leading* current. A strongly excited synchronous motor thus behaves as if it possessed capacity.

The fact that by sufficiently increasing the excitation of a synchronous motor it may be made to take a large *leading* current, has been practically applied in a number of instances, notably in the United States. In cases of power transmission over long distances, the load has not unfrequently a comparatively low power factor, so that a large lagging current has to be transmitted along the line. Not only does this reduce the efficiency of transmission, but—a more important matter—it causes a large drop along the line, rendering satisfactory regulation very difficult. The regulation might be improved by using larger generators at the generating station. But a cheaper solution of the problem has in some cases been found by installing a synchronous motor at the receiving end of the line, the excitation of the motor being adjusted so that the leading current which is taken by it exactly balances the lagging current taken by the load. The wattless current is thereby entirely confined to the local circuit formed by the synchronous motor and the load, while the line current is in phase with the p.d. at the receiving end of the line. The synchronous motor here plays the part of a compensator for the wattless current of the load.

§ 83. Hunting of Synchronous Motor

A trouble which sometimes arises in connection with synchronous motors is that of *hunting*, or *phase-swinging*. By this are meant the persistent periodic fluctuations in the speed and armature current of the motor which are observed under certain conditions. It is not difficult to see how such fluctuations may be started. Let us suppose that the generator supplying the p.d. runs at an absolutely uniform speed, so that the vector OV of Fig. 113 has a perfectly constant speed. Let the motor be also running at a steady speed under a constant load, the angle θ remaining constant. Suppose now that a sudden change of load takes place—say an increase. A momentary retardation of the motor results, and this retardation will go on until the increase in the driving torque (or the driving power), consequent on the decrease of θ (see power curves, Fig. 115), becomes equal to the increase in the resisting torque due to the larger load. When the retardation * ceases, however, the motor is running at a lower speed than that of synchronism. Hence θ will *go on* decreasing; the driving torque will accordingly increase above the value required to overcome the resisting torque, acceleration will take place, and the speed of the motor will increase; when the speed of synchronism is reached, θ will cease to decrease,† and will then begin to increase, since acceleration is still taking place. The vector OE (Fig. 113) now gains on OV. As θ increases, the driving torque decreases, and at a certain stage becomes equal to the resisting torque. Acceleration now ceases; but at this point the motor is running at a speed above synchronism, so that θ will *go on* increasing, and the driving torque decreasing: retardation takes place and synchronous speed is reached, when θ becomes a maximum. Further retardation now takes place, θ begins to decrease, and so on.

We see, then, that any sudden change of load will cause the speed of the motor and the current taken by it to undergo fluctuations. Such fluctuations would be indicated by an ammeter in the armature circuit. The irregular motion of rotation which causes the fluctuations may be regarded as consisting of a uniform motion of rotation at synchronous speed, combined with a to-and-fro or pendular motion. The pendular motion is, in the vector diagram of Fig. 113, represented by the swaying to and fro of the vector OE relatively to the vector OV. Under ordinary circumstances, the pendular motion will die out after a time, owing to the resistances

* Retardation = rate of decrease of angular velocity.

† OV and OE (Fig. 113) now having the same angular velocity—that of synchronism—and hence θ becoming momentarily constant.

encountered by it; these resistances are due to friction, hysteresis, and eddies.*

A sudden change of load is thus seen to start oscillations, which become superposed on the uniform rotation of the motor. There are, however, other ways in which such oscillations may be started. Let, for instance, the load remain quite constant, but let the speed of the generator supplying the p.d. undergo a sudden increase. This corresponds, in the vector diagram of Fig. 113, to a sudden advance of OV towards OE (*i.e.* to a decrease of θ), and it is evident, from what has already been said, that this will start oscillations.

Again, a sudden change in the exciting current of either generator or motor will have a similar effect.

It must be clearly understood that the oscillations under consideration are very *slow* in comparison with the frequency of the alternating current which drives the motor. The vector diagram of Fig. 113 has to revolve a considerable number of times before a single to-and-fro oscillation of OE relatively to OV is completed. Hence it is that such oscillations are readily observed on the ammeter, whose pointer sways to and fro in time with the oscillations.

As already mentioned, oscillations started by any sudden disturbance, such as those we have considered, will gradually subside, their energy becoming dissipated by friction, hysteresis, and eddy currents. If, however, before the oscillations have been damped out a fresh disturbance arises, of such a nature as to reinforce the already existing oscillations, then their amplitude may be considerably increased. With a rapidly and suddenly fluctuating load on the motor, or a generator whose speed fluctuates regularly during each revolution, the disturbances will be repeated at intervals, a fresh disturbance starting new oscillations before those due to the previous disturbance have been damped out. Now, in general, the interval between two disturbances will not bear any definite relation to the natural period of the oscillations; as a result, the disturbances will act in such a way that sometimes they re-inforce the existing oscillations, while at other times they weaken them. An ammeter in the circuit will show this effect very clearly; its pointer sometimes swinging violently—showing that the disturbances are re-inforcing the natural oscillations—while at other times it remains nearly stationary.

A particularly troublesome condition arises when the disturbances have a definite period not differing greatly from the natural period of

* The frictional, hysteresis, and eddy-current losses here referred to are merely the *additional* losses brought about by the *oscillations* of the rotor, and must be carefully distinguished from the *ordinary* losses of this nature which are due to the rotation at constant angular speed. Any change in θ causes a shifting of the flux across the pole-face, so that as θ oscillates the magnetic flux sways to and fro across the pole-pieces, giving rise to *additional* hysteresis and eddies, over and above the normal losses occurring when θ has a constant value.

the oscillations. The time during which a series of reinforcing disturbances is received may then become so considerable, and the amplitude of oscillation of θ may increase to such an extent, as ultimately to carry it beyond the top of the power curve (Fig. 115), into the region of instability, when the motor will "topple over," dropping out of synchronism.

One of the most troublesome consequences of the pendular motion we have just considered, which is variously known as "hunting," "pumping," "surging," or "phase-swinging," are the comparatively large fluctuations in the generator p.d. caused by the current fluctuations. If the generator supplying the motor is also used for feeding incandescent lamps, the hunting of the motor will cause the brightness of the lamps to vary periodically, rendering the lighting extremely unsatisfactory.

§ 84. Prevention of Hunting

One method of reducing the amplitude of the oscillations is immediately suggested by an inspection of the power curves of Fig. 115. From these power curves it is evident that a given change of torque will be obtained with a smaller change of θ when the field is strong than when it is weak. Thus the use of strong fields is favourable in checking hunting.

A highly effective and largely used method of preventing hunting

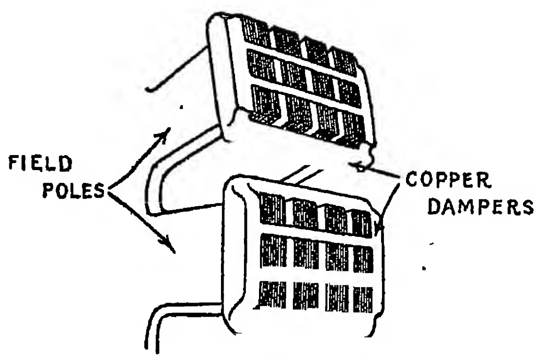


FIG. 118.—Damping Grids fitted to Alternator Poles.

and its troublesome accompaniments is that in which special devices, known as "damping coils" or "dampers," are employed to dissipate the energy of the oscillations.

One of the earliest forms of damper consisted of a solid band of copper closely surrounding the top of the pole-piece. This device

was not very effective, as it is merely capable of resisting a change in the total flux—*i.e.* fluctuations in the demagnetizing effect of the armature current—but offers no resistance to the periodic distortion of the field, or the swaying of the flux inside the pole-piece (corresponding to fluctuations in the cross-magnetizing effect of the armature current).

A more effective form of damper is that illustrated in Fig. 118. It consists of a regular grid of copper embedded in the pole-piece, the outer bars forming a closed band around the pole-piece.

Leblanc's damper ("amortisseur") is shown in Fig. 119. A series of thick rods of copper is embedded in each pole-shoe, and these rods

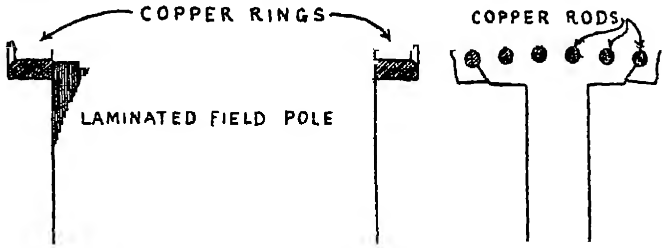


Fig. 119.—Leblanc's "Amortisseur."

are connected at each end by heavy rings of copper. The system practically forms a squirrel-cage rotor winding like that used in induction motors, and any oscillations of the flux relatively to the field-poles are damped so powerfully that the arrangement is capable of preventing hunting even in the most troublesome cases.

§ 85. *Starting of Synchronous Motor*

In starting a single-phase synchronous motor, it is necessary to run the machine up to the speed of synchronism—*i.e.* up to the speed at which the frequency of the motor e.m.f. equals that of the supply p.d.—and to adjust the phase of the motor e.m.f. so that it is in opposition to the p.d. The excitation is conveniently adjusted to such a value as to make the e.m.f. equal to the p.d. At the instant of closing the armature switch, no current will in that case pass through the armature. If the supply of power to the auxiliary motor used in starting up the synchronous motor be now cut off, the motor will begin to undergo retardation, and receive power as already explained (§ 79). The load may now be put on, and the excitation adjusted to correspond to minimum armature current—*i.e.* to maximum power factor.

The auxiliary motor employed in starting up the synchronous motor may be either an induction motor having a smaller number of poles than the synchronous motor, so that it is capable of running up to a speed slightly above the synchronous speed of the main motor, or it may be the exciter used as a continuous-current motor, and supplied with continuous current from any available source.

The operation of running the motor up to synchronous speed and phase opposition of its e.m.f. relatively to the p.d. is termed *synchronization*. The operation of synchronizing will be considered in detail in connection with the parallel running of alternators (§ 86).

Polyphase synchronous motors may also be started and synchronized by means of suitable auxiliary motors. A polyphase synchronous motor may, however, be made self-starting by providing its field with a squirrel-cage winding,* as shown in Fig. 120. A number of copper bars are threaded through slots in the pole-shoes, and are on each side bolted to copper rings. The arrangement resembles the squirrel-cage winding of an ordinary induction motor, except that there are gaps (in the interpolar spaces) between the consecutive groups of conductors embedded in the pole-shoes. When an alternating p.d. is impressed on the armature, the motor starts as an induction motor, and gradually gains speed. As the polar regions of the armature core sweep past the pole-shoes of the field, the latter experiences an alternately accelerating and retarding torque, due to the fact that each field pole-shoe tends to approach the nearest armature pole.† Hence superposed on the driving torque due to the squirrel-cage winding we shall have an alternating torque caused by the fact that the field has well-defined projecting or salient poles. So long as the speed of the field falls considerably below that of synchronism, the effect of this alternating torque is slight, as each half-wave of torque is of comparatively short duration, and the corresponding impulse (mean torque × time) is insufficient to alter the speed materially. But as synchronism is approached, the impulses due to the alternating torques greatly

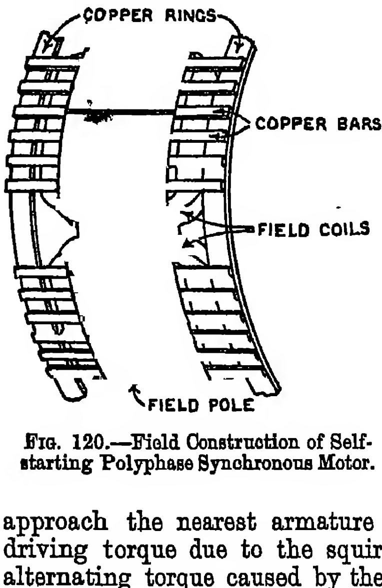


FIG. 120.—Field Construction of Self-starting Polyphase Synchronous Motor.

* *The Electric Journal*, vol. vi p 347 (1909).

† No appreciable effect of this kind would take place with a cylindrical type rotor.

increase in value, and oscillations are superposed on the uniform motion of rotation of the field. Finally, at a certain speed a driving impulse becomes powerful enough to pull the field into synchronism, after which synchronous rotation is maintained. The field current may then be switched on.

In order to prevent the induction of a dangerously high e.m.f. in the field winding at starting, it is best to close this winding through a resistance.

The squirrel-cage winding forms an excellent damping winding when the motor has run up to synchronism.

A similar method of starting may be employed with polyphase synchronous motors which are not provided with any special starting winding, the starting torque being in this case due to eddy-currents and hysteresis, and being much feebler than in the case of a motor provided with a squirrel-cage starting winding.

Since the polarity of the motor field after synchronism has been reached is uncertain, the field switch should be of the throw-over reversing type, and should be provided with auxiliary or pilot contacts, so arranged that a sufficiently high resistance is introduced into the field circuit when the pilot contacts are closed. The effect on the armature current of closing these contacts is noticed; if the armature current is increased by exciting the field through the pilot contacts, the field is excited the wrong way; but if the armature current is decreased, the excitation is in the right direction, and the field switch may be pushed home so as to close the main contacts and cut out the resistance.

A serious disadvantage of starting a motor in this manner is the very large starting current required. In order to limit this, auto-transformers are used, and in the case of large motors the p.d. is raised by several steps.

§ 86. Paralleling or Synchronization of Alternators *

Alternators are capable of being run in parallel like continuous-current generators. In order to throw an additional alternator into parallel with a number of others, it is necessary to adjust its excitation, speed, and phase so that its e.m.f. is about equal to the 'bus bar p.d., is of the same frequency, and in phase opposition to the p.d.†

* See note at end of chapter.

† *I.e.* so related to the p.d. that if the switch were closed the e.m.f. would just balance the p.d., and no current would flow. But since the e.m.f. and p.d. both tend to send a current in the *same* direction around the *external circuit*, it is usual to speak of the p.d. and e.m.f. as being *in phase* with each other (with respect to the external circuit), instead of in opposition of phase (which they are when considered with reference to the local circuit of the incoming machine).

This process of adjustment is known as *paralleling* or *synchronization*, and the instrument or collection of apparatus by means of which the proper phase relation is ascertained is termed a *synchronizer*.

An arrangement of synchronizer connections suitable for a single-phase machine is shown in Fig. 121. The synchronizer consists of two small transformers and an incandescent lamp or voltmeter. The primary P_1 of one transformer is across the 'bus bars, while that of the other—P_2—is across the terminals of the incoming machine. The secondaries S_1 and S_2 are connected in series with each other and with a lamp L (or voltmeter), the connections being so arranged that

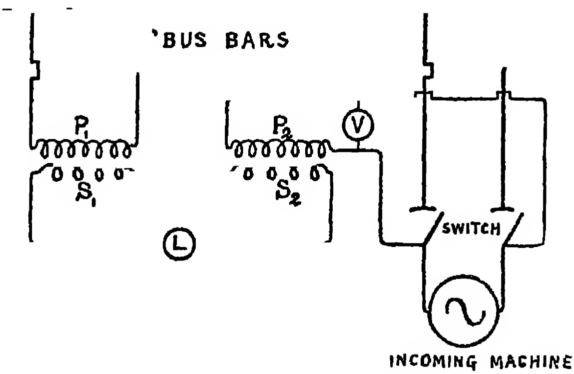


FIG. 121.—Connections of Synchronizer.

when the e.m.f. of the incoming machine is in phase opposition to the 'bus bar p.d., the e.m.f.s in the secondaries are added, and the lamp L glows brightly.* The e.m.f. of the incoming machine is adjusted to the desired value by means of the voltmeter V. When the machine is first started, the e.m.f. in the Secondary S_2 is very feeble and of low frequency, and the lamp L is barely aglow. As the amount and frequency of the e.m.f. in S_2 gradually increase, very rapid fluctuations in the light of the lamp become noticeable, and as synchronous speed is approached, the frequency of the fluctuations is reduced, the lamp slowly passing from a brilliant glow to complete darkness, and then slowly lighting up again. These fluctuations are easily explained by the fact that when the frequency of the e.m.f.s in S_1 and S_2 is nearly, but not quite, the same, one of the e.m.f. vectors (in the vector diagram of e.m.f.s) slowly gains on the other, gradually passing from coincidence of phase to phase opposition, and then again to coincidence

* In some instances, the connections are arranged so that with phase opposition of the 'bus bar p.d. and the machine e.m.f. the secondary e.m.f.s oppose each other and the lamp L is dark.

of phase, and so on. The resultant e.m.f., represented by the diagonal of the parallelogram constructed on the two e.m.f. vectors as sides, will thus slowly fluctuate between a maximum and a zero value. When the pulsations are as slow as it is practicable to make them, and the lamp is at its maximum brilliancy, the main switch is closed.

The lamp L forms a convenient visual signal for the engine-driver. In order to enable the switchboard attendant to ascertain more accurately the correct moment of closing the switch, a synchronizing voltmeter is frequently provided in addition to the lamp, the two being connected in parallel; the switch is closed when the voltmeter gives its maximum reading.

In Fig. 121 the primary P_1 of the sychronizing transformer is shown permanently connected across the terminals of the incoming machine. In a large generating station it would not, of course, be necessary to provide a synchronizing transformer for each machine, as the same synchronizer could be used for several machines in succession as required, the transformer being switched off as soon as the main switch has been closed. In order to enable the same synchronizing transformer to be used for all the generators, a set of "synchronizer 'bus-bars" is frequently provided.

Such a synchronizer might also be used in connection with three-phase machines, the Primary P_1 of the synchronizing transformer being connected across two of the 'bus bars, and the primary P_2 across the two corresponding terminals of the incoming machine.

A synchronizer of this form, although indicating any difference of frequency or phase, is incapable of showing whether the incoming alternator is running above or below the speed of synchronism. Arrangements for doing this are provided in the more elaborate types of synchronizer.

§ 87. Everett-Edgcumbe Rotary Synchronizer

The principle underlying the action of this instrument is as follows. Imagine a small induction motor, whose stator and rotor are both provided with polyphase (two- or three-phase) windings, the rotor winding being connected to slip-rings. If polyphase currents of the *same frequency* be supplied to stator and rotor, the magnetic fields due to them will revolve at the same speed, the rotor taking up a position which gives coincidence of the two fields, and remaining *stationary*. It is evident that any relative displacement of the fields will result in a couple tending to turn the rotor one way or the other according to the direction of displacement. If, therefore, we suppose the frequency of the rotor currents to be reduced below that of the

stator currents, the rotor field will tend to lag behind the stator field, and the rotor will be pulled round by the stator *in the direction of rotation* of the stator field, so that the two fields will keep in step as before, the speed of rotation of the rotor corresponding to the difference of the speeds of the stator and rotor fields. Similarly, if the frequency of the rotor currents be increased above that of the stator currents, the rotor field will tend to gain on the stator field, and the rotor will be *pulled back* by the stator field, so as to make both fields keep in step as before. The arrangement, therefore, furnishes an extremely accurate method of comparing two nearly equal frequencies, the rotor rotating one way or the other according as the frequency of the rotor currents is below or above that of the stator currents, and its speed of rotation being directly proportional to the frequency difference.

The stator and rotor cores of the Everett-Edgcumbe rotary synchronizer are shown at the top of Fig. 122. The two-phase currents for the four-pole rotor are obtained by adopting the arrangement of connections shown in Fig. 123. It will be seen that the two phases are connected in parallel across the mains,* the necessary phase difference being obtained by the insertion of a high non-inductive resistance (an incandescent lamp) in series with one phase, and a high reactance (a choking coil) in series with the other.† The core of the choking coil is shown in the middle of Fig. 122. The rotor runs in ball bearings, and a pointer attached to it, and arranged to move over the face of the dial-plate, as shown in Fig. 124, indicates by the direction and speed of its rotation whether, and by how much, the incoming machine is running fast or slow. The proper time of closing the main switch is when the pointer is vertical and stationary, or moving with extreme slowness.

A very useful feature of this instrument is the ingenious visual engine-driver's signal. In the upper part of the dial-plate, Fig. 124, is a circular opening, behind which is placed an incandescent lamp. In front of this lamp, and immediately behind the dial-plate, is a pivoted vertical aluminium swing-plate of the shape shown in the lower part of Fig. 122. This plate is fitted with two coloured transparent screens, a red and a green one, and is free to swing to one side or the other, so as to place one or other of the screens in front of the lamp. A red light indicates to the engine-driver that his engine is running too fast, a green light that it is running too slowly. The motion of the plate is limited by means of a ∏-shaped block of brass

* The connections are, however, made through two synchronizing transformers, one for the 'bus bars and the other for the incoming machine.

† The four-pole stator is provided with a single-phase winding. The interaction between the rotor and stator will be readily understood by supposing the single-phase or oscillating stator field replaced by two oppositely rotating fields (§ 21).

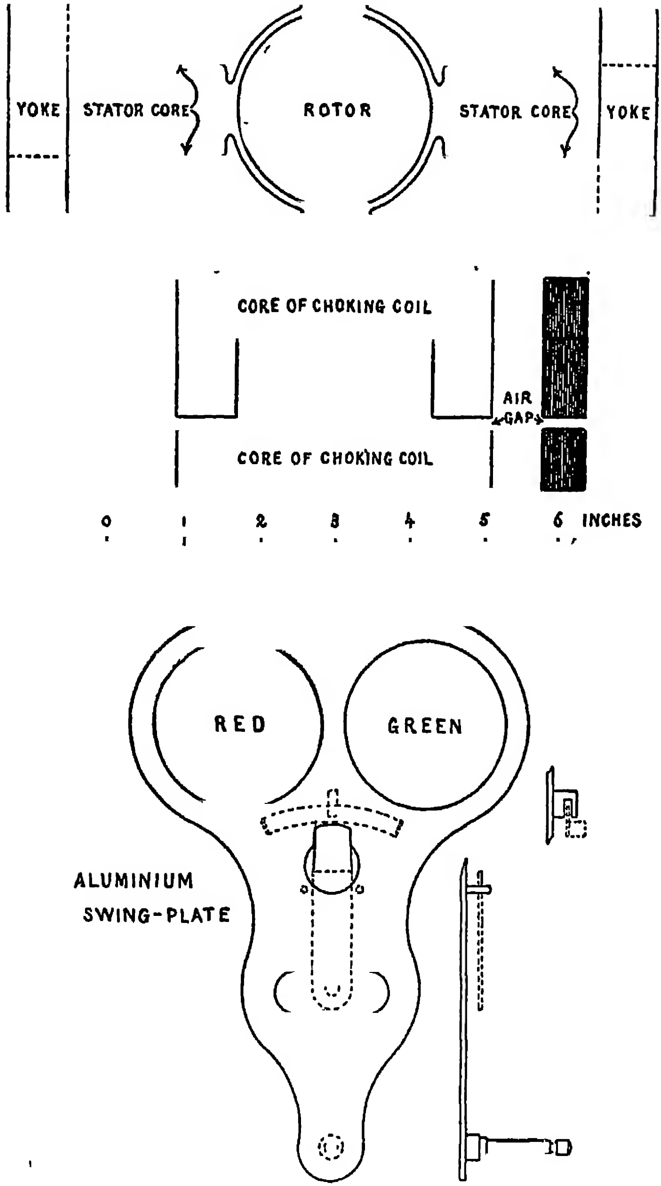


FIG. 122.—Details of Rotary Synchronizer.

attached to the back of the plate (Fig. 122), which embraces a circular strip of brass (shown dotted in Fig. 122), whose ends are bent so as

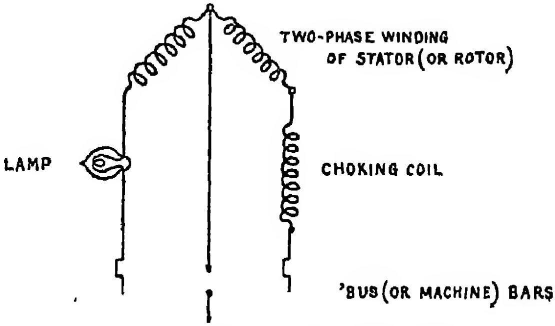


FIG. 123.—Connections of Rotary Synchronizer.

to form two stops. This strip is more clearly shown to a larger scale in Fig. 125; it is screwed to the heavy brass plate which contains the front ball bearing of the rotor. The mechanism which throws

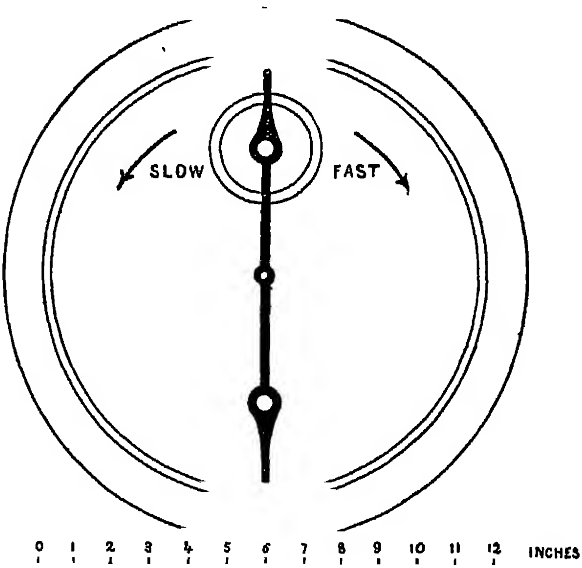


FIG. 124.—Dial-plate and Pointer of Rotary Synchronizer.

the swing-plate against one or other of its stops will be understood by reference to Fig. 125. Mounted *loosely* on the rotor spindle is a striker-plate (this is also shown dotted in the lower part of Fig. 122)

which is placed between two pins projecting from the back of the swing-plate (Fig. 122). The swing-plate is thereby carried round to one side or the other, according to the motion of the striker-plate.

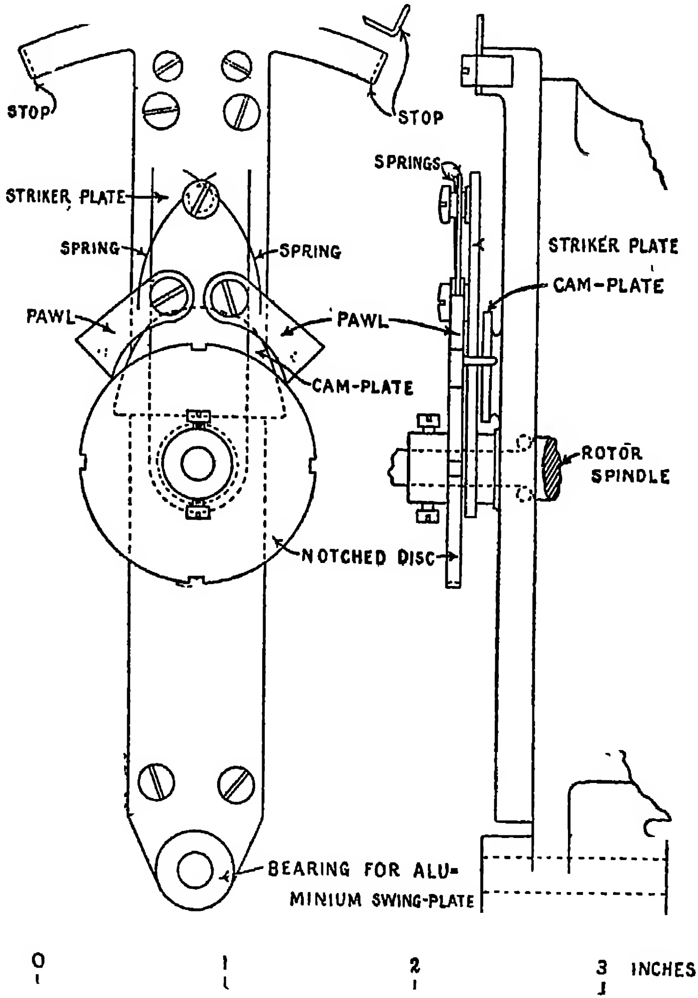


Fig. 125.—Mechanism of Visual Signal.

The latter is fitted with two pawls, which by the action of two light phosphor-bronze springs fixed into them, press against the edge of a notched disc rigidly mounted on the rotor spindle (Fig. 125). Each

pawl is provided with a pin, which, in the extreme position of the striker-plate, comes into contact with a cam-plate, thereby lifting the pawl out of the notch in the disc and allowing the disc to revolve without carrying the striker-plate beyond that position. In Fig. 125 the striker-plate is shown in the halfway position; but as soon as the rotor begins to move, the striker-plate will be thrown to the right or left, according to the direction of rotation, and will communicate its motion to the swing-plate.

When this synchronizer is used on a three-phase circuit, its stator is connected across two of the 'bus bars, and its rotor across two of the terminals of the incoming machine.*

§ 88. Siemens and Halske Three-phase Synchronizer

Messrs. Siemens and Halske have introduced a form of synchronizer for three-phase machines which is used very largely. The principle of this synchronizer will be understood by reference to Fig. 126. Let OA, OB, OC be the secondary windings of a transformer whose primaries are across the 'bus bars, and O′A′, O′B′, O′C′ the secondary windings of a transformer whose primaries are across the terminals of the incoming alternator; the connections being such that when the machine is ready to be switched in, the e.m.f.s in OA and O′A′, OB and O′B′, and OC and O′C′ respectively, are in phase with each other. In that case, it is evident, there will be no p.d. between A and A′, B and B′, and C and C′, so that three lamps connected between these pairs of points would all remain dark. If the incoming machine were running at a speed either slightly below or slightly above synchronism, the lamps would periodically and *simultaneously* brighten up and grow dark.

Let us next suppose that one lamp is connected between A and A′, another between B and C′, and a third between C and B′, as shown in Fig. 126. For the sake of simplicity, we may suppose the neutral points O and O′ of the two secondaries connected together. Let in each case the instantaneous value of the e.m.f. be reckoned positive when acting away from the neutral point. Then for any phase relation of the secondary e.m.f.s, the e.m.f. acting around the entire circuit which contains any lamp is clearly the vectorial difference of the e.m.f.s in the two phases of the star-connected windings on either side of the lamp; thus, the e.m.f. acting around OAA′O′O, the circuit which contains the lamp L_1, is the vectorial difference of the e.m.f.s in OA and O′A′; similarly for the remaining lamps L_2 and L_3. Consider now the instant at which all the e.m.f.s

* See Appendix VII. for description of Weston Synchroscope.

are in phase, as shown by the vector diagrams (*a*) and (*b*) in Fig. 127. The vector difference of OA and O′A′ is clearly zero, so that L_1 is dark. The vectorial difference of OB and O′C′ is OL_2, as shown

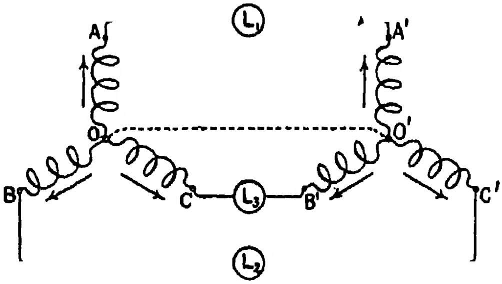


FIG. 126.—Three-phase Synchronizer.

in the vector diagram (*c*), and the vectorial difference of OC and O′B′ is OL_3 in diagram (*d*). It is thus evident that since $OL_2 = OL_3$, the lamps L_2 and L_3 will be partly incandesced. Thus at the proper instant for closing the switch L_1 is dark, while L_2 and L_3 are equally bright, but duller than when fully incandesced.

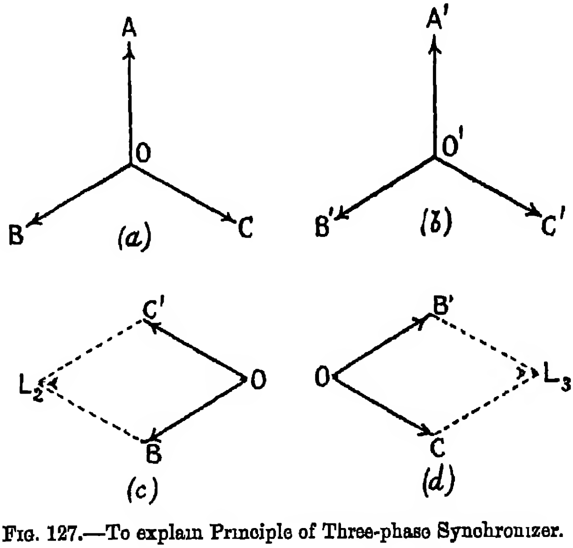


FIG. 127.—To explain Principle of Three-phase Synchronizer.

Let us now consider the changes which take place when the incoming machine is running below the speed of synchronism, so that the three vectors of the diagram (*b*), Fig. 127, gradually lag

behind those of diagram (*a*). It is evident that as the lag increases* the vector difference of OA and OA′ increases, the lamp L_1 beginning to glow. At the same time, the vector difference of OB and O′C′ decreases, owing to the increase of the angle BOC′ in diagram (*c*); the lamp L_2 therefore grows duller. Lastly, the vector difference of OC and O′B′ increases, owing to the decrease in the angle COB′ in diagram (*d*), hence the lamp L_3 grows brighter. Thus L_1 begins to glow, L_3 approaches maximum brightness, while L_2 grows less bright. This goes on until the vector O′B′ in (*b*) comes into phase opposition with OC in (*a*), corresponding to a lag of 60°; at this instant, L_3 is at its maximum brightness, while L_1 and L_2 are both equally bright, but much below maximum brightness. After another increase of 60° in the lag, O′C′ comes into phase coincidence with OB, and L_2 is dark. A further increase of 120° in the lag will bring O′B′ into phase coincidence with OC, and L_3 will become dark.

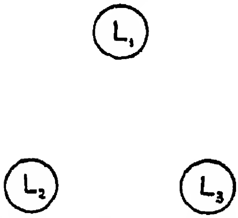


Fig. 128.—Showing Arrangement of Lamps of Three-phase Synchronizer.

We thus see that when the machine is running too slowly, the lamps darken and brighten in the order L_1, L_2, L_3. Similarly, it may be shown that when it is running too fast, the order of brightening or extinction is L_1, L_3, L_2.

Hence, by noticing the order in which the lamps go out we can ascertain whether the speed is too low or too high. In order to make this observation as easy as possible, the lamps are in practice mounted at the corners of an equilateral triangle, as shown in Fig. 128. The light will appear to travel round the triangle, counter-clockwise if the speed is too low, clockwise if it is too high.

In addition to the lamps, two voltmeters are generally provided: one for adjusting the e.m.f. of the incoming machine to the required value, and the other for ascertaining the exact instant of phase opposition with greater accuracy than is afforded by the darkening of the top lamp.

When synchronizing large alternators controlled by oil-insulated switches which are operated by solenoids or motors, it must be carefully borne in mind that such switches do not act instantly, but have a certain time lag, and due allowance for this must be made in the process of synchronization.

* The vectors are supposed, as usual, to revolve *counter-clockwise*, so that a lag of (*b*) relatively to (*a*) implies a *clockwise* rotation of (*b*) *relatively to* (*a*).

§ 89. Parallel Running of Alternators. Starting of New Machines

Let us suppose that an alternator has been duly synchronized and switched into parallel with other machines. By increasing the supply of power to the engine driving the alternator, it may be made to take part of the load. The phase relation of the alternator e.m.f. to the current and the 'bus bar p.d. is determined by (1) the mechanical power supplied to the alternator, and (2) the excitation.

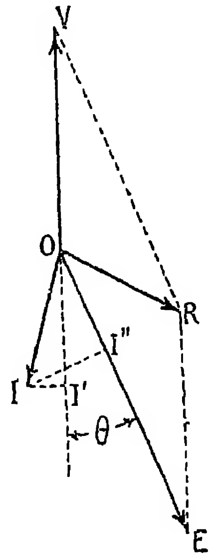


FIG. 129.—Vector Diagram of Parallel-connected Alternator.

In Fig. 129 is given a vector diagram—similar to that of Fig. 113—showing the connection between the e.m.f., p.d., and current, and their phase relations. OV, as before, denotes the p.d., V; OE the e.m.f., E; OR the impedance drop in the alternator armature, and OI the current, which lags behind OR by an angle $\tan^{-1}\frac{L\omega}{r}$. The total electrical power developed by the alternator is given by E × OI″, while the useful power transmitted to the 'bus bars is V × OI′.

By assuming different values for the angle θ, and supposing E to be constant, we can find the relation connecting θ with the useful or the total electrical power. This relation may be plotted in the form of a curve. Corresponding to each value of the exciting current, or to each value of E, we can find such a power curve. By this means a series of power curves similar to those shown in Fig. 115 for a synchronous motor may be obtained. From these curves we see that, within the limits of stability, an increase of θ results in an increase of power. If the torque exerted by the engine fluctuates during each revolution, it will tend to set up oscillations such as those we have already considered in connection with the running of synchronous motors. Any tendency towards hunting or phase-swinging may be counteracted by the use of damping circuits or of sufficiently heavy fly-wheels.

In order to test the connections of the synchronizer and those of the armature coils of a newly installed alternator, a good plan is to couple this alternator in parallel with another which is not running,

and to start both machines very slowly and simultaneously. Any error in the coupling of the armature circuits of the new machine will cause the circuit-breakers between the machines to open, while an error in the synchronizer connections will cause this instrument to give wrong indications when the machines are running smoothly in parallel.

Note on Synchronization of Large Alternators.

When alternators of large size are being synchronized, serious damage to the projecting ends of the coils may be done if the paralleling switch is not closed at precisely the right moment; since the momentary interchange of current is sufficient to call into play enormous mechanical forces tending to bend and crush the projecting coil ends (even if these are mechanically supported). In order to obviate all risk, it is advisable to use the method proposed by M. Brooks and M. K. Akers.* This consists in interposing a (coreless) choking or reactance coil between the 'bus bars and the incoming machine, the reactance of the coil being large enough to prevent serious damage should the switch be closed at the wrong moment, but small enough to take sufficient current to pull the machine into phase. The choking coil may be connected between the 'bus bars and the incoming machine either directly or through transformers. The cost of such a coil would not amount to more than about 1 per cent. of the cost of the alternator.

* *Proceedings of the American Institute of Electrical Engineers*, vol. xxv. p. 439 (1906).

CHAPTER X

90. Regulation of alternators and transformers—§ 91. Behn-Eschenburg and Kapp's method—§ 92. Kapp's diagram—§ 93. Algebraical method of determining drop—§ 94. Rothert's ampere-turn method—§ 95. Analysis of armature self-inductance into two components—§ 96. Experimental determination of two components of armature reactance—§ 97. Potier's method of predetermining the regulation of an alternator—§ 98. Numerical values of regulation. Saturation factor.

§ 90. Regulation of Alternators and Transformers

One of the most important problems connected with the behaviour of an alternator or transformer is that of *regulation*. By this is meant the ability of the apparatus to maintain a more or less constant terminal p.d. with varying load. In the case of an alternator, the exciting current is supposed to be constant; in that of a transformer, the primary p.d. is supposed constant. The regulation is generally expressed as the percentage rise of voltage from full load to no load. This rise depends very largely, as we shall see, on the nature of the load, *i.e.* on its power factor.

In the case of an alternator, two different methods of specifying the regulation have been used. According to one, the regulation is defined as the percentage *drop* of voltage when full load is *switched on*, the exciting current being adjusted to give the normal p.d. *on open circuit;* while, according to the other, the regulation is given by the percentage *rise* of voltage when full load is *switched off*, the value of the exciting current being such as to give the normal p.d. at *full load.* The second method of specifying the regulation is the one which has now come into general use.

The regulation of either an alternator or transformer may be determined by direct experiment if a suitable load and a sufficient amount of power be available for carrying out the test. In many cases, however, the power available is insufficient. Some indirect mode of arriving at the regulation thus becomes desirable. Again, the indirect method of determining the regulation leads to much more accurate results in the case of transformers than could be obtained, under ordinary testing conditions, by a direct test. Numerous attempts have been made to devise such indirect methods in connection with alternators, which present much more difficulty than transformers.

§ 91. Behn-Eschenburg and Kapp's Method

The earliest of these methods is one which is equally applicable to alternators and transformers. Its application to alternators we owe to Behn-Eschenburg,* while Kapp † devised a similar method for transformers. In this method it is assumed that the apparatus under test behaves as if it possessed a definite constant resistance, and a definite constant self-inductance, under all possible conditions of load. These assumptions are practically correct in the case of transformers, but by no means so in the case of alternators.

The values of the resistance and self-inductance are determined by the *short-circuit test.* This consists in short-circuiting the apparatus —the armature of the alternator in the one case, the secondary of the transformer in the other—through an ammeter of negligible resistance and self-inductance, and adjusting the exciting current in the one case, and the primary p.d. in the other, until the full-load current is obtained.‡ A second test, the *open-circuit test,* is then carried out. The apparatus is open-circuited, and a voltmeter connected across the armature in the one case, the transformer secondary in the other. The reading of this voltmeter is noted when the exciting current in the one case, and the primary p.d. in the other, has the same value as it had in the short-circuit test. Then the ratio $\frac{\text{voltmeter reading in open-circuit test}}{\text{ammeter reading in short-circuit test}}$ gives the impedance of the apparatus, and in the method about to be explained this impedance is assumed to remain constant under all conditions of load.

We have next to analyze the impedance into its resistance and reactance components. The resistance of the alternator armature will be approximately equal to its resistance as measured by means of continuous currents (owing to eddy currents, an apparent increase of resistance may be produced amounting, perhaps, to 20 per cent. in extreme cases). In order to find the equivalent resistance r of the transformer, § due to the resistances r_1 and r_2 of its primary and secondary respectively, we notice that, since for a given winding space and given mean length of turn, the resistance varies as the *square* of the number of turns, a resistance r_1 in the primary is equivalent to a resistance $r_1\left(\frac{S_2}{S_1}\right)^2$ in the secondary, S_1 and S_2 denoting the primary and secondary turns respectively. Thus the total equivalent resistance component of the transformer impedance is $r_2 + r_1\left(\frac{S_2}{S_1}\right)^2$.

* *The Electrician,* vol. xxxv. p. 424 (1895).
† *Elektrotechnische Zeitschrift,* vol. xvi. p. 260 (1895).
‡ See, however, § 101.
§ Referred to its secondary circuit.

The reactance component is given by $\sqrt{(\text{impedance})^2 - (\text{resistance})^2}$. Since the resistance component seldom exceeds $\frac{1}{10}$th of the reactance component, its exact determination—involving the allowance to be made for eddy currents—is not a matter of very great importance.* The best method of determining the total equivalent resistance of a *transformer* is to measure the total power taken by it during the short-circuit test, and to divide this by the square of the secondary current.

Having determined the (equivalent) resistance and reactance of our apparatus as explained above, we may construct the vector diagram connecting the constant open-circuit p.d. corresponding to any constant excitation (*i.e.* any constant value of the exciting current in the case of an alternator, and any constant primary p.d. in the case of a transformer) with the terminal p.d. for a given load current of known power factor. Let us provisionally assume the terminal p.d. to be known, and to be represented by OA in Fig. 130. Let ϕ be the angle of lag ($\cos\phi$ = power factor of load) of the current

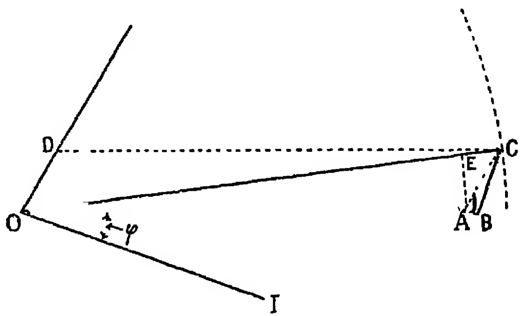


FIG 130.—Vector Diagram of Loaded Alternator or Transformer.

behind the terminal p.d., so that OI gives the direction of the current vector. We obtain the open-circuit p.d. (e.m.f. in case of alternator) by adding *vectorially* to the terminal p.d. OA the resistance drop rI = AB (which is parallel to the current vector OI) and the reactance drop ωLI = BC (this is perpendicular to OI).† We thus get the open-circuit p.d. OC. Instead of considering the resistance and reactance drops separately, we may consider the impedance drop

* It may be noted that the *alternator* reactance here considered is practically the *total* reactance of the armature (not merely its *leakage* reactance, § 95), *i.e.* it is practically identical with the value of the reactance which would be obtained if the (synchronously rotating) field were open-circuited and an alternating p.d. applied to the armature from some external source; whereas, in the case of the *transformer*, the reactance here considered is merely its *leakage reactance*, which is enormously smaller than the total reactance (or the reactance obtained by open-circuiting the primary and applying an alternating p.d. to the secondary).

† r = equivalent resistance, L = equivalent inductance, I = load current.

AC, which makes an angle $\tan^{-1}\frac{L\omega}{r}$ with the current vector. By reversing the construction, we can easily obtain the terminal p.d.s corresponding to various load currents, as follows. With O as centre, and radius = constant open-circuit p.d., describe an arc. Draw OA to represent the direction of the terminal p.d. vector, and OI to represent the direction of the current vector. From O draw a line making an angle $\tan^{-1}\frac{L\omega}{r}$ with OI, and along it lay off OD to represent the impedance drop corresponding to the given current. Through D draw DC parallel to OA, and through C draw CA parallel to DO. Then OA = OE gives the required terminal p.d., while EC represents the drop corresponding to the given current (the *percentage* drop is $100\frac{\text{EC}}{\text{OC}}$). It is evident from the diagram that the value of EC will, for a given load current, depend very largely on ϕ, and that when ϕ assumes a sufficiently large *negative* value—*i.e.* when the current becomes a *leading* one, such as might be obtained with a condenser or over-excited synchronous motor (§ 82) in circuit—EC may become negative, the drop becoming *added* to the open-circuit p.d. to give the terminal p.d. This happens when the angle which OD makes with OA becomes sufficiently obtuse to cause CA to intersect OA *outside* the circle.

§ 92. Kapp's Diagram

The variation of the drop with varying power factor for a given constant value of the load current is best studied by the aid of a very elegant construction due to Kapp.*

If the value of the load current remains constant, the vector OD in Fig. 130 will remain of constant length. Let us assume the directions of OI and OD as the fixed lines of reference in our diagram. As the power factor varies, the point C moves along the circumference of a circle having its centre at O. Now, the position of A is obtained by displacing C through a distance CA equal and parallel to DO. But since DO remains fixed in magnitude and direction, it follows that the locus of A may be derived from that of C by displacing the latter through a distance equal and parallel to DO. It is thus evident that the locus of A is a circle whose centre is at D′, as shown in Fig. 131, where OD′ = DO. The angle IOA is the angle of lag. The diagram shows very clearly the rapid increase in the drop with increasing angle of lag, the drop being represented by the *intercept between the two circles* of the radius vector drawn from O to A.† If

* *Loc. cit.*

† When speaking of the drop in connection with voltage regulation, we mean simply the *arithmetical* difference between the open-circuit p.d. and the terminal p.d.

the current is a *leading* one, *i.e.* if OA falls *below* OI, the drop steadily decreases with increasing angle of lead, vanishes at the

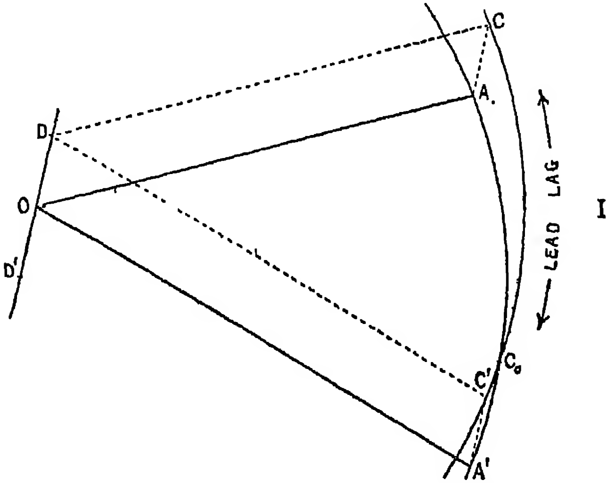


FIG. 131.—Kapp's Construction for Variation of Drop with Power Factor of Load.

intersection C_0 of the two circles, and beyond this point changes sign, becoming a *rise* instead of a drop.

§ 93. Algebraical Method of determining Drop

Although the graphical methods explained above are useful in exhibiting the relations connecting the various quantities in a manner which enables them to be more easily remembered than would be the case with a purely algebraical method, yet for the purpose of

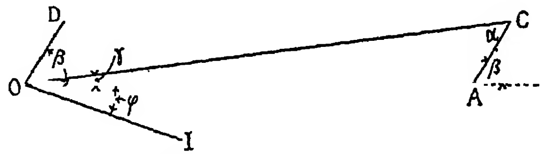


FIG. 132.—To illustrate Algebraical Method of determining Drop

actually determining the drop a purely algebraical method is preferable. Purely graphical methods are, in fact, incapable of giving very accurate results, unless the diagrams are drawn to an inconveniently large scale, and with extreme care. It may be mentioned in this connection that the drop in Figs. 130 and 131 is greatly

of the loaded apparatus; we do *not* mean the vectorial drop AC, which represents the voltage required to overcome the impedance of the apparatus.

exaggerated for the purpose of making the diagram clear, and that in actual practice the angle AOC, in the case of transformers, may be less than 1°. The value of the terminal p.d. OA is easily calculated from the known values of OC and AC, as follows:—

In the triangle OAC, reproduced for the sake of clearness in Fig. 132, we have—

$$\beta = \tan^{-1}\frac{L\omega}{r} - \phi$$

$$\sin\gamma = \frac{AC}{OC}\sin\beta$$

and—

$$OA = \frac{\sin\ \alpha}{\sin\beta}.\ OC = \frac{\sin(\beta - \gamma)}{\sin\beta}.\ OC$$

These three equations enable us to find OA, and so to determine the percentage drop $100\ \frac{OC - OA}{OC}$.

§ 94. Rothert's Ampere-turn Method

The method just explained yields very satisfactory results with transformers, but in the case of alternators it is unsatisfactory, and can only be depended on in cases where the alternator field is well below saturation. With a strongly saturated field, it yields excessive values for the drop, so that it is, at all events, a *safe* method, and has, for this reason, been termed the *pessimistic* method by Behrend. A different method, involving the vectorial composition of *ampere-turns* instead of e.m.f.'s, has been proposed by Rothert.* For the purpose of applying this method, two curves, the *open-circuit* and the *short-circuit* curves or characteristics, are required. The open-circuit curve is the curve connecting the field current (or field ampere-turns) with the e.m.f. or open-circuit p.d., while the short-circuit curve is the curve connecting the field current with the short-circuit current. In the construction, it is immaterial whether we use field current or field ampere-turns, since the latter are proportional to the former. The armature ampere-turns corresponding to a given current are taken to denote that value of the field ampere-turns which is necessary in order to produce a short-circuit current equal to the given armature current; the *armature ampere-turns are thus obtainable from the short-circuit curve.*

When the armature current is in phase with the e.m.f., armature reaction consists of a cross-magnetizing or distorting effect pure and simple; a lagging current gives, in addition to the cross-magnetizing

* *Elektrotechnische Zeitschrift*, vol xx. p. 619 (1899).

effect, a demagnetizing one, while a leading current gives a magnetizing effect (§ 54). Hence Rothert, regarding the actual or effective ampere-turns (to which the terminal p.d. is regarded as due) as the vectorial resultant of the field ampere-turns and the armature ampere-turns, compounds these two vectorially, making the angle between the resultant and the armature ampere-turns 90° for zero phase difference, and $90° \pm \phi$ for a phase difference of ϕ degrees, the + sign being taken for a lagging current and the − sign for a leading one.

In this construction, ϕ should, strictly speaking, be taken as the angle of phase difference between the current and the *e.m.f.* But since this angle is not known, we may, as an approximation, assume it to be equal to the angle of phase difference between the current and the p.d. The actual diagram is then constructed as follows. Lay off OR (Fig. 133) to represent the *direction* of the resultant or

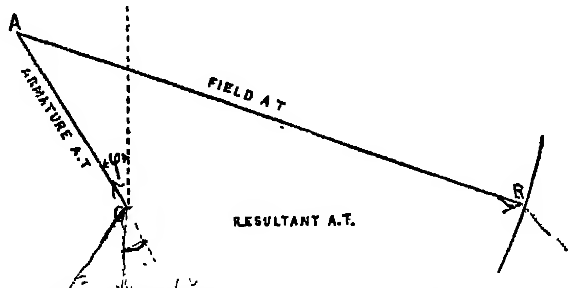


FIG. 133.—Rothert's Ampere-turn Method of determining Drop.

effective ampere-turns, and draw OA, making an angle $90° + \phi$ with OR, $\cos \phi$ being the given power factor of the load (the current is assumed to lag; for a leading current, the angle AOR would be acute, and equal to $90° - \phi$). Find from the short-circuit curve the value of the armature ampere-turns corresponding to the given load current, and lay off a length OA in the diagram to represent these ampere-turns. With A as centre, and radius AR equal to the given field ampere-turns, describe an arc cutting OR at R. Then OR is taken to correspond to the resultant ampere-turns. Now refer to the open-circuit curve, and find the e.m.f. corresponding to these resultant ampere-turns; this e.m.f. is taken to be equal to the required terminal p.d. corresponding to the given load current, power factor, and excitation.

Rothert's method is only capable, like Behn-Eschenburg's, of giving approximately correct results so long as the field is well below saturation. Beyond a certain value of the exciting current, it yields values of the p.d. which are far too high. In this respect, it is an

extremely unsafe method, unlike Behn-Eschenburg's, which yields wrong values, but on the safe side. Behrend has termed Rothert's method the *optimistic* method.

§ 95. Analysis of Armature Self-inductance into Two Components

Practical experience having shown that neither of the two methods described yields reliable results, numerous attempts have been made to establish some more satisfactory method. In many of these, the total armature self-inductance is regarded as made up of (1) that portion of the flux due to the armature current which penetrates the field and produces a weakening of it; and (2) the remaining portion, which is made up of armature leakage lines, *i.e.* lines linked with the armature windings, but not passing into the field cores, and hence incapable of affecting the main field strength. Where such a splitting up of the total armature self-inductance is adopted, the first portion of the self-inductance is generally spoken of as *armature reaction*, while the second is termed *armature self-inductance* (in reality, it is only the *leakage* self-inductance).

Looking at the matter from this point of view, and considering the special case in which the armature current lags 90° behind the e.m.f.*—when the drop will have its greatest possible value—it is obvious that the leakage self-inductance e.m.f. will be simply subtracted arithmetically from the open-circuit e.m.f. (the two being in direct opposition of phase), while armature reaction will be equivalent in its effect to a definite reduction in the field ampere-turns. So long as the armature current is constant, both these effects will remain independent of the exciting current. Hence it follows that if we plot, on the same sheet of paper—as in Fig. 134—the open-circuit curve OP and the curve LP′ connecting the terminal p.d. with the exciting current when the wattless armature current is maintained constant (by suitably adjusting the reactance which constitutes the load), the two curves will be so related that any point P′ on the latter may be obtained from a certain point P on the former by the subtraction of a constant amount PR (representing leakage self-inductance drop) from the e.m.f., and the addition of a constant amount RP′ to the field ampere-turns or exciting current (RP′ representing the amount required to balance armature reaction). Now, this is equivalent to a bodily displacement of the open-circuit curve through a distance PP′. Thus LP′ should simply represent OP

* This, of course, is an ideal case, as we can never get rid of resistance in the circuit. But with a load of very low resistance and large self-inductance it may be closely approached in practice.

displaced through this distance. Potier * found experimentally that such is the case.

If I be taken to represent the constant wattless current corresponding to the curve LP′, then we may write—

$$PR = E_s I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and—

$$RP' = A_r I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where E_s and A_r are two constants.† In order to find the values of these constants, we determine the curve LP′ ‡ for any convenient value

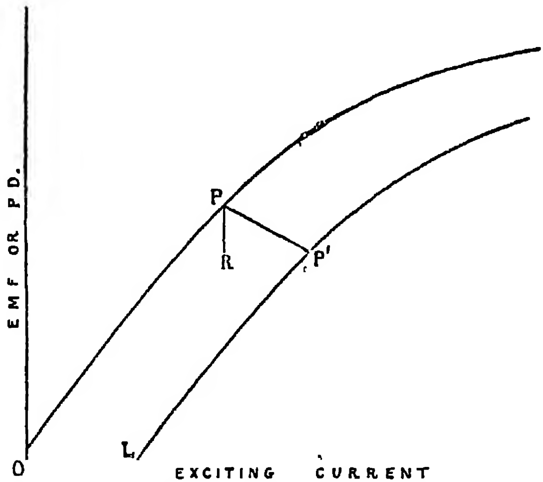


Fig 134.—Analysis of Armature Reactance into Two Components.

of I, and, having made a tracing of LP′ and the axis of exciting current, and marked the position of a convenient point P′ on LP′, we displace the tracing, keeping the axis of exciting current parallel to its original direction, until a position is found in which the tracing of LP′ exactly fits over OP. We then prick the position of P′ through the tracing, thus obtaining the corresponding position of P on OP. Through P and P′ lines are drawn parallel to the two axes, and the lengths PR and P′R thus obtained enable us, by using equations (1) and (2), to find E_s and A_r.

* *Éclairage Électrique*, vol. xxiv. p 133 (1900)

† E_s = reactance corresponding to leakage self-inductance, A_r = armature reaction constant.

‡ LP′ is frequently termed the *load characteristic* corresponding to a given armature current and power factor.

§ 96. Experimental Determination of Two Components of Armature Reactance

The method just explained of finding E_s and A_r involves the complete determination of the "load curve" LP′. Blondel * and Fischer-Hinnen † have, however, indicated methods by means of which E_s and A_r may be found from a single point on LP′, involving the taking of a single reading instead of the determination of the entire curve.

Let in Fig. 135 PBA be the open-circuit curve, P′A giving the value of the e.m.f. which corresponds to the excitation OP′, and let us provisionally assume that we know the position of P, where P is that point on the open-circuit curve which corresponds to P′ on the load

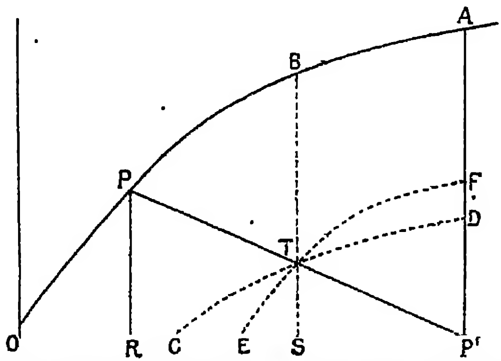


FIG. 135.—Blondel's Determination of Two Components of Armature Reactance.

curve for a wattless armature current equal to the armature short-circuit current at excitation OP′. If the position of P were actually known, E_s and A_r could be found at once.‡ Let I_s = short-circuit armature current at excitation OP′; this may be found from the short-circuit curve. Further, let V = the experimentally determined p.d. corresponding to excitation OP′ and any convenient wattless armature current I (this is the single additional reading required for determining E_s and A_r, besides the open and short-circuit curves). Corresponding to a wattless armature current I, the reduction in the ampere-turns is $P'S = \frac{I}{I_s} . P'R$, and the leakage self-inductance voltage drop is

* *Elektrotechnische Zeitschrift*, vol. xxii. p. 474 (1901).

† *Ibid.*, vol. xxii. p. 1061 (1901).

‡ $E_s = \frac{PR}{I_s}$, and $A_r = \frac{RP'}{I_s}$, by equations (1) and (2) of § 95; I_s being the short-circuit current at excitation OP′.

$ST = \frac{I}{I_s} \cdot RP$. Hence the p.d. is equal to $SB - ST = TB$, so that the point T obviously lies on a curve CTD, obtained by subtracting from each of the ordinates of the open-circuit curve the constant value $TB = V$. Again, since $P'T = \frac{I}{I_s} \cdot P'P$, it follows that T also lies on a curve ETF, similar to PBA, the centre of similarity being at P′, and the radii drawn from P′ to ETF being in the constant ratio $\frac{I}{I_s}$ to those drawn to PBA. It is now obvious that the position of T may be obtained by the following construction. Draw the curve CTD, by lowering the open-circuit curve through a vertical distance $= V$. Next, draw ETF similar to the open-circuit curve, the centre of similarity being at P′, and the ratio of the radii vectores drawn from P′ to the two curves being $\frac{I}{I_s}$. The intersection of CTD and ETF gives T, and we then find $E_s = \frac{ST}{I}$, $A_r = \frac{SP'}{I}$.

The method given by Fischer-Hinnen is as follows. In Fig. 136, let $P'V = V$ be the p.d. corresponding to a wattless current I at

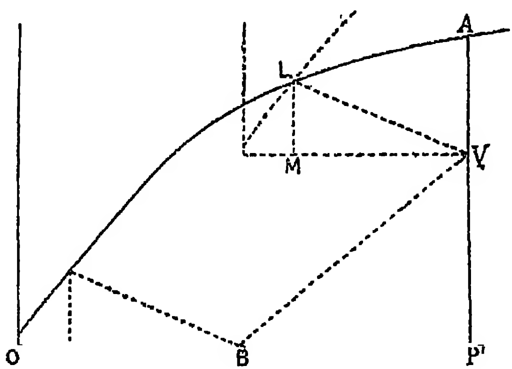


FIG 136.—Fischer-Hinnen's Method of determining the Two Components of Armature Reactance.

excitation OP′. Lay off OB to represent the excitation (obtained from the short-circuit curve) required to produce a short-circuit current I. Join BV, and displace the open-circuit curve (by using a tracing) parallel to itself along BV until B coincides with V. The intersection L of the original and the displaced curve enables us to find the direction

LV (corresponding to PP′ in Fig. 134), and we have $E_s = \frac{LM}{I}$, $A_r = \frac{MV}{I}$.*

In the case of a three-phase armature whose neutral point is accessible, the following method, due to Kapp,† of analyzing armature reactance into its two components may be employed. The open-circuit characteristic is first determined, and then two short-circuit characteristics, one being taken with only one phase of the star winding short-circuited (by connecting an ammeter between the neutral point and one of the terminals), and the other with all three phases short-circuited. Let I be any value of the short-circuit current, and let X_1 and X_3 denote the values of the ampere-turns (or of the exciting currents) required to produce this current when the short-circuit is (*a*) limited to one phase and (*b*) extended to all three phases respectively. Then, since in case (*b*) the demagnetizing effect is thrice as great as in case (*a*), we have, if A_s denote the ampere-turns (or the exciting current, as the case may be) required to generate the e.m.f. E_sI necessary to balance the leakage reactance e.m.f. of the armature,

$$X_1 = A_rI + A_s$$
$$X_3 = 3A_rI + A_s$$

whence $A_r = \frac{X_3 - X_1}{2I}$. Either of the above equations then gives A_s, and by referring to the open-circuit curve we find the value V of the e.m.f. corresponding to A_s. Since $V = E_s I$, we have $E_s = V/I$, which gives the leakage reactance of the armature.

§ 97. Potier's Method of Predetermining the Regulation of an Alternator

Potier ‡ determines the p.d. corresponding to given excitation, armature current, and external power factor cos ϕ as follows. Rothert's method (Fig. 133) is first used to determine the effective excitation, and from the open-circuit curve the e.m.f. corresponding to this excitation is found. From this e.m.f. is next subtracted vectorially the combined drop corresponding to armature resistance and *leakage* self-inductance, as in Behn-Eschenburg's method.§

* In order to secure accuracy, the point A must (in either method) be chosen well above the knee of the open-circuit curve, otherwise the intersection (T in Fig. 135, and L in Fig. 136) will take place at a very acute angle, and accuracy will be impossible.

† *Journal of the Institution of Electrical Engineers*, vol. xlii. p. 703 (1909).

‡ *Elektrotechnische Zeitschrift*, vol. xxii p. 1061 (1901).

§ Using, however, the *leakage* self-inductance drop instead of the *total* self-inductance drop.

This gives a rough value for the p.d., and also for the angle of phase difference θ between the *e.m.f.* and the current. If θ differs appreciably from ϕ, the entire construction is repeated, using θ in place of ϕ, and a more accurate value of the p.d. is obtained.

Potier's method has been found to give much better results than either Behn-Eschenburg's or Rothert's. This is due to the fact that the variation in the reluctance of the magnetic circuit is taken into account.

Where accuracy is required, it is advisable to calculate the drop, making use of the diagram only for the purpose of deducing the necessary simple formulæ.*

§ 98. Numerical Values of Regulation. Saturation Factor

In the case of transformers, the regulation on a non-inductive load is very largely dependent on the output of the transformer, and varies from about 3 per cent. for a 1 k.w. transformer to about 1·5 for a 100 k.w. transformer. For very large transformers, it may be below 1 per cent. If the load is an inductive one, the drop is largely influenced by the exact arrangement and subdivision of the windings.

The following limits are assigned by Kloss† to the regulation of large low-speed alternators working at different power factors:—

Power factor	1	0·9	0·8	0·7
Regulation, per cent.	3 to 3·5	8 to 9	11 to 12·5	13·5 to 16

The corresponding figures for turbo-alternators are:—

Power factor	1	0·9	0·8	0·7
Regulation, per cent.	5 to 7	12 to 15·5	15 to 18	20·5 to 22·5

The ratio of the short-circuit current at full-load excitation to the normal full-load current is generally of the order of 3 in low-speed alternators, and about 2 to 3 in turbo-alternators.

It is easy to see that the regulation—*i.e.*, the percentage voltage rise on switching off full-load—will depend on the degree of saturation of the magnetic circuit, *i.e.* on the steepness of the open-circuit characteristic. The less the steepness of this curve (or the higher the saturation), the less will be the voltage rise. In order to take this fact into account, Kloss has proposed the use of the term *saturation*

* See Appendix VIII.

† *Journal of the Institution of Electrical Engineers*, vol. xlii. p. 156 (1908).

factor, which he defines as follows. Let a tangent be drawn to the open-circuit characteristic at the point corresponding to the

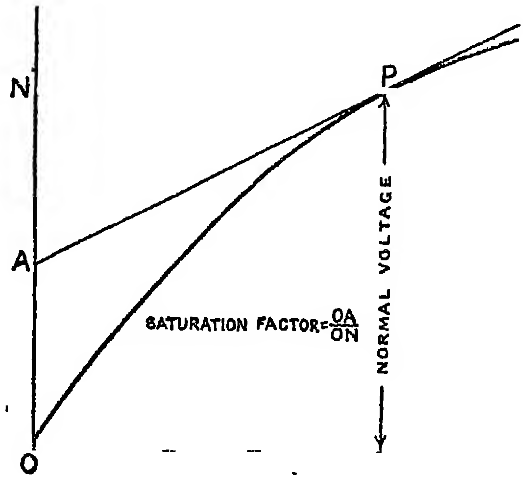


FIG. 137 —Construction for Saturation Factor.

normal voltage. Then the ratio of the length cut off by this tangent along the axis of ordinates, to the normal voltage, is the saturation factor (Fig. 137). Its value generally lies between 0·5 and 0·6.

CHAPTER XI

§ 99. Resistance measurements of alternating-current machinery—§ 100. Insulation tests—§ 101. Experimental determination of equivalent impedance of transformer—§ 102 Direct experimental method of determining transformer regulation—§ 103. Transformer efficiency test—§ 104. Separation of core losses—§ 105. Heating test of transformer—§ 106. Ageing of transformer core—§ 107. Alternator efficiency tests—§ 108. Heating test of alternator. Goldschmidt's method—§ 109. Hobart and Punga's method—§ 110. Current rushes at instant of switching on transformers—§ 111. Sudden short-circuit of large generator.

§ 99. Resistance Measurements of Alternating-current Machinery

SOME of the tests which have to be carried out in connection with alternators and transformers are common to almost every class of electrical machinery. Thus we have to determine the resistances of the windings and the insulation resistances, and to test for dielectric strength. We have, further, to find the maximum temperature rise under normal working conditions.

The conductor resistances are most conveniently determined by sending a suitable current from a battery of secondary cells through the coil or coils under test, and measuring the drop of potential. In the case of three-phase apparatus, we may have either a mesh or a star connection of the windings. With the former arrangement of circuits, the resistance of each phase (*i.e.* of each side of the triangle formed by the windings) is clearly equal to 1½ *times* the measured resistance between two terminals, while with a star connection the resistance of each phase is *one-half* of the measured resistance between two terminals.*

The voltage absorbed by the resistance of the windings if expressed as a percentage of the total voltage may, in the case of a *balanced*

* Let x = resistance of each phase of a Δ system The resistance between any two terminals is then the joint resistance of one phase connected in parallel with the other two joined in series. It is, therefore, $\frac{x \cdot 2x}{x + 2x} = \frac{2}{3}x$, so that $x = \frac{3}{2} \times$ measured resistance. In the case of a star-connected system, it is obvious that between any two terminals there are two phases in series with each other, so that the measured resistance is twice that of one phase.

system, be calculated from the measured resistance between two terminals and the current in each line wire, without any knowledge of how the windings are connected. For if V = voltage between line wires, I = current in each line wire, and r = measured resistance between any two terminals, then in a mesh-connected system we have for the resistance per phase $\frac{3}{2}r$, for the current per phase $\frac{I}{\sqrt{3}}$, and for the voltage per phase V, so that the percentage drop is

$$100\frac{\frac{3}{2}r\frac{I}{\sqrt{3}}}{V} = 100 \,.\, \frac{\sqrt{3}}{2V}rI.$$

In a star-connected system, the resistance per phase is $\frac{1}{2}r$, the current per phase is I, and the voltage per phase is $\frac{V}{\sqrt{3}}$, the percentage drop amounting to $100\frac{\frac{1}{2}rI}{\frac{V}{\sqrt{3}}} = 100 \,.\, \frac{\sqrt{3}}{2V}rI$—an expression identical with that obtained for a mesh-connected system.

It must be remembered, in dealing with alternating-current machinery, that the resistance as measured by means of a continuous current is always *less* than the true resistance corresponding to an alternating current, on account of eddy-current loss. With conductors of small cross-section, the difference is inappreciable; but with large conductors the eddy-current loss may be considerable, amounting to as much as 20 per cent. of the loss calculated from the resistance as determined by means of continuous currents.

§ 100. Insulation Tests

A considerable amount of importance was formerly attributed to insulation resistance tests. Such tests are now, however, regarded as of little value. The insulation resistance is an extremely variable quantity, and measurements of it are liable to be misleading, both when the results are very high and somewhat low.* A much more important test is that for *dielectric strength*. By means of a suitable testing transformer, a voltage considerably in excess of the normal working voltage is applied to the winding for one minute, and the test may be regarded as satisfactory if no breakdown takes place during that time. The dielectric strength test is applied to the insulation between the winding and the core, and also to the insulation

* According to the rules of the British Engineering Standards Committee, the insulation resistance, in megohms, should not be less than $\frac{\text{rated voltage}}{1000 + \text{rated kva}}$.

between different windings if there are several on the same core. The test should be applied to the machine when *hot*, after a load run of several hours' duration.

The value of the testing voltage should, according to the British Engineering Standards Committee, be equal to *twice the rated voltage* + 1000.

§ 101. Experimental Determination of Equivalent Impedance of Transformer

For the purpose of determining the regulation * of a transformer, we have to find its equivalent *reactance*. This may be done by short-circuiting the secondary, and applying a p.d. to the primary sufficient to produce the full-load currents in the windings. In order to eliminate any error due to the resistance and reactance of the ammeter, it is preferable to place this in the primary circuit. If V_s = primary p.d. required to produce the full-load current I_1 in the primary of the short-circuited transformer, then $\frac{V_s}{I_1}$ gives us the equivalent impedance of the short-circuited transformer, the equivalent resistance being $r_1 + \left(\frac{S_1}{S_2}\right)^2 r_2$, where r_1, r_2 are the resistances, and S_1, S_2 the turns in the primary and secondary respectively. The transformer behaves as if there were no magnetic leakage and no resistance in its coils, but as if connected in series with its primary there were an impedance $\frac{V_s}{I_1}$, the resistance component of this impedance being $r_1 + \left(\frac{S_1}{S_2}\right)^2 r_2$. Or, if we prefer to refer everything to the secondary circuit,† the transformer behaves as if magnetic leakage and coil resistance were absent, and as if in series with the secondary there were connected an impedance $\left(\frac{S_2}{S_1}\right)^2 \frac{V_s}{I_1}$, whose resistance component is $r_2 + \left(\frac{S_2}{S_1}\right)^2 r_1$.

When the transformer is short-circuited, the induction in its core is very small, and hence the magnetic leakage is not quite the same as that occurring at full load. In order to overcome this objection to the method of measuring leakage reactance just explained, Berry

* See §§ 90–93.

† As was done in § 91.

recommends * measuring the impedance which corresponds to normal working conditions, the arrangement of connections being as shown in Fig. 138. The method can only be used if *two* similar transformers are available. The secondaries S_1 and S_2 are connected in series with each other, the connections being so arranged that the e.m.f.s induced in them *oppose* each other. One of the primaries, P_2, is connected directly across mains between which the normal working primary p.d. is maintained. If the primary P_1 of the other transformer were similarly connected, the secondary e.m.f.s would exactly balance each other, there would be no secondary current, and each primary would take only its small magnetizing current; the transformers would, in fact, behave as if their secondaries were open-circuited. In order to cause the full-load currents to flow in the

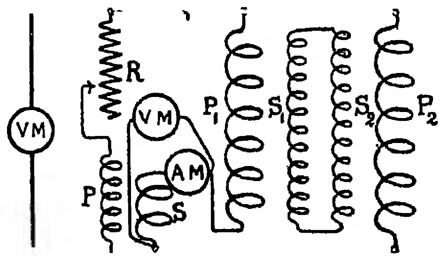


FIG. 138.—Berry's Method of determining the Equivalent Impedance of a Transformer.

transformer windings, a small auxiliary transformer is used, whose primary P is connected in series with an adjustable resistance R, while its secondary S is connected in series with P_1, the e.m.f. induced in S being added to the p.d. which exists across the mains. *Half* the reading of the voltmeter across S gives the impedance drop in *one* transformer.

Another method of measuring the equivalent impedance, and of determining its two components—the equivalent resistance and the leakage reactance—of a transformer under normal load conditions has been given by A. Shane.† The method involves the use of two similar transformers. The connections are arranged as shown in Fig. 139. The primaries of the transformers are across the mains, and one of the secondaries is loaded, the load current being measured by the ammeter AM. The secondaries are connected on one side in

* "Modern Electric Practice," vol. ii. p. 89.

† *Proceedings of the American Institute of Electrical Engineers*, vol. xxix. p. 1089 (1910). A direct experimental method of finding the equivalent resistance and leakage reactance of a loaded transformer has also been devised by Dr. C. V. Drysdale (*The Electrician*, vol. lxv. p. 648 (1910)).

such a manner as to oppose each other, so that the voltmeter VM connected across their remaining ends gives the vector drop due to the impedance of the loaded transformer; by dividing this drop by the load current, we obtain the impedance. In order to find the equivalent resistance, we divide the wattmeter reading by the square of the load current. The current coil of the wattmeter is, it will be

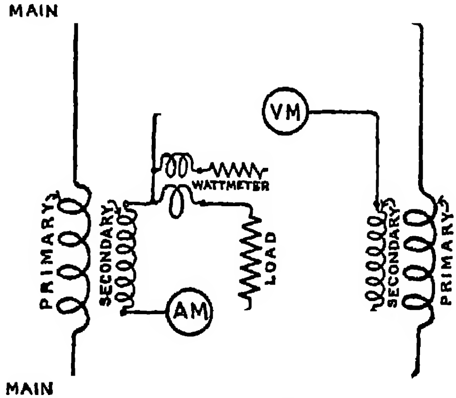


Fig. 139.—Connections for Shane's Method of finding Impedance and Equivalent Resistance of Transformer.

noticed, traversed by the load current, while its fine-wire coil and non-inductive resistance are across the same ends of the secondaries as VM, so that the instrument measures the power spent in maintaining the load current through the impedance of the transformer. The equivalent impedance and resistance being known, the impedance triangle of the transformer is completely determined, and so its regulation may be calculated by the method of § 93.

§ 102. Direct Experimental Method of Determining Transformer Regulation

The following ingenious method of directly measuring the regulation drop of a transformer is due to A. Shane.* The principle of the method will be best understood by considering the diagram of Fig. 130, the portion OAC of which is reproduced in Fig. 140. In this diagram, OC is the secondary e.m.f., OA the secondary terminal p.d., AC the vector impedance drop, and EC the regulation drop ($AE \perp OC$). Suppose now that we have the means of opposing to the e.m.f. OC a variable e.m.f. CM which is in direct opposition of phase to OC, and

* *Loc. cit.*

that we are able to measure both CM and the vector difference AM between OM and OA. As the magnitude of CM is gradually increased the point M in the diagram moves along CO, and AM decreases

FIG. 140.—Vector Diagram to illustrate Shane's Method of determining Drop.

until the point E is reached, beyond which it increases. Let us suppose that CM has been adjusted so as to make AM a minimum (*i.e.* so as to make AM coincide with AE); then MC (which becomes EC) gives the regulation drop (the percentage regulation drop is $100 \frac{EC}{OC}$).

In order to carry out this test, we require two similar transformers, and an auxiliary transformer of special construction which

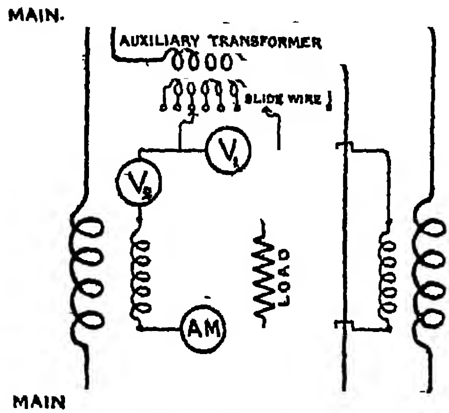


FIG. 141.—Connections for Shane's Method of determining Regulation.

enables us to obtain the variable e.m.f. CM. The arrangement of connections is shown in Fig. 141. The auxiliary transformer has a secondary with a variable number of turns, and exact adjustment of the e.m.f. CM of Fig. 140 is obtained by the aid of a slide-wire connected across the last section of the secondary. The e.m.f. CM of Fig. 140 is measured by the voltmeter V_1 in Fig. 141. The vector OA of Fig. 140 is in Fig. 141 represented by the p.d. across the secondary of the loaded transformer, while OC corresponds to the p.d.

across the secondary of the unloaded transformer. The voltmeter V_2 measures AM. The reading of V_1 when V_2 gives a minimum reading is the regulation drop.

It may be noted that it is not permissible to alter the magnitude of CM (*i.e.* the reading of V_1) by the aid of a variable resistance in series with the primary of the auxiliary transformer (as in Fig. 138), as this would have the effect of altering the *phase* of CM; the latter must be directly opposed to the p.d. OC across the unloaded transformer.

§ 103. Transformer Efficiency Test

Although the efficiency of a transformer might be determined by measuring the useful power across its secondary terminals, and the total power supplied to the primary, and dividing the former by the latter, this method is neither so exact nor so convenient as methods in which the *lost power* is measured directly. A very simple method of carrying out the efficiency test, originally suggested by Dr. Sumpner, consists in finding, by means of a wattmeter, (1) the core loss and (2) the copper loss at full load. Since the core loss remains sensibly constant at all loads,* its value at full load may be taken to be the same as on open circuit. Hence the core loss is equal to the power supplied to the primary when the secondary is open-circuited.† The most satisfactory method of measuring the copper loss is that already explained in § 101 in connection with the determination of the equivalent resistance of the transformer. The sum of the core and copper losses being known, the efficiency is easily calculated.

§ 104. Separation of Core Losses

In many cases, it is interesting to separate the core losses into their hysteresis and eddy-current components. Such a separation may be effected by a method due to Carhart.‡ The principle of the method is based on the fact that, for a given constant value of the maximum induction B, the eddy-current loss varies as the square of the frequency (see § 61), while the hysteresis loss varies in simple proportion to the frequency. Hence the total core loss at any frequency f may be written—

$$w = \mathrm{E}f^2 + \mathrm{H}f$$

w denoting the total watts lost in the core, $\mathrm{E}f^2$ the eddy-current

* See § 61.

† Strictly speaking, this power also includes the (extremely small) copper loss in the primary due to the magnetizing or no-load current.

‡ *Electrical World*, vol. xxxi p. 306 (1898).

watts, and Hf the hysteresis watts; E and H being two constants for a constant value of B.

If, then, we carefully determine w at two different frequencies, f_1 and f_2, using the same value of B, the two equations—

$$w_1 = Ef_1^2 + Hf_1$$
$$w_2 = Ef_2^2 + Hf_2$$

enable us to find E and H, and thus to calculate separately the hysteresis and eddy-current losses occurring at any frequency—always assuming B to remain constant.

Instead of determining only two values—w_1 and w_2—of w at two different frequencies, it is advisable to take an entire series of readings connecting w and f, to plot the results on squared paper, and select two well-defined points on the curve for the purpose of finding E and H.

A practically constant value of B with varying frequency is easily secured by keeping the exciting current of the alternator used in the experiment constant, and running the machine at different speeds, so as to obtain different frequencies.

§ 105. Heating Test of Transformer

An important test—which applies to all classes of electrical machinery—is the heating test. This is carried out for the purpose of ascertaining the temperature to which the windings of a transformer will ultimately rise when the transformer is kept fully loaded. The importance of this test is due to the fact that when the temperature exceeds a certain limit, the insulation of the windings undergoes rapid deterioration, and frequent renewals will be necessary. The safe temperature limit for the windings may be taken as 95° C. Since the temperature of the surroundings may at times be as high as 40°, it follows that 55° C. must be regarded as the maximum permissible rise of temperature for any portion of the windings.

The most reliable method of determining the temperature of a coil is the increase of resistance method. The resistance is measured before commencing the test, and is then periodically measured as the test proceeds. If t_c = temperature of the coil just before the commencement of the test; r_c = measured resistance at this temperature; α = temperature coefficient at the same temperature t_c,* and

* See Note at end of chapter for values of α.

r = measured resistance at some higher unknown temperature, then the temperature rise is given by $\frac{r - r_c}{a r_c}$.

The most convenient method of determining the resistance of the transformer windings as the heating test proceeds is that explained in § 101 in connection with the determination of the equivalent resistance of the transformer. By using this method, the heating test is allowed to proceed uninterruptedly, whereas if the ordinary continuous current method were employed, a temporary interruption of the heating test would be necessary each time the resistance was measured.

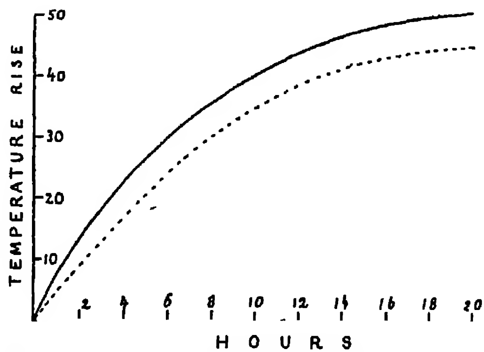


FIG. 142.—Showing Temperature Rise in Transformer.

In Fig. 142 are shown the results of a heating test for an oil-cooled transformer, the upper or full-line curve giving the temperature rise of the windings as determined by the increase of resistance method, and the dotted curve giving the temperature rise of the oil as measured by a thermometer.

In carrying out this test, the most economical method—where two similar transformers are available—is to employ the arrangement of connections shown in Fig. 138.

§ 106. Ageing of Transformer Core

It was originally noticed by Mr. Partridge that the core loss of certain transformers gradually increased in course of time. The effect has since been dealt with by a large number of investigators.* The slow increase in the core loss, or the *ageing* of the core stampings, has been shown to be due to the maintenance of the core at a

* The most complete investigation of the ageing effect is that carried out by Roget, see *The Electrician*, vol. xli. p. 182 (1898), also *Roy. Soc Proc.*, vol. lxiv. p. 150 (1899).

moderately high temperature by the losses taking place in it, and a precisely similar effect, it was found, could be brought about by simply baking the core in an oven maintained at the proper temperature.

Manufacturers of transformer sheets have now succeeded in producing a *non-ageing* material, *i.e.* one whose hysteresis loss is unaffected by long-continued baking. The only method of applying the ageing test is to go on measuring the core loss periodically for some time after the transformer has been installed. If after a few months' use no appreciable increase has taken place in the core loss, the transformer may be regarded as fulfilling the non-ageing requirement.*

§ 107. Alternator Efficiency Tests

As in the case of dynamos, the efficiency test of an alternator is most economically and conveniently carried out by the use of an indirect method, the power lost in the alternator being measured directly.

Where two similar alternators are available, the Hopkinson test may be used. The shafts of the alternators are coupled to each other and to a continuous-current motor, which is large enough to supply the power corresponding to the losses. This motor is first carefully tested for efficiency, so that the exact amount of mechanical power transmitted by it to the alternators is known for any given values of the p.d. and current supplied to the motor. By weakening the field of one of the alternators, any desired current may be made to circulate in the local circuit of the two armatures, which are short-circuited on each other through an ammeter; the machine with the stronger field acting as a generator, while that with the weaker field acts as a motor.

In cases where only a single machine is available, the most generally used method for determining the efficiency is the *separate loss* method. In this the losses are measured directly, and on adding them to the output we obtain the input. The ratio $\frac{\text{output}}{\text{input}}$ then gives the efficiency. The losses consist of (*a*) frictional losses; (*b*) core losses; and (*c*) copper losses. (*a*) may be determined by driving the machine under test unexcited by means of a standardized motor; the sum of (*a*) and (*b*) is similarly found by driving the machine excited with a current corresponding to the normal magnetic flux under load; while (*c*) may be determined by calculation. The first test (*a*) is obviously only required if it is desired to separate the frictional from the core losses.

* See Appendix IV. for magnetic properties of core sheets.

The efficiency so determined is not the true efficiency, as no account is taken of the so-called *stray load losses*. These are additional losses arising under actual load conditions due to field distortion and eddy currents in the copper conductors. An approximate value for these may be obtained by driving the machine on short-circuit with the full-load current flowing through the armature. On subtracting from the total power the frictional losses and the calculated copper losses, we obtain, roughly, the stray load losses.*

§ 108. Heating Test of Alternator. Goldschmidt's Method

In order to determine the temperature rise of an alternator, the machine must be kept running under normal full-load conditions until the temperature approaches a steady value. Since the period during which an alternator of large output would have to be run for this purpose is very considerable, such a heat test would prove expensive if the machine were actually run on a dead load. Numerous methods have, therefore, been devised by means of which the heat test may be carried out with the expenditure of only a relatively small amount of energy. One method of carrying out this test economically consists in combining it with the Hopkinson efficiency test described in § 107.

A very ingenious method has been devised by Goldschmidt.† The alternator is run at normal full-load excitation, so that the iron and field copper losses are approximately the same as those occurring at full-load. The armature copper loss is supplied by sending a *continuous* current from a battery or dynamo around the armature winding, the continuous current being made equal to the r.m.s. value of the full-load armature current. It need hardly be pointed out that the connections must be arranged so as to prevent the high e.m.f. of the alternator armature from reaching the source of continuous current. Various ways of securing this result are available. In the case of a single-phase machine, the armature coils may be divided into two equal groups, connected so as to oppose each other; the resultant alternating e.m.f. will then be zero, and the armature circuit will form a simple resistance so far as the source of continuous current is concerned. In a two-phase machine, each phase may be similarly treated, a *separate* source of continuous e.m.f. being used for each phase. With three-phase machines having a Δ-connected armature, the Δ may be opened at one corner to allow

* This method of stray load loss determination applies to *polyphase* machines only; it is incapable of giving reliable results with *single-phase* machines

† *Elektrotechnische Zeitschrift*, vol. xxii. p. 682 (1901). For later developments of this method, see Appendix IX

of the introduction of the continuous current. If the windings are star-connected, they may be temporarily altered to a Δ connection for the purposes of the test. Another method may also, however, be used. A suitable star-connected three-phase transformer is joined across the alternator terminals, and the continuous current led into the system at the neutral point of the transformer windings, and out of it at the neutral point of the armature winding—each phase carrying in this case a current equal to $\frac{1}{3}$ of the continuous current. In order to balance the magnetic effect of the continuous current on the transformer core, a continuous current producing an equal and opposite number of ampere-turns must be sent through the secondary windings of the transformer.*

§ 109. Hobart and Punga's Method

A convenient method of carrying out the heat test of a large generator has been described by Hobart and Punga.† The method consists essentially in running the machine alternately on open circuit and on short circuit, the periods of the two sets of runs and the values of the exciting current corresponding to them being so chosen that the mean values of the various losses during the entire time of the test are equal to the actual values of these losses at full load.

The losses may be regarded as made up of (1) mechanical friction losses; (2) armature copper loss, w_a; (3) iron loss, w_i; (4) field copper loss, or excitation loss w_e.

The friction losses are independent of the excitation, and their value may be determined by driving the unexcited alternator at its normal speed by means of an auxiliary standardized motor. The armature copper loss is easily calculated, as is also the field copper loss, for a given value of the current. The iron loss is found by driving the alternator on open circuit by means of a standardized motor, the exciting current having its normal full-load value; if from the total power employed in driving the alternator we subtract the friction losses as previously determined, we obtain the iron loss. For the purposes of the heat test under consideration, it is necessary to know the relation connecting the iron loss with the excitation loss, so that a set of readings must be obtained connecting these two, and the results plotted in the form of a curve as in Fig. 143.‡

* Goldschmidt's method is, with suitable modifications, applicable to large transformers (single- and polyphase), provided two similar transformers are available.

† *Electrical World and Engineer*, vol xlv. p. 759 (1905).

‡ On account of armature reaction, which produces a distortion of the field, the full-load iron loss w_i will generally be in excess of the open-circuit iron loss with the same exciting current.

Let the alternator be run for t_1 minutes on open circuit with certain excitation, then for t_2 minutes on short circuit with a different ccitation, then again for t_1 minutes on open circuit, for t_2 minutes on iort circuit, and so on, the open-circuit and short-circuit runs ternating with each other. During the open-circuit runs, there e only frictional, excitation, and iron losses taking place in the iachine, while during the short-circuit runs we have only frictional, ccitation, and armature copper losses, the iron loss being negligible. he machine running at its normal speed, it is evident that the iotional loss is identical, all the time, with the full-load frictional ss. The other losses will depend on the excitation, and the problem so to choose the values of the exciting current corresponding to the en-circuit and short-circuit runs, and the relative values of t_1 and t_2, to make the *average* values of all the losses equal to those which cur at full load.

Let w_e, w_i, and w_a stand for the excitation, iron, and armature pper losses respectively under normal full-load conditions, and let = duration of a cycle, so that $T = t_1 + t_2$. Further, let w_e' and w_i' e the excitation and iron losses respectively during the open-circuit ins, and w_e'', w_a' the excitation and armature copper losses respecvely during the short-circuit run. Then in order to make the erage values of all the losses equal to those which occur at full ad, we must have

$$w_i' t_1 = w_i T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$w_a' t_2 = w_a T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$w_e' t_1 + w_e'' t_2 = w_e T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Using the values of $\frac{t_1}{T}$ and $\frac{t_2}{T}$ given by (1) and (2) respectively in), we find

$$w_e' \times \frac{w_i}{w_i'} + w_e'' \times \frac{w_a}{w_a'} = w_e$$

$$\frac{w_e'}{w_i'} = \frac{w_e}{w_i} - \frac{w_e''}{w_i} \times \frac{w_a}{w_a'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The short-circuit curve of the machine (assumed to be known) eing a straight line, the ratio of the exciting current to the shortrcuit current is constant, and the square of this ratio is therefore so constant. Let the square of the ratio be denoted by k. Then $\frac{w_e''}{w_a'} = k$, and (4) becomes

$$\frac{w_e'}{w_i'} = \frac{w_e}{w_i} - k\frac{w_a}{w_i} = \frac{w_e - kw_a}{w_i} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Let the alternator be run for t_1 minutes on open circuit with a certain excitation, then for t_2 minutes on short circuit with a different excitation, then again for t_1 minutes on open circuit, for t_2 minutes on short circuit, and so on, the open-circuit and short-circuit runs alternating with each other. During the open-circuit runs, there are only frictional, excitation, and iron losses taking place in the machine, while during the short-circuit runs we have only frictional, excitation, and armature copper losses, the iron loss being negligible. The machine running at its normal speed, it is evident that the frictional loss is identical, all the time, with the full-load frictional loss. The other losses will depend on the excitation, and the problem is so to choose the values of the exciting current corresponding to the open-circuit and short-circuit runs, and the relative values of t_1 and t_2, as to make the *average* values of all the losses equal to those which occur at full load.

Let w_e, w_i, and w_a stand for the excitation, iron, and armature copper losses respectively under normal full-load conditions, and let T = duration of a cycle, so that $T = t_1 + t_2$. Further, let w_e' and w_i' be the excitation and iron losses respectively during the open-circuit runs, and w_e'', w_a' the excitation and armature copper losses respectively during the short-circuit run. Then in order to make the average values of all the losses equal to those which occur at full load, we must have

$$w_i' t_1 = w_i T \quad \ldots \quad (1)$$

$$w_a' t_2 = w_a T \quad \ldots \quad (2)$$

$$w_e' t_1 + w_e'' t_2 = w_e T \quad \ldots \quad (3)$$

Using the values of $\frac{t_1}{T}$ and $\frac{t_2}{T}$ given by (1) and (2) respectively in (3), we find

$$w_e' \times \frac{w_i}{w_i'} + w_e'' \times \frac{w_a}{w_a'} = w_e$$

or

$$\frac{w_e'}{w_i'} = \frac{w_e}{w_i} - \frac{w_e''}{w_i} \times \frac{w_a}{w_a'} \quad \ldots \quad (4)$$

The short-circuit curve of the machine (assumed to be known) being a straight line, the ratio of the exciting current to the short-circuit current is constant, and the square of this ratio is therefore also constant. Let the square of the ratio be denoted by k. Then $\frac{w_e''}{w_a'} = k$, and (4) becomes

$$\frac{w_e'}{w_i'} = \frac{w_e}{w_i} - k\frac{w_a}{w_i} = \frac{w - kw_a}{w_i} \quad \ldots \quad (5)$$

The quantities on the right-hand side of (5) being all known, the value of $\frac{w_e'}{w_i'}$ becomes known. In order to determine w_e' and w_i' separately, we make use of the curve of Fig. 143 which connects these two quantities. From the origin let a straight line be drawn making an angle with the horizontal axis whose tangent is equal to $\frac{w_e'}{w_i'}$; then the point of intersection with the curve gives w_e' and w_i'.

By means of (1) we next find $\frac{t_1}{T}$, and hence also $\frac{t_2}{T}$, which latter is equal to $1 - \frac{t_1}{T}$. Equation (2) then yields w_a', and a reference to the short-circuit curve gives w_e''. Thus all the data required for carrying out the test become known.

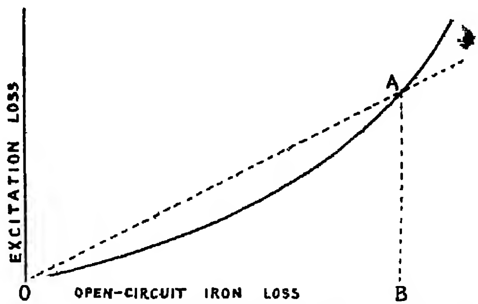


FIG 148.—Relation connecting Iron with Excitation Loss.

As regards the actual values of t_1 and t_2, we may make $t_1 + t_2$ anything between 10 and 20 minutes, say. If, for example, we find $\frac{t_1}{T} = 0{\cdot}6$, and we make $t_1 + t_2 = 15$, the duration of each open-circuit run will be 9 minutes, and that of each short-circuit run 6 minutes.

As in other heat tests, the most reliable method of ascertaining the rise of temperature is the increase of resistance method (§ 105).

§ 110. Current Rushes at Instant of Switching on Transformers.

It has been known for a long time that when the primary of an unloaded transformer is first connected to the supply mains, a very large rush of current may frequently take place at the instant of connection. The effect is easily observed by including an ammeter in the primary, and repeatedly closing and opening the switch, when

the ammeter will sometimes give a violent swing, indicating the presence of a momentary current which is many times the normal no-load current of the transformer. This effect was first studied by J. A. Fleming,* and subsequently investigated in great detail by A. Hay.†

The theory of such initial current rushes will be best understood by considering the ideal case of a resistanceless inductive circuit. In such a circuit, the induced e.m.f. must at every instant be equal and opposite to the impressed e.m.f. Hence if e = instantaneous value of impressed e.m.f., and f = instantaneous value of total flux linked with circuit, we must have—

$$e = 10^{-8}\frac{df}{dt},$$

e being measured in volts, and f in c.g.s. units. If we take the instant of closing the circuit for our era of reckoning, then the value of the flux f at time t is given by—

$$f = 10^{8}\int_{0}^{t} edt,$$

and it is obvious that the value of f will go on increasing numerically with t so long as e retains the algebraical sign which it had at the instant of switching on. The highest value of f—and hence also the highest value of the current—will be reached when e passes through its first zero value. The actual value of f at this instant will be determined by the initial value of e (*i.e.* the value of e at $t = 0$), and it is clear that the integral $\int_{0}^{t} edt$ will have its maximum possible value if e is initially zero, the upper limit of integration for the maximum value of f becoming in this case $\frac{1}{2}$T (half the period), and the integral representing the area of a complete half-wave of e.m.f.—which is the largest numerical value of the area of the e.m.f. curve that it is possible to obtain between any two limits. For this case, then, we have—

$$f = 10^{8}\int_{0}^{\frac{1}{2}T} edt.$$

Let us now compare this maximum value reached by the flux during the first half-wave of flux with the normal or steady maximum value to which it settles down after the irregular waves characteristic

* The possibility of large initial current rushes was first pointed out by Dr W. E. Sumpner (*Phil Mag*, June, 1888). Dr. Fleming's paper will be found in the *Journal of the Institution of Electrical Engineers*, vol xxi (1892).

† *The Electrician*, vol. xxxiii. (1894) and *The Electrical Review*, vol. xliii. (1898).

of the initial stages have died away. We know that after a steady state has been reached, the current in a circuit whose resistance is negligible in comparison with its reactance lags nearly 90° behind the e.m.f. (§ 6). Hence the zero value of f occurs when e is at its maximum value, and f will go on increasing numerically only during $\frac{1}{4}$T, the highest numerical value reached by the integral $\int edt$ thus corresponding to the area of a *quarter*-wave of e.m.f., instead of the

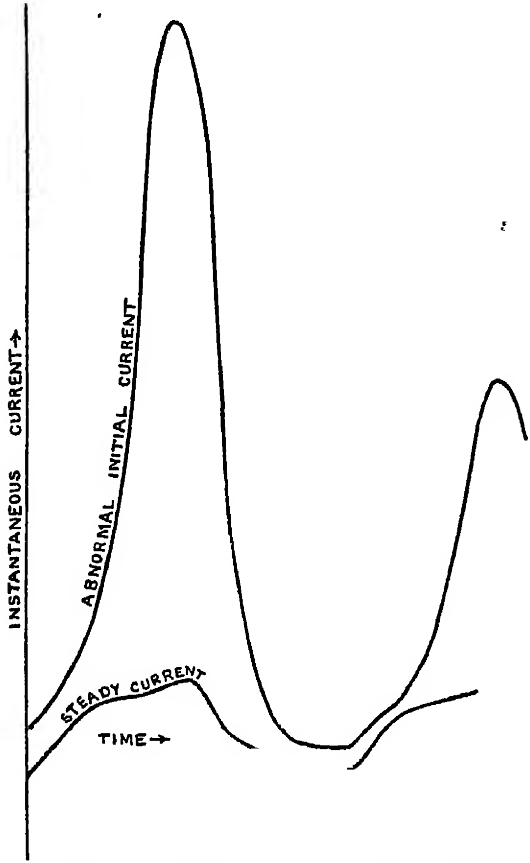


FIG. 144.—Normal and Abnormal Current Waves of Transformer.

half-wave which it may reach during the first abnormal half-wave of flux. We thus arrive at the result that (in a circuit of negligible resistance) *the maximum change of flux which can occur during the*

initial stages is equal to twice the normal maximum value of the flux after a steady state has been reached.

If the circuit contains no iron cores, the current is proportional to the flux, and it follows that the maximum value reached by the current during the initial stages under the most favourable conditions (*i.e.* when the switch happens to be closed at zero value of the e.m.f.) can never exceed *twice* the normal maximum. But it is otherwise in the case of a magnetic circuit of variable permeability, such as a transformer core. For if the normal maximum induction be fairly high, then in order to double this induction, the current may require to be increased many times, and so a very powerful initial rush of current may take place. There is, in addition to this, another important factor which very materially affects the initial current rush in a transformer—viz. the residual induction in the core. If the transformer has previously been switched off at an instant such as to leave the core strongly magnetized, and if on again closing the circuit (at zero value of the e.m.f.) the initial current tends to magnetize the core in the *same* direction, the first abnormal maximum of f may rise to nearly three times its normal value, and owing to the greatly reduced permeability at high inductions an enormous initial current-rush may take place.

The nature of such abnormal current rushes is shown in Fig. 144, in which both the steady or normal and the initial or abnormal waves of current are drawn. The abnormal initial waves consist of a large positive half-wave followed by a small negative half-wave, the large half-waves gradually decreasing, and the small ones increasing, until complete equalization takes place, corresponding to the strictly periodic or steady state.

It is to be particularly noted that these initial current rushes are not accompanied by any voltage rises, and do not endanger the insulation of the transformer. Danger to the insulation at the instant of switching on arises from a totally different cause—viz. the great concentration of p.d. over the end turns (§ 59). But a very violent rush of current in the case of very large transformers might prove dangerous on account of the severe mechanical stresses on the coils to which it would give rise.

§ 111. Sudden Short-circuit of Large Generator

When an alternator which is running under normal conditions is suddenly short-circuited, the transition from the steady state of normal load to the steady state of normal short-circuit takes place

by a series of abnormal or non-periodic current waves during which the current may run up to values which are many times the maximum value corresponding to the steady state of short-circuit. The existence of such abnormal transition waves of current was not suspected for a long time, until their disastrous effects on generators of large output became known. In order to guard against the destructive effects of the large mechanical stresses to which the abnormal waves give rise, the coil ends of large generators are now provided with strong mechanical supports (§ 47).

The existence of the large abnormal current waves during the period of transition from the steady state of normal load to the steady state of short-circuit is easily accounted for. In the steady state of normal load, the alternator field is strong: in the steady state of short-circuit, it is weak—in spite of the fact that the field current has the same value in each case. The great weakening of the field which takes place under normal short-circuit conditions is due to the demagnetizing action of the armature ampere-turns. When a sudden short-circuit takes place, the field is initially several times as strong as after the steady short-circuit state has been reached. Hence the first few abnormal current waves will have amplitudes greatly in excess of the normal short-circuit waves. It might be thought that the powerful demagnetizing action of these abnormal waves would immediately pull down the field to its normal short-circuit value; but it must be remembered that the magnetic circuit of an alternator consists largely of solid masses of iron or steel, in which powerful eddy currents are induced by any change of flux, and the magnetizing effect due to such eddy currents largely neutralizes the strong demagnetizing action of the armature ampere-turns. Even if the whole of the magnetic circuit were laminated, no sudden change of flux could take place, owing to the fact that the circuit is surrounded by the field coils, in which a powerful e.m.f. would be induced, giving rise to a very large temporary increase in the exciting current. The field flux therefore decreases more or less gradually, and so long as its decrease continues, we get the abnormal waves of large but steadily decreasing amplitude, until normal short-circuit conditions are reached.

NOTE.—The following table gives the values of the temperature coefficient α and of its reciprocal for various temperatures :—

Temperature, C°	0	5	10	15	20	25	30	35	40
α	00427	00418	00409	·00401	·00393	·00385	·00378	·00371	00364
$\frac{1}{\alpha}$	234	239	244	249	254	260	265	270	275

CHAPTER XII

§ 112. Assumptions underlying approximate theory of induction motors—§ 113. Leakage coefficients—§ 114 Behaviour of motor independent of number of turns in rotor winding. Rotor slip—§ 115. Induction motor replaced by equivalent transformer—§ 116. Torque of induction motor—§ 117. Study of transformer equivalent of induction motor. Vector diagram—§ 118. Transformation of vector diagram. Deduction of circle diagram—§ 119. Construction for slip and torque—§ 120. Case of negligible primary resistance. Heyland's circle diagram—§ 121. Effect of resistance in (1) stator and (2) rotor—§ 122. Use of starting resistances—§ 123. Effect of stator core loss. Efficiency of motor.

§ 112. Assumptions underlying Approximate Theory of Induction Motors

In dealing with the theory of induction motors, it is convenient to make certain assumptions which, though not corresponding exactly to the actually existing conditions, lead to results sufficiently near the truth for all practical purposes. We shall assume the stator and rotor windings to be identical in every respect, each consisting of the same number of conductors arranged in the same number of similar slots.* We shall, further, assume that the reluctance of the iron path is negligible in comparison with that of the air-gap; this assumption is equivalent to supposing the flux to be proportional to the current. As in § 20, we shall assume the rotating waves of magnetic flux produced by the stator and rotor currents to be distributed in space according to the simple sine law, and we may also conveniently assume all the p.d.s, e.m.f.s, and currents to follow the same law with respect to time.

Since (apart from the question of phase difference) the phases of an induction motor may be regarded as practically identical, we shall confine our attention to one phase, and study the changes taking place in the current, power factor, etc., of that phase as the load changes. A precisely similar series of changes will take place in the remaining phases.

When an induction motor is running, we may regard the systems of polyphase currents in the stator and rotor windings as each giving

* In practice, the number of slots in the stator is always different from that in the rotor; otherwise the torque corresponding to different relative positions of stator and rotor would fluctuate very considerably. See § 76.

rise to its own wave of magnetic flux, the actually existing wave being obtained by the superposition of the two hypothetical waves corresponding to the stator and rotor currents. In considering the theory of induction motors, we may either deal with the hypothetical waves which the currents in each winding would produce if those in the other winding were non-existent, or we may take as the basis of our investigation the actually existing flux due to the superposition of the hypothetical fluxes (§ 11). Both modes of investigation have been employed, and according to the method adopted different writers on the subject have defined and used various so-called "leakage factors" or "leakage coefficients," whose meaning and mutual relationship we now proceed to explain.

§ 113. Leakage Coefficients

Let the secondary winding be open, and let polyphase currents be supplied to the primary, the maximum value of the current in each phase being I_1. Let F_1 stand for the maximum value of the total flux linked with the windings of each phase. Then, according to our assumption, F_1 is proportional to I_1, so that we may write—

$$F_1 = L_1 I_1$$

and in addition the instantaneous values of these two quantities are in phase with each other. We may term L_1 the *virtual self-inductance* of each phase of the winding, since it represents the maximum value of the total flux linked with one phase of the winding when *all* the phases are traversed by currents of unit amplitude.*

Similarly, if we suppose the primary open-circuited, and polyphase currents supplied to the secondary, we may write—

$$F_2 = L_2 I_2$$

where F_2 is the maximum value of the flux linked with one phase of the secondary when all its phases are supplied with (polyphase) currents of amplitude I_2. The constant L_2 we may term the virtual self-inductance of one phase of the secondary.

In dealing, in § 22, with the stationary or alternating flux waves produced around the rotor periphery by the alternating currents in

* In the case of a two-phase winding, L_1 is identical with the true self-inductance of either phase, *i.e.* with the flux which becomes linked with that phase when supplied with unit current; for, as has been shown in § 22, the amplitude of the resultant rotating wave is the same as that of the oscillating wave due to one phase only; but in a three-phase motor L_1 is greater than the true self-inductance, and includes, in addition to the true self-inductance, the effect of the mutual inductance of the neighbouring phases (§ 22).

the various phases—waves whose superposition gives rise, as we have seen (§ 22), to a rotating wave of magnetic flux in the air-gap—we considered only those lines of induction which actually cross the gap and penetrate into the rotor core, there becoming linked with the rotor conductors. In addition to such effective or useful lines, however, there will be others which simply become linked with the stator windings, without undergoing any linkage with the rotor windings. All such lines, which do not contribute anything towards the magnetic link connecting the two sets of windings, are spoken of as leakage lines, and the corresponding flux as leakage flux. Out of the total maximum flux L_1I_1 linked with the primary, a certain proportion, MI_1, will become linked with the secondary, and the remainder will form the primary leakage flux. We may term M the *virtual mutual inductance* of the two windings.

The currents circulating in the secondary similarly give rise to a flux L_2I_2 in each phase, an amount MI_2 becoming linked with the primary, and the remainder forming the secondary leakage flux.*

The ratios $v_1 = \frac{L_1}{M}$ and $v_2 = \frac{L_2}{M}$ are termed the Hopkinson leakage coefficients. They were first introduced by A. Blondel.†

The reciprocals of the Hopkinson leakage coefficients, viz. $\nu_1 = \frac{1}{v_1} = \frac{M}{L_1}$ and $\nu_2 = \frac{1}{v_2} = \frac{M}{L_2}$, are known as Behrend's leakage coefficients.

The ratios $\tau_1 = \frac{L_1 - M}{M}$ and $\tau_2 = \frac{L_2 - M}{M}$ of the leakage flux to the useful flux in the two windings are known as Heyland's leakage coefficients.

Lastly, a very important coefficient, which has been used by Heyland, Behrend, Blondel, Hobart, and many other writers, and which is known as the *dispersion coefficient*, is the ratio—

$$\sigma = \frac{L_1L_2 - M^2}{M^2} = \tau_1 + \tau_2 + \tau_1\tau_2 = v_1v_2 - 1 = \frac{1}{\nu_1\nu_2} - 1 \ddagger$$

In the special case under consideration, in which the stator and

* In the special case under consideration, in which the primary and secondary windings are supposed to be identical in every respect, $L_1 = L_2$; this, however, would not hold generally.

† It must be clearly understood that the definitions $v_1 = \frac{L_1}{M}$ and $v_2 = \frac{L_2}{M}$ are based on the assumption of *equal numbers of turns* in the stator and rotor windings. If the turns are different, we must imagine the rotor winding replaced by one having the same number of turns as the stator winding.

‡ Behn-Eschenburg uses σ in a slightly different sense, viz $\sigma = \frac{L_1L_2 - M^2}{L_1L_2}$ (*Journal of the Institution of Electrical Engineers*, vol. xxxiii. p. 239).

rotor windings are supposed to be identical, it is evident that $L_1 = L_2$, $\tau_1 = \tau_2$, $v_1 = v_2$, and $\nu_1 = \nu_2$, so that—

$$\sigma = \frac{L^2 - M^2}{M^2} = 2\tau + \tau^2 = v^2 - 1 = \frac{1}{\nu^2} - 1$$

Since in a well-designed motor the difference between L and M would always be small, we may write, approximately, $L + M = 2M$, so that—

$$\sigma = 2\frac{L - M}{M} = 2\tau = 2(v - 1) = 2\left(\frac{1}{\nu} - 1\right)$$

§ 114. Behaviour of Motor Independent of Number of Turns in Rotor Winding. Rotor Slip

We shall next show that, so long as the number and shape of the slots in the rotor core, and the total cross-section of copper in each slot, remain unaltered, no difference whatever is produced as regards the behaviour of the motor by altering the number of turns in the rotor winding.

For, let us suppose that the turns have been increased m-fold. This will have the effect of an m-fold increase in the e.m.f. induced in the secondary winding. But the resistance and self-inductance of this winding having both been increased m^2 times, the impedance will be increased in the same ratio, while the angle of lag, whose tangent is given by $\frac{\text{reactance}}{\text{resistance}}$, will remain unaltered. Thus the current will be reduced in the ratio $\frac{1}{m}$, but its phase will remain unaltered. The ampere-turns on the rotor, and the rotating field due to them, will obviously be the same as before, for while the current has been reduced to $\frac{1}{m}$th of its original value, the turns have been increased m-fold; the phase of the current having undergone no change.

On account of the rotation of the rotor, the frequency of the rotor currents is much less, under normal running conditions, than that of the stator currents. Let f_1 be the primary frequency, and f_2 the secondary. The slip s (§ 70) is defined by the equation—

$$s = \frac{f_2}{f_1}$$

and is frequently expressed as a percentage, in the form $100\frac{f_2}{f_1}$.

§ 115. Induction Motor replaced by Equivalent Transformer

Let us now suppose that the motor is running with a slip s, exerting a definite torque, and that while the torque remains unaltered, resistance is introduced into the rotor windings, the slip increasing until ultimately the rotor comes to rest—still exerting its original torque. Let r_2 be the resistance of each phase of the rotor winding, and let r be the additional resistance which must be introduced into each phase in order to reduce the rotor to rest. Then the total resistance R_2 of each phase, when the rotor has been reduced to rest, is $R_2 = r_2 + r$. If f_2 denote the original frequency of the rotor currents, then it is clear that by reducing the rotor to rest we have increased the e.m.f. induced in its windings (by the hypothetical field due to the stator currents) in the ratio $\frac{f_1}{f_2}$; the frequency and reactance of the rotor windings have each been increased in the same ratio. It is, therefore, clear that in order that the secondary current and its phase relatively to the induced secondary e.m.f. may remain unaltered, the total resistance of the rotor windings must also be increased in the same ratio $\frac{f_1}{f_2}$, *i.e.* we must have—

$$R_2 = \frac{f_1}{f_2} r_2 = \frac{r_2}{s}$$

or—

$$r = \frac{1 - s}{s} r_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The primary and secondary currents, their phase relations, and the total power supplied to the primary, remain unaltered. But the rotor is now at rest, and although still exerting its original torque, does not develop any mechanical power. It is evident that the power which formerly corresponded to mechanical power is now employed in producing heat in the additional or external resistance r, and is represented by rI_2^2 per phase, I_2 being the secondary current in each phase.

We have thus reduced our induction motor to an equivalent three-phase transformer, the total mechanical power developed by the motor corresponding to the power absorbed by the resistances r external to the rotor windings, and the slip s of the motor corresponding, according to equation (1), to—

$$s = \frac{r_2}{r + r_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

§ 116. Torque of Induction Motor

If T = torque exerted by motor,* in *lb.-feet*, and P = number of pairs of poles in motor, then corresponding to a slip s the speed of the rotor is $\frac{(1-s)f_1}{P}$ revs. per sec., and the total mechanical power is—

$$2\pi\frac{(1-s)f_1}{P}\cdot\frac{T}{550}\text{ horse-power, or }2\pi\frac{(1-s)f_1}{P}\cdot\frac{T}{550}\times 746\text{ watts.}$$

But if N = number of phases in rotor winding, we must have—

$$2\pi\frac{(1-s)f_1}{P}\cdot\frac{746}{550}T = NrI_2^2$$

whence—

$$T = 0{\cdot}737\frac{P}{(1-s)\omega}\cdot NrI_2^2$$

or, using (1)—

$$T = 0{\cdot}737\frac{PNr_2}{\omega}\cdot\frac{I_2^2}{s}\quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $\omega = 2\pi\times$ primary frequency.

§ 117. Study of Transformer Equivalent of Induction Motor. Vector Diagram

We may now study the behaviour of the motor by substituting for it the equivalent transformer shown in Fig. 145, which replaces each phase of the winding; λ_1 and λ_2 denoting those portions of the primary and secondary self-inductances which do not contribute anything towards the mutual inductance.† The non-inductive resistance h, shown as a shunt across T′T″, is intended to represent a load equivalent to the hysteresis and eddy-current loss in the stator core. This loss will not be quite constant, for with increasing load the p.d. across T′T″ will decrease, and with it also the hysteresis and eddy-current loss. As an approximation, however, we may suppose this loss to remain constant at all loads, which is equivalent to supposing h connected across TT′.

Provisionally, however, we shall neglect the iron loss entirely,

* T stands for the *total* torque developed by the rotor, and not merely for the *useful* torque available at the pulley

† λ_1 and λ_2 may be termed the *leakage self-inductances* of the windings; by many writers they are, incorrectly, called the self-inductances of the windings simply.

and investigate the law according to which the primary and secondary currents, the power factor, etc., vary with increasing load when the primary p.d. across the terminals of the transformer is kept constant.

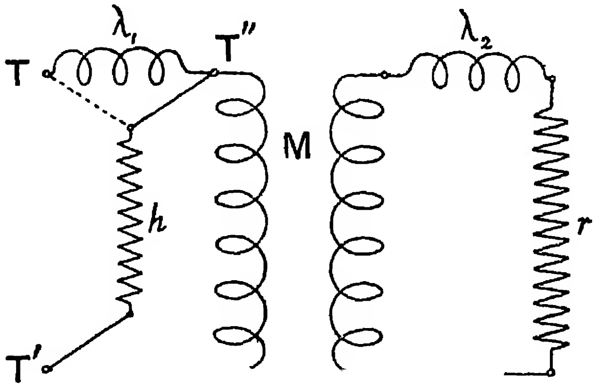


Fig. 145.—Transformer Equivalent of Induction Motor.

The method which we shall follow is one originally given by F. Bedell,* and subsequently applied to the induction motor by J. Bethenod.†

Let I_1 be the (r.m.s.) primary current, and r_1 the primary resistance (per phase). The primary phase p.d.,‡ which we shall denote by V, may be regarded as made up of the following components :—

(1) The resistance component, r_1I_1, in phase with I_1.

(2) The primary self-inductance component, ωL_1I_1, 90° ahead of I_1.

(3) The component corresponding to the mutual inductance of the two circuits. In order to find the value of this, let us assume the instantaneous value of the secondary current to be $i_2 = I_m \sin \omega t$. The hypothetical flux through the primary, due to this secondary current, is Mi_2, and the e.m.f. induced by it in the primary is given by—

$$-\frac{d}{dt}(Mi_2) = -M\frac{di_2}{dt} = -\omega MI_m \cos \omega t$$

In order to balance this, the primary phase p.d. must provide a component $+\omega MI_m \cos \omega t$, *i.e.* a component whose r.m.s. value is, say, ωMI_2, and which is 90° ahead of I_2.

Considering next the secondary circuit, we have in it—

(1) The e.m.f. induced by the hypothetical field through the

* *Proceedings of the Physical Society of London*, vol. xiv. p. 327 (1896).

† *L'Éclairage Électrique*, vol. xl. p. 253 (1904).

‡ *I.e.* the p.d. per phase, or $\frac{1}{\sqrt{3}} \times$ line p.d.

secondary due to the current I_1 in the primary. The magnitude of this e.m.f. is ωMI_1, and it is 90° behind the primary current. We may regard it as an e.m.f. impressed on the secondary, and as made

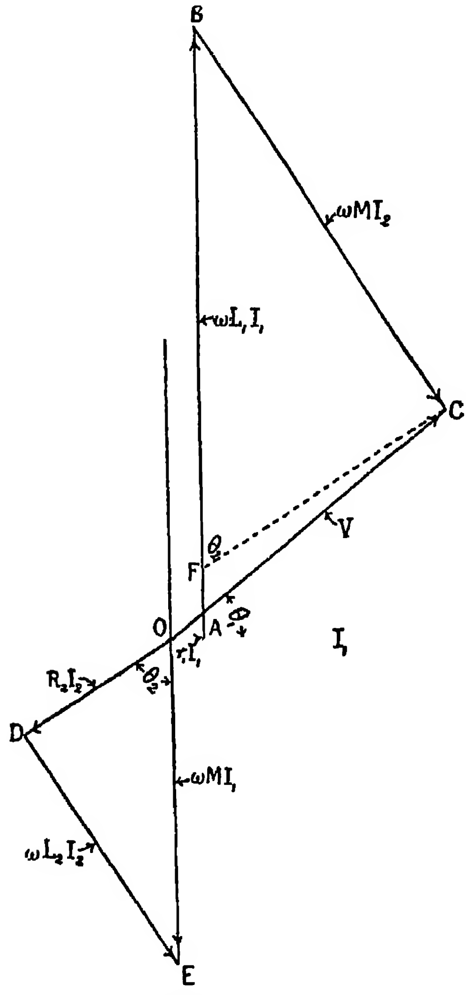


Fig. 146.—Vector Diagram of Equivalent Transformer.

up of (2) the R_2I_2 component, in phase with I_2, and (3) the $\omega L_2 I_2$ component, 90° ahead of I_2.

These relations are graphically exhibited in Fig. 146, in which

we take the primary current I_1 as our vector of reference in a horizontal direction. The resultant of $OA = r_1I_1$, $AB = \omega L_1I_1$, and $BC = \omega MI_2$ in the primary circuit gives us V, the primary phase p.d., the angle $AOC = \theta_1$ being the angle of lag of the primary current behind the p.d., so that $\cos \theta_1$ = power factor of motor. In the secondary, the resultant of $OD = R_2I_2$ and $DE = \omega L_2I_2$ gives us the e.m.f. ωMI_1, while the angle $EOD = \theta_2$ is the angle by which I_2 lags behind ωMI_1. From C let CF be drawn parallel to OD. Then angle $BFC = \theta_2$, while BCF is a *right angle.*

§ 118. Transformation of Vector Diagram. Deduction of Circle Diagram

Let us now transform our vector diagram by dividing the length of each vector in it by I_1. We then obtain the diagram of Fig. 147. A, B, and E now become *fixed points* ($OA = r_1$, $AB = \omega L_1$, and $OE = \omega M$ being constant). Hence, since, as the load varies, D describes a semicircle on OE as diameter, and since BC is parallel and proportional to ED, it follows that C *is constrained to move along a circle described on* BF *as diameter.*

Since O is fixed with respect to the circle, it follows by a well-known geometrical proposition that $OP \times OC$ = constant for all possible positions of C. Hence—

$$OP = \frac{\text{constant}}{OC} = \frac{\text{constant}}{V}I_1 = \text{constant} \times I_1$$

since V is by supposition maintained constant. OP is, therefore, proportional to the primary current, and we have the important result that as the load changes *the extremity of the primary current vector moves along a circle.* Since the angle which this vector makes with the horizontal is θ_1, we may, in our new diagram, take the horizontal direction to be that of the primary phase p.d. vector.

Let the line OB be drawn, intersecting the circle at Q. Then we have $OQ \,.\, OB = OP \,.\, OC$, or $\frac{OQ}{OP} = \frac{OC}{OB}$. Hence, the triangles OQP and OCB are similar, and $\frac{QP}{OP} = \frac{BC}{OB}$, or—

$$QP = \frac{OP}{OB} \,.\, BC = \text{constant} \times I_2 \quad . \quad . \quad . \quad . \quad . \quad (4),$$

since OB, ω, and M are all constant. Thus, QP is proportional to the secondary current.

We may now write—

$$I_1 = K_1 . OP \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$
$$I_2 = K_2 . QP \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

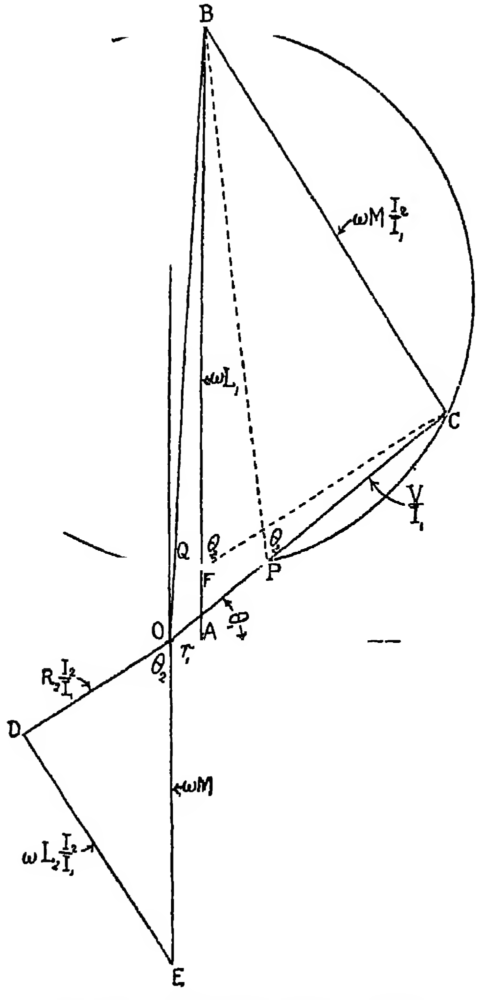


FIG. 147.—Transformed Vector Diagram.

where K_1 and K_2 are constants. We proceed to determine their values.

The value of r_1 being, in any well-designed motor, extremely

small in comparison with ωL_1, we may assume that O is practically at the same distance from the centre of the circle as A, so that $OP \cdot OC = AF \cdot AB$.

Now—

$$FB = \frac{BC}{\sin \theta_2} = \omega M \frac{I_2}{I_1} \cdot \frac{I_1 \cdot \omega M}{\omega L_2 I_2} = \frac{\omega M^2}{L_2}$$

and—

$$AF = AB - FB = \omega \left(L_1 - \frac{M^2}{L_2} \right) = \frac{\omega}{L_2}(L_1 L_2 - M^2)$$

Thus—

$$AF \cdot AB = \omega^2 \frac{L_1}{L_2}(L_1 L_2 - M^2)$$

and—

$$OP = \frac{AF \cdot AB}{OC} = \omega^2 \frac{L_1}{L_2} \cdot \frac{L_1 L_2 - M^2}{V} \cdot I_1 \quad \ldots \ldots \quad (7)$$

On comparing this with (5), we find—

$$K_1 = \frac{V \cdot L_2}{\omega^2 L_1 (L_1 L_2 - M^2)} \quad \ldots \ldots \quad (8)$$

Considering next equation (4), and putting, approximately, $OB = \omega L_1$, and utilizing (7), we find—

$$QP = \frac{\omega^2 M}{L_2} \cdot \frac{L_1 L_2 - M^2}{V} \cdot I_2 \quad \ldots \ldots \quad (9)$$

and on comparing this with (6) we get—

$$K_2 = \frac{V L_2}{\omega^2 M (L_1 L_2 - M^2)} \quad \ldots \ldots \quad (10)$$

The scales according to which the two currents are to be measured are thus different, the ratio of the corresponding constants being—

$$\frac{K_1}{K_2} = \frac{M}{L_1} \text{*}$$

* Always supposing the secondary winding to contain the same number of turns as the primary.

§ 119. Construction for Slip and Torque

We shall now show that the *slip* and *torque* may also be represented by a very simple graphical construction, shown in Fig. 148.

If, in Fig. 147, we join BP, then the angle BPC, which stands on the same arc as BFC, is equal to this latter, *i.e.* to θ_2. Thus, the angle between the primary current vector and the line PB is equal to θ_2

In Fig. 148 part of the diagram of Fig. 147 has been reproduced.

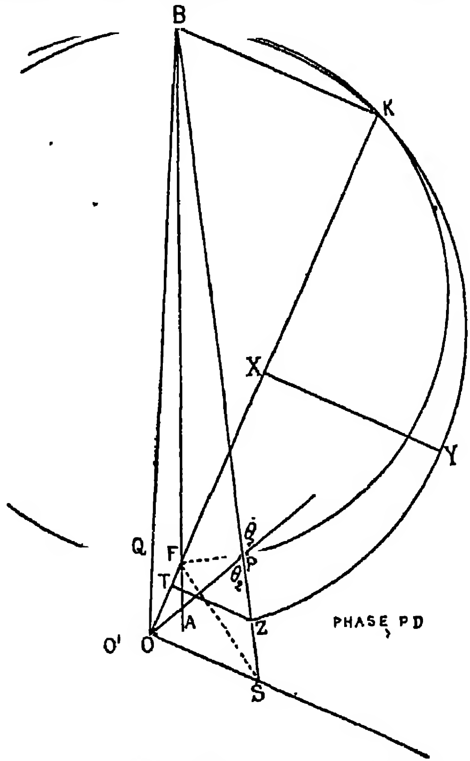


Fig. 148.—Construction for Slip and Torque of Induction Motor.

Referring once more to Fig. 147, we see that $\tan \theta_2 = \frac{\omega L_2}{R_2}$. But since (§ 115) $R_2 = \frac{r_2}{s}$, we have—

$$\tan\,\theta_2 = \frac{s\omega L_2}{r_2}$$

or—

$$s = \frac{r_2}{\omega L_2}\,.\tan\,\theta_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

so that the slip is proportional to $\tan\,\theta_2$.

In Fig. 148, join OF, and draw $OS \perp OF$. Join BP, and produce it until it intersects OS at S. Since the angle FPS is a right angle, both the triangles FSP and FSO will be right-angled, and hence a circle described on FS as diameter will pass through P and O. Since the angle OFS stands on the same arc of this circle as OPS, it follows that $\angle OFS = \theta_2$, hence $\tan\,\theta_2 = \frac{OS}{OF}$, and by (11)—

$$s = \frac{r_2}{\omega L_2}\cdot\frac{1}{OF}\,.\,OS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

but OF being constant, it follows that the *slip is proportional to* OS.

We may notice that the point K where OF intersects the circle corresponds to an infinite value of the slip, BK being parallel to OS, *i.e.* intersecting it at an infinite distance.

We shall next explain the graphical construction for the *torque.*

On OB as diameter, describe a semicircle intersecting BS in Z, and from Z let fall the perpendicular ZT on OK.

Equation (3), § 116, given above may be written—

$$T = K_8\frac{I_2^2}{s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

K_8 being a constant whose value (if the torque is expressed in lb.-feet) is given by—

$$K_8 = 0{\cdot}737\frac{PNr_2}{\omega}$$

Now in the triangle QPB—

$$\frac{QP}{QB} = \frac{\sin\,QBP}{\sin\,QPB}$$

so that—

$$I_2 = K_2\,.\,QP = K_2\frac{QB}{\sin\,QPB}\,.\sin\,OBZ$$

or—

$$I_2 = K_2 \frac{QB}{OB} \cdot \frac{1}{\sin QPB} \cdot OZ^*$$
$$= K_4 \cdot OZ$$

where K_4 is a constant whose value is $K_4 = K_2 \frac{QB}{OB \sin QPB}$.†

Hence—

$$I_2^2 = K_4^2 \cdot OZ^2$$

Again, by (12), $s = K_5 \cdot OS$, where $K_5 = \frac{r_2}{\omega L_2 \cdot OF}$. Substituting these values for I_2^2 and s in (13), we find—

$$T = \frac{K_3 \cdot K_4^2}{K_5} \cdot \frac{OZ^2}{OS} = K_6 \frac{OZ^2}{OS}$$

where K_6 is the constant $\frac{K_3 \cdot K_4^2}{K_5}$. Since $\frac{OZ^2}{OS} = OZ \times \frac{OZ}{OS} = OZ \cos ZOS = OZ \cos OZT = ZT$, we have—

$$T = K_6 \cdot ZT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

i.e. the torque is *proportional to the length of the perpendicular let fall from Z on* OK.

It is evident that the maximum torque which the motor is capable of exerting is given by the length XY of the perpendicular erected at the middle point X of OK.

Since (Fig. 147) $FB = \frac{BC}{\sin \theta_2} = \frac{\omega M^2}{L_2}$, and $AF = AB - FB = \omega\left(L_1 - \frac{M^2}{L_2}\right)$, we see that—

$$\frac{AF}{FB} = \frac{L_1 L_2 - M^2}{M^2} = \sigma, \text{ by § 113} \quad . \quad . \quad . \quad . \quad (15)$$

and—

$$\frac{AB}{AF} = \frac{1 + \sigma}{\sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

So long, therefore, as L_1, L_2, and M remain unaltered, AF and FB, and the circle described on FB as diameter, remain unchanged. We

* The straight line joining O and Z is, for the sake of clearness, not shown in the diagram.

† The angle QPB remains constant, because QB is a fixed chord of the circle described by P.

may now investigate—assuming L_1, L_2, and M to remain constant—what effect is produced by a change in the primary (or stator) resistance.

§120. Case of Negligible Primary Resistance. Heyland's Circle Diagram

We shall in the first place consider the ideal case of a motor whose stator winding is of absolutely negligible resistance. The point O (Fig. 148) in this case moves up to A, K moves up to B, the diameters BF and BO of the two circles coincide, and the diagram assumes the simple form shown in Fig. 149, U and X being the

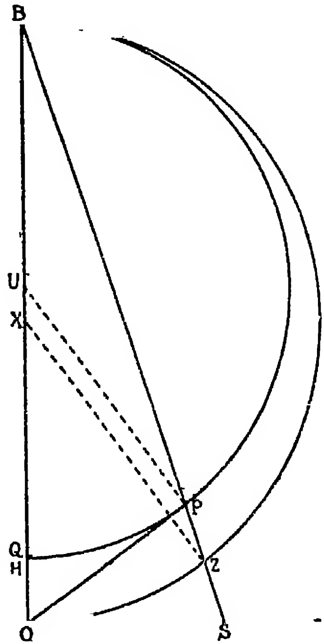


FIG. 149.—Heyland's Circle Diagram.

centres of the two circles. We can easily express the maximum value of the power factor in this ideal case in terms of the dispersion coefficient σ. The maximum power factor $\cos\,\theta_0$ is obtained when the primary current vector OP becomes tangential to the smaller circle. We have in this case—

$$\text{maximum power factor } \cos\theta_0 = \cos \text{POS} = \cos \text{PUO}^{*}$$
$$= \frac{\text{UP}}{\text{UO}} = \frac{\text{UQ}}{\text{UQ} + \text{QO}}$$
$$= \frac{1}{1 + 2\sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

since $\frac{\text{QO}}{\text{UQ}} = \frac{\sigma\text{QB}}{\text{UQ}} = 2\sigma$, by (15).

Again, the point K (Fig. 148) being now at B (Fig. 149), the perpendicular XY of Fig. 149, which is a measure of the maximum torque, has become equal to the radius of the larger circle. Hence—

$$\frac{\text{maximum torque}}{\text{torque at maximum power factor}} = \frac{\text{XZ}}{\text{HZ}} = \frac{1}{\sin\theta_0} = \frac{1 + 2\sigma}{2\sqrt{\sigma(1 + \sigma)}} \quad (18)$$

It must be understood that (17) and (18) only hold in the ideal case of a motor whose stator winding is quite negligible.

The construction shown in Fig. 149 was first given by Heyland, and is known as Heyland's Circle Diagram. It only applies, however, to the ideal case of a motor the resistance of whose stator windings is negligible.

§ 121. Effect of Resistance in (1) Stator and (2) Rotor

Let us next suppose that the resistance of the stator winding is steadily increased while L_1, L_2, and M remain unaltered. Then in Fig. 148 the point O will travel away from A, the line OFK will swing round, the point K gradually moving away from B. The most important effect of this change is to *reduce the maximum torque*, for as K moves away from B, XY shortens. And since in every well-designed motor $\sigma = \frac{\text{AF}}{\text{FB}}$ is very small, a comparatively small increase in the stator resistance (the length OA) may produce a relatively large reduction in the maximum torque (*i.e.* the overload capacity) of the motor.

In § 119, we obtained an expression for the torque of the form $T = K_6 . ZT$, where $K_6 = \frac{K_3 . K_4^2}{K_5}$. Now, of the three constants K_3, K_4, and K_5, one—K_4—does not involve r_2 at all, while K_3 and K_5 both contain r_2 as a factor. Thus K_6 is independent of r_2, that is, the torque scale in our diagram does not depend on the resistance

* $\angle$ POS = complement of $\angle$ POU = $\angle$ PUO.

of the rotor windings; so that a given stator current will produce a definite torque, no matter what the resistance of the rotor may be. Increasing the rotor resistance does not, therefore, affect the value of the torque corresponding to a given value of the stator current.

Again, on referring to equation (12) for the slip, we notice that corresponding to a given value of OS, and hence also OP (*i.e.* a given value of the stator current), the slip is directly proportional to the rotor resistance. For a given value of the stator current, therefore, an increase in the rotor resistance has the effect of *increasing the slip.*

The introduction of resistance into the motor windings has thus a totally different effect, according as the resistance is introduced into the stator or the rotor windings. For a given value of the stator current, the introduction of resistance into the stator windings reduces the torque and lowers the efficiency; whereas its introduction into the rotor windings leaves the torque unaltered, but increases the slip and lowers the efficiency.*

§ 122. Use of Starting Resistances

Let us next consider the effect of introducing resistance into the rotor windings at the moment of starting, when the slip is unity. By equation (12), § 119, we have, putting $s = 1$—

$$r_2 \times \mathrm{OS} = \text{constant}$$

Hence OS will vary inversely as r_2. Now, in order to secure high efficiency, it is desirable to make the rotor windings of very low resistance. If, therefore, we simply short-circuit the rotor at starting, r_2 being very small, OS will be very large, and hence the point P in our circle diagram (Fig. 148) will lie in the neighbourhood of K. As a result, the length of the perpendicular let fall from Z on OK will be small, *i.e.* the starting torque will be small, in spite of the fact that the current OP taken by the motor is very large. Let us now suppose that r_2 is momentarily increased by the introduction of starting resistances (§ 72) into the rotor windings. Increasing r_2 will cause OS to shorten, the point P travelling downwards from its original position in the neighbourhood of K. This will obviously increase the torque, while at the same time the current is reduced. If necessary, the maximum torque, corresponding to XY, may be obtained at starting by giving to r_2 a value such that the point Z falls on Y.

We thus see that large starting torque can only be obtained by

* Since there is a decrease of speed without any change of torque.

temporarily introducing resistances into the rotor windings, and that incidentally we thereby gain the advantage of a smaller starting current.

In the case of rotors having permanently short-circuited windings, of which the best-known and most widely used type is the squirrel-cage winding (§ 70), the introduction of starting resistances into the rotor windings is, of course, impossible. Such motors are, therefore, incapable of starting against a heavy load. Further, if the output of such a motor is large, the starting current would reach a dangerous value if the full p.d. were applied to the stator windings. Hence it becomes necessary to introduce resistances into the *stator* windings, or to use an auto-transformer (§ 65) in order to limit the current; but this, as we have seen, reduces the torque still further. The squirrel-cage type of winding is, therefore, totally unsuitable for motors which are required to start under load.

§ 123. Effect of Stator Core Loss. Efficiency of Motor

We have hitherto neglected the stator core loss. In the diagram (Fig. 145) of the equivalent transformer, this is represented by the power absorbed by the shunting non-inductive resistance h. The presence of this resistance has the effect of adding to the primary current a component I_h which is in phase with the p.d. The addition of this to the primary current vector in Fig. 148 would displace all points on the primary current locus a distance representing I_h to the right. It is simpler, however, to leave the diagram unaltered, and to measure the primary current not from O, but from a point O′ to the left of O, such that O′O represents, on the scale of primary current, I_h. The primary angle of phase difference will be PO′A instead of POA.

The total *mechanical power* is easily calculated from the known value of the torque and the speed. This latter is easily determined from the slip, as it is equal to $(1 - s) \times$ speed of synchronism. If we express the total mechanical power in watts, then its ratio to the total watts supplied to the primary gives us the *electrical efficiency* of the motor. The *useful* power is obtained by subtracting the rotor friction loss from the total mechanical power, and the ratio of the useful power to the total power supplied to the primary gives us the *commercial efficiency*.

CHAPTER XIII

§ 124. Experimental data required for construction of circle diagram. Determination of stator iron loss—§ 125 Short-circuit test—§ 126. Construction of torque circle and slip line. Scales for torque and slip—§ 127. Applications of circle diagram—§ 128. Load Tests. Measurement of slip and of friction loss—§ 129. Efficiency tests. Methods of Meunier, Sumpner, and Weekes—§ 130. Alexanderson's method.—§ 131. Induction motors with large starting torque.

§ 124. Experimental Data required for Construction of Circle Diagram. Determination of Stator Iron Loss

THE circle diagram furnishes us with an easy method of completely investigating the properties of an induction motor, without requiring any very elaborate tests. In order to be able to construct the diagram, we have to carry out the following measurements.

The motor is supplied at the normal p.d. and frequency, and is allowed to run light. The current I_0 and total power W_0 are carefully measured. If V_l = normal line p.d., then phase p.d. for a three-phase star winding* $= \frac{V_l}{\sqrt{3}} = V$ (§ 17). The power per phase is $\frac{1}{3}W_0$, and hence the power factor at no load is—

$$\cos \theta_0 = \frac{W_0}{V_l \cdot I_0\sqrt{3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We may now commence the construction of our diagram by drawing a horizontal line O′V (Fig. 151), and from O′ drawing a line $O'P_0 = I_0$, making an angel θ_0 with O′V, θ_0 being given by (1); P_0 will obviously be a point on the circle.

The next step consists in determining the position of the point O in Fig. 148. Since O′O is that part of the power component of the current which corresponds to the stator core loss, we must measure this loss. Now, when the motor is running light, the power taken

* In what follows, we suppose that we are dealing with a three-phase motor; the necessary modifications in the formulæ for a two-phase motor are quite obvious

by it is not all absorbed by the core loss, a certain amount going towards overcoming the frictional losses. If W_h = core loss, and W_f = frictional loss, we have—

$$W_h = W_0 - W_f$$

This equation enables us to find W_h if W_f is known. In order to determine W_f, we take a set of readings connecting the power absorbed by the motor (when running light) with the line p.d. As the p.d. is steadily decreased, the power decreases, on account of the decrease in the hysteresis and eddy-current losses in the stator core. By producing the curve connecting power with p.d. backwards until it intersects the axis of power, we obtain the power corresponding to zero p.d., *i.e.* the power when the core loss is vanishingly small.* This power will then simply represent W_f.

The distance O′O is now given by—

$$O'O = I_h = \frac{W_0 - W_f}{V_l \cdot \sqrt{3}}$$

and on laying this off horizontally from O′ we obtain the point O in our diagram.

The following alternative method of finding W_h, due to W. Linke,† may be used in the case of motors provided with *wound* rotors. The motor to be tested is coupled to an auxiliary motor, and is run at various speeds, the stator of the motor under test being supplied at the normal p.d., and its rotor being kept open-circuited. The power taken by the stator at the various speeds is measured, and a curve is plotted as shown in Fig. 150, connecting this power with the speed. It will be found that the curve exhibits a discontinuity at the speed of synchronism, at which a sudden drop, represented by AB, takes place in the power. This discontinuity is due to the change in sign of the hysteresis couple exerted by the stator on the rotor. *Below* synchronism, the couple due to hysteresis is a *driving* couple, and the stator transmits power to the rotor; *above* synchronism, this couple becomes a *retarding* couple, and the rotor begins to transmit power to the stator. At exact synchronism, there is no interchange of power between stator and rotor, so that the whole of the power supplied to the stator (and corresponding to the middle point C of AB) is dissipated in it, and thus represents the stator loss. This power when corrected for the stator copper loss gives us W_h.

We next measure the resistance r_1 of one phase of the primary. The effect due to eddy currents in the stator windings is equivalent to an increase in the resistance of the windings. It is difficult to

* See Appendix X.

† *Elektrotechnische Zeitschrift*, vol. xxviii. p. 964 (1907).

estimate this equivalent increase of resistance,* but as a rough value we may, in order to be on the safe side, take it to amount to 30 per cent. We shall suppose r_1 to be a resistance which is 30 per cent. greater than the resistance of one stator phase as measured by using

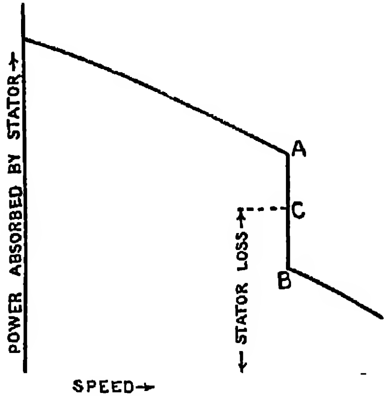


Fig. 150.—Method of finding Stator Iron Loss.

a continuous current (§ 99). Then $r_1 I_0$ gives us the resistance drop at no load.

Now, on referring to the diagram of Fig. 147, we notice that OB makes an angle a with the vertical such that—

$$\sin a = \frac{r_1}{\sqrt{r_1^2 + \omega^2 L_1^2}} = \frac{r_1 I_0}{I_0\sqrt{r_1^2 + \omega^2 L_1^2}} = \frac{\text{resistance drop at no load}}{\text{primary phase p.d.}}$$

Returning now to Fig. 151, we erect a perpendicular at O, and draw OB making an angle a with the vertical, such that $\sin a = \dfrac{r_1 I_0}{\frac{V}{\sqrt{3}}}$. Along OB lay off a length $OQ = I_0 \sin \theta_0$ equal to the wattless component of I_0, and from Q draw a line QY making an angle a with OB ($2a$ with the vertical). On again referring to Fig. 147, we see that the centre of the circle must lie on QY.

* See, on this subject, a paper by M. B. Field in the *Journal of the Institution of Electrical Engineers*, vol. xxxvii. p. 83 (1906).

§ 125. Short-circuit Test

We now have one point on the circle—P_0—and the line QY, which must contain the centre of the circle. The circle could be drawn if we knew another point on it. Such a point is obtained by determining the magnitude and phase of the *short-circuit current*, *i.e.* the current which would be obtained by clamping the short-circuited rotor and applying the normal p.d. to the stator.

In this form, however, the experiment could only be carried out with extremely small motors, as the currents would be excessive, and a dangerous rise of temperature would result in a very short time. Further, the value so obtained would not be quite correct, as the permeability of the cores would be different from that corresponding to normal working conditions. Hence the plan generally adopted consists in applying to the stator a p.d. of amount V_s which is sufficient to give rise to a short-circuit current I'_s about equal to the normal full-load current of the motor, the power W_s being measured at the same time. If $\cos\theta$ denote the corresponding power factor, we have—

$$\cos\theta_s = \frac{W_s}{V_s I'_s \sqrt{3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

V standing for the *line* p.d. during the short-circuit test. The short-circuit current I_s, which would have been obtained at the full line p.d. of V_l volts, is then calculated by means of the equation—

$$I_s = I'_s \frac{V_l}{V_s}$$

the assumption being made that the short-circuit current is directly proportional to the p.d.

As it is found that the value of the short-circuit current depends on the position of the rotor relatively to the stator, it is best to allow the rotor to run *very slowly* during this test.

We can now construct the vector of short-circuit current in our diagram, by laying off $O'P_s = I_s$ (Fig. 151), making an angle θ_s, given by equation (2), with the horizontal. This determines a second point P_s on the circle. The construction is then completed by joining P_0P_s, and at the middle point of P_0P_s erecting a perpendicular to it. The intersection of this perpendicular with QY gives the centre U of the circle, and the circle may now be drawn.

In determining the no-load and short-circuit current vectors, it is best to find a number of points on the curve connecting p.d. with current, and to read off the required value from the curve, rather than rely on an isolated reading. This procedure is in any case

necessary, in the no-load test, for the purpose of determining the frictional loss as already explained.

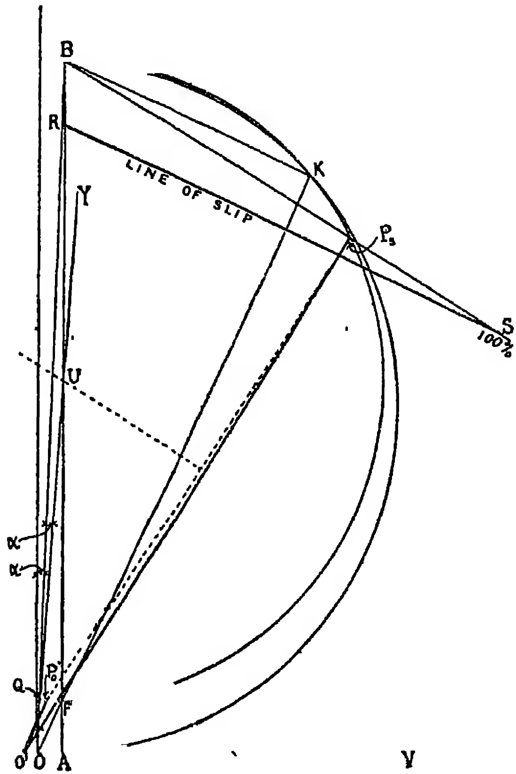


FIG. 151.—Construction of Circle Diagram from Experimental Data.

By drawing a vertical line through U, we are able to determine the dispersion coefficient σ of the motor, given by (§ 119)—

$$\sigma = \frac{AF}{FB}$$

§ 126. Construction of Torque Circle and Slip Line. Scales for Torque and Slip

We next describe the torque circle on OB as diameter, join OF and produce it to meet the circles in K. The torque is, to a certain scale, given by the length of the perpendicular let fall on OK from

that point on the torque circle where it is intersected by the line drawn from B to the extremity of the primary current vector (ZT in Fig. 148). If we wish to measure the torque in lb.-feet—*i.e.* in terms of the pull which the motor would exert at the circumference of a pulley one foot in radius—then we may fix the scale of torque very simply, as follows. Assuming, for the moment, that there are no losses at all in the stator, we must have the power transmitted to the rotor equal to that supplied to the stator. Now the former is, in terms of the torque T,* equal to $\frac{\omega}{P} \cdot \frac{T}{550} \cdot 746$ watts, ω being $2\pi \times$ frequency and P = number of pairs of poles; while the latter is, for a three-phase motor, $V_l I \cos\theta\sqrt{3}$ (I being primary current and $\cos\theta$ = power factor). Thus—

$$\frac{\omega}{P} \cdot \frac{T}{550} \cdot 746 = V_l I \cos\theta\sqrt{3}$$

or—

$$T = \frac{550}{746}\,\frac{P}{\omega} V_l\sqrt{3}\,.\,I\cos\theta$$

$$= kI\cos\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

k being the constant $\frac{550}{746} \cdot \frac{P}{\omega} \cdot V_l\sqrt{3}$. Now when, as we have momentarily assumed, there are no losses in the stator, the points O′, O, and A in our diagram become coincident, and the maximum torque T_m is represented by $\frac{1}{2}$AB. We may thus write—

$$T_m = \tfrac{1}{2}c\,.\,AB\,\dagger$$

c being the required constant of proportionality, so that—

$$c = \frac{2T_m}{AB}$$

The value of T_m may be determined by means of (3); in this, k has a known value, and I and $\cos\theta$ may be found by describing a circle on AB as diameter, from its centre drawing a horizontal radius, and joining its extremity to B. The point in which this line intersects the current circle determines I and $\cos\theta$ in equation (3).

The slip might be found as in Fig. 148, but if we wish to consider large values of the slip, the line OS there shown would correspond to an inconveniently large scale for the slip. It is, therefore, more convenient to choose a point R in OB nearer B, and to draw RS parallel to BK. Since P_s corresponds to the short-circuit current when the slip is unity (or 100 per cent.), the point of intersection S of

* In lb.-feet

† AB being measured on the scale of *currents*.

BP, produced with RS fixes the slip scale, the length RS representing a slip of 100 per cent. The slip corresponding to any other value of the current is, on the same scale, equal to the distance from R of the point of intersection of the line drawn from B to the extremity of the current vector.

§ 127. Applications of Circle Diagram

Having constructed our diagram, we may use it to find the relation connecting any two out of the various quantities which we have to consider—current, power factor, slip, torque, mechanical power, and electrical efficiency. These relations are conveniently represented in

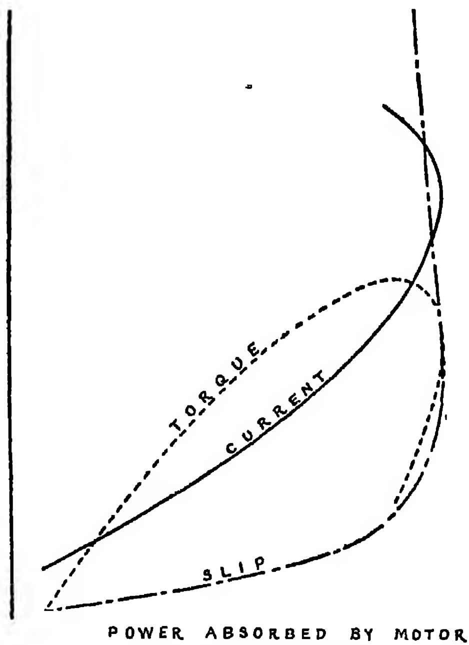


FIG. 152.—Curves derived from Circle Diagram.

the usual way by curves, and a very large number of different modes of representation may be adopted. Two of the most commonly employed are those in which the electrical power supplied to the motor, and the mechanical power developed by it, are taken as abscissæ, the remaining quantities being plotted as ordinates. An

example of this mode of representation is shown in Fig. 152, where the current, torque, and slip are plotted against the power supplied to the motor.

§ 128. Load Tests. Measurement of Slip and of Friction Loss

The experiments required for finding the constants which determine the circle diagram may all be carried out with the expenditure of a comparatively small amount of power, and hence the method of testing which depends on the use of the circle diagram is of particular value in connection with motors of large size, when the power necessary for carrying out brake tests becomes a serious consideration. When dealing with motors of small size, it may be desirable to supplement the no-load and short-circuit tests by tests under load and measurements of the slip, which furnish additional checks on the accuracy of the circle diagram. We shall now consider some of these supplementary tests. For testing the motor under load, and measuring the brake-h.p. corresponding to various stator currents, any one of the numerous forms of absorption dynamometer * may be employed. A very convenient method is to couple the motor to a standardized continuous-current generator.

We shall next consider the measurement of *slip*. For the sake of securing high efficiency, the slip is arranged to be small at all ordinary loads, not exceeding, as a rule, some 5 per cent. at full load even in small motors, and being much less in larger motors (§ 76). Hence its measurement by a direct comparison of the actual speed with the speed of synchronism is incapable of yielding accurate results, as the error involved in the speed measurement is quite comparable with the slip, and may even exceed it considerably at light loads. Special methods have, therefore, been devised for measuring *small* values of the slip.

In one of these, the shaft of the motor carries a cardboard disc divided into a number of sectors, alternately black and white, the total number of sectors being equal to twice the number of poles. The disc is illuminated by means of an arc lamp, which derives its current from the same source as the motor. Now, an alternating-current arc is extinguished twice during each complete wave of current, so that the light coming from it will in reality consist of a succession of flashes. If we suppose that the motor is being driven at synchronous speed, then since the time taken by a white sector to

* Descriptions of the Soames brake and the eddy-current brake will be found in the author's *Introductory Course of Continuous Current Engineering* (Constable & Co.).

move into the position of the white sector next in advance of it is equal to a half-period, it follows that as each flash reaches its maximum brightness the white and black sectors are in the same relative positions, and thus the disc appears to be stationary. If slip is allowed to take place, the positions of the sectors will be retarded relatively to the flashes, and the disc will appear to rotate.* By counting the number of apparent revolutions of the disc during any convenient interval of time, we obtain the number of slip revolutions. If we at the same time determine the number of revolutions of the motor (during the same interval of time, while counting the slip revolutions), then the slip is given by—

$$\frac{\text{slip revolutions}}{\text{motor revolutions} + \text{slip revolutions}}$$

Instead of a cardboard disc divided into white and black sectors, a simple radial chalk line drawn across the web of the pulley is sufficient, provided the pulley is illuminated mainly by the light of the arc. The experiment may be conducted in daylight.

A disadvantage connected with the use of the arc is that as it draws a fairly heavy current from the mains, it is liable to unbalance the three-phase system, so that the phase p.d's across the stator windings are not alike, and the motor is not running under entirely normal conditions.

Samojloff† has, however, found that if the rotating disc be in darkness, an incandescent lamp (whose periodic partial extinctions, due to alternate heating and cooling of the filament, are ordinarily quite imperceptible to the eye) may be successfully employed for the same purpose. With an incandescent lamp, which only takes a small current, no serious unbalancing can, of course, take place.

Although by the above means *small* values of the slip are easily determined with a high degree of accuracy, the method fails when the slip is considerable, since it then becomes impossible to count the rapid revolutions of the stroboscopic disc. The range of the stroboscopic method may, however, be greatly extended by various special devices, one of the most successful of which is that embodied in the direct-reading slip indicator devised by Dr. C. V. Drysdale,‡ and shown in Fig. 153. The instrument consists of a boxwood cone mounted on a spindle whose end may be applied to the motor shaft, so that the cone will be driven at the speed of the motor. Resting on the cone is a pivoted disc mounted at the end of

* In a direction opposed to the direction of rotation of the motor.

† *Annalen der Physik*, 3 2. p. 353 (1900).

‡ *The Electrician*, vol. lv. p. 734 (1905) For the use of Fig. 153 the author is indebted to Dr. Drysdale and *The Electrician* Printing and Publishing Co., who kindly supplied the block for this illustration.

a light fork, which is pivoted in a slider. The position of the slider may be varied by means of a screw, and may be read off on a scale which gives the corresponding value of the slip directly. Attached to the surface of the disc is a stroboscopic disc of paper, the number of black or white sectors being equal to the number of poles in the motor. The disc is illuminated as usual by a source of light connected to the same mains as those supplying the motor. If the position of the disc is such that its diameter is equal to that of the cone, then disc and cone will revolve at the same speed, and if this

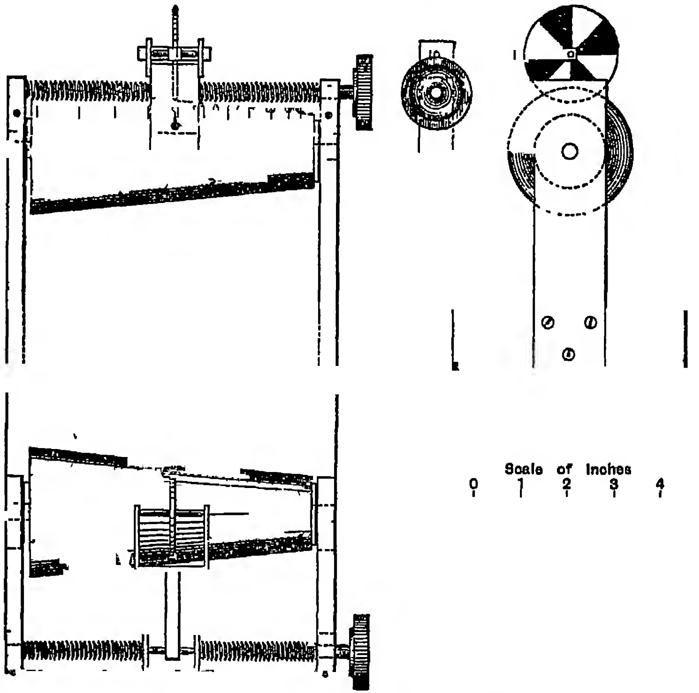


FIG. 153.—Drysdale Direct-reading Slip Indicator.

speed were that of synchronism, the disc would appear to be stationary. This position gives the zero of the scale. Since the speed of the motor is, however, below that of synchronism, the disc will appear to rotate if placed at the zero of the scale. But its speed may be increased by traversing it towards the thicker end of the cone, until it appears stationary; the method is thus a *zero* method. In constructing the direct-reading scale, the value of the slip corresponding to a given displacement of the disc from the zero position is easily calculated from the angle of the cone.

Another method of determining the slip is the intermittent contact method. Several varieties of this have been devised. In its earliest form, the method is due to G. Seibt.* Mounted on the motor shaft is a disc of insulating material, which carries a complete metallic ring of width equal to half that of the disc; this ring has a projection which is let into the uncovered portion of the disc, as shown in Fig. 154 (*a*). Two brushes are arranged to press, one against the metal ring, the other against the uncovered portion of the disc, so that during rotation the circuit between the brushes is closed once in each revolution, and is maintained closed for a time depending on the width of the metal tongue or segment let into the uncovered portion of the disc. The stator terminals are connected through a suitable

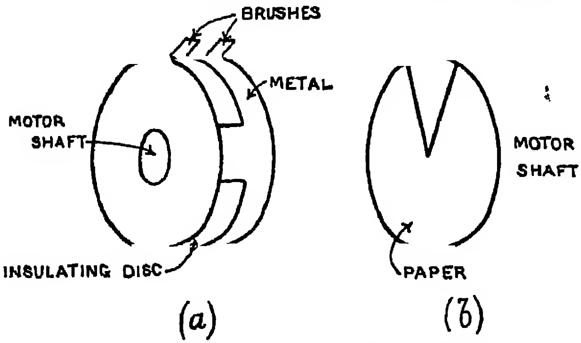


Fig. 154.—Intermittent Contact Methods of measuring Slip.

resistance or transformer, and a moving-coil instrument (with permanent magnet, as used for continuous currents), to the brushes. Now, if the rotor were running synchronously, it is evident that contact would each time be made and broken at the same points of the stator p.d. wave, such as the points *a* and *b* in Fig. 155. Hence the moving-coil instrument would receive the same impulse during each revolution (the quantity discharged through it being proportional to the area of the curve included between the ordinates at *a* and *b*), and these impulses would blend to produce a steady deflection, whose value would depend on the position of the brush bearing on the uncovered portion of the disc. For a certain brush position, the deflection would vanish, viz. for that corresponding to make at *c* and break at *d* in Fig. 155, since in this case the instrument would receive two equal impulses in opposite directions during each revolution. By moving the brush along steadily we could continuously change the amount and direction of the deflection, and cause the pointer of the indicating instrument to oscillate to and fro. It is evident,

* *Elektrotechnische Zeitschrift*, vol. xxii. p. 194 (1901).

however, that the same result would be obtained if—as is actually the case—the brush were fixed and the rotor were running asynchronously. Thus, if during a certain revolution contact is made at *a* (Fig. 155), then during the next revolution it will be made at a point *in advance* of *a*, since the time of a revolution is *longer* than that corresponding to a revolution of the stator field. Thus the points *a* and *b* travel along the base line, and the pointer of the indicating instrument oscillates with a frequency equal to that of the rotor currents, so that by counting the oscillations during any convenient interval of time we can directly determine the slip frequency. As regards the length of the contact segment, which determines the distance apart of the points *a* and *b*, this may have any convenient value, provided the amplitude of the oscillations of the indicating

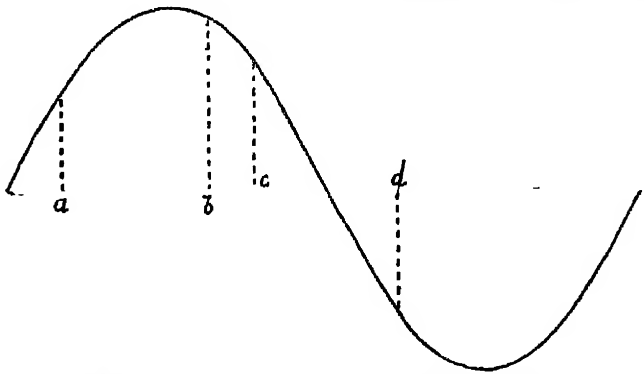


FIG. 155.—To illustrate Theory of Intermittent Contact Methods of determining Slip.

instrument is sufficiently large. The greatest amplitude is obviously obtained by making the points *a* and *b* include half a wave-length—*i.e.* by making the angular span of the contact segment equal to the angular pole-pitch of the motor.

Instead of a polarized indicating instrument, a hot-wire instrument or an incandescent lamp of suitable voltage may be employed. Since the indicating device in this case does not discriminate between positive and negative impulses, its indications will oscillate between a certain maximum and minimum, and the frequency of the oscillations will be *twice* the slip frequency. From this it will be seen that for the measurement of large slips a polarized instrument is more suitable.

In some cases, the motor shaft may not be long enough to allow of the attachment of the contact-disc, and for such cases the following extremely simple device, due to C. F. Guilbert,* and illustrated in

* *La Lumière Électrique*, vol. iv. p. 297 (1908).

Fig. 154 (*b*), may be used. A disc of paper with a sector of suitable angle cut out of it is stuck on the end of the shaft (or the boss of the pulley), and a contact brush—which may be either held by hand or suitably supported in a fixed holder—is pressed against it. The brush forms one contact of the intermittent contact circuit, while the other is formed by the shaft, permanent connection with which may be obtained by means of another brush.*

We have already, in considering the circle diagram, explained one method of finding the friction loss in a motor. Another very convenient method, which may be used as a check on the former, is to take one set of wattmeter readings when the motor is running light at the normal p.d. with its rotor short-circuited as usual, and another set when the rotor has been suddenly open-circuited, the second set of readings being obtained as quickly as possible after open-circuiting the rotor. The difference of power in the two cases practically represents the frictional loss.†

F. Blanc ‡ finds that for well-constructed motors of $\frac{1}{10}$ to 300 h.p. the friction loss may be represented by the formula—

$$\text{friction loss} = C\sqrt{\text{h.p.}} \times D$$

D being the rotor diameter in cms., and C a constant whose value ranges from 20 to 30.

§ 129. Efficiency Tests. Methods of Meunier, Sumpner, and Weekes

A very accurate method of determining the efficiency of two motors of about the same output is the following modification of the well-known Hopkinson test for dynamos, which appears to have been first suggested by M. P. Meunier,§ and independently by Dr. Sumpner and Mr. R. W. Weekes,‖ who have tested the method practically with very satisfactory results. The arrangement of connections is shown in Fig. 156. The two motors are belted together, the sizes of the pulleys being different, so that while one of the machines—which has the larger pulley—is running as a *motor*, the other—which has the smaller pulley—is being driven mechanically above the speed of synchronism. It will be shown (§ 132) that when the rotor of an induction motor whose stator is connected across the supply mains is driven *above* the speed of synchronism, the motor will act as a *generator*, supplying power to the mains.

* See Appendix XI.
† See Appendix X.
‡ *Elektrotechnische Zeitschrift*, vol. xxi. p 131 (1900).
§ *Éclairage Électrique*, vol. xxxiv. p 228 (1903).
‖ *Electrical Engineer*, vol. xxxiii. p. 918; see vol xxxiv. p. 310 (1904)

Thus one of the machines shown in Fig. 156 will act as a motor, receiving power partly from the mains, partly (the greater amount) from the second machine, which acts as a generator. The stator terminals of the generator are marked G_1, G_2, G_3, those of the motor, M_1, M_2, M_3. The difference in the diameters of the pulleys must be sufficient to give slips corresponding to the maximum load when the rotors are short-circuited. Smaller loads may be obtained by introducing suitable resistances into the rotor circuits.* Let w = power supplied from mains (algebraical sum of readings of wattmeters W_1 and W_2 in Fig. 156). This power is required to make up for the total losses. Now, one source of loss is that represented by the power w_b required to bend the belt and drive it

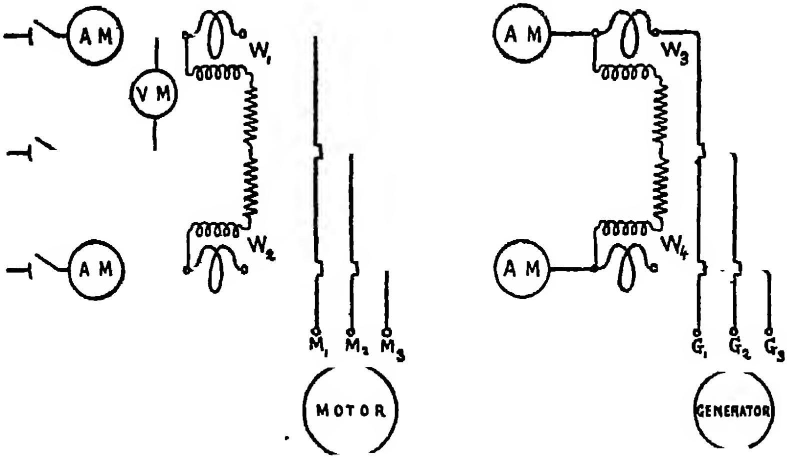


FIG. 156.—Efficiency Test of Two Similar Induction Motors.

against air friction. In order to find w_b, the wattmeters W_1 and W_2 are read (1) when both stators are across the mains, and the rotor of the motor is short-circuited, driving the open-circuited rotor of the generator by belt; and (2) when both machines are running light with the belt off. The difference in the power drawn from the mains in the two cases gives w_b. Thus $w - w_b$ gives us the power actually wasted *in the machines*. Part of this waste will occur in the motor, the remainder in the generator. Now, since the motor is necessarily more heavily loaded than the generator, it will waste a larger amount of power; the power wasted in each machine may be taken to be

* An allowance can easily be made for the power wasted in the regulating resistances

proportional to the slip. Thus, if s_m, s_g stand for the slips of motor and generator respectively, the power lost is—

$$\frac{w - w_b}{s_m + s_g} . s_m \text{ in the motor}$$

and—

$$\frac{w - w_b}{s_m + s_g} . s_g \text{ in the generator}$$

Now, if w_g = power developed by generator (algebraical sum of readings of W_3 and W_4 in Fig. 156) and transmitted to motor, the total power received by motor is $w_g + w - w_b$. On subtracting from this the power wasted in the motor, we obtain the useful power transmitted to the generator, and on dividing this latter by $w_g + w - w_b$, we obtain the motor efficiency. Similarly, the generator efficiency is obtained by dividing its useful output w_g by the power transmitted to it by the motor.*

There is, however, an additional source of loss due to the belt drive which we have so far left out of consideration. This is due to the slipping of the belt on both pulleys. So long as the machines are lightly loaded, the belt slip is inappreciable, and w_b represents the only loss taking place outside the machines. But with a heavy load, the rate of heat production at the pulleys due to slipping of the belt over their surfaces may represent an appreciable fraction of the brake-power of the motor. A correction thus becomes necessary. If d_m = dia. of motor pulley, d_g = dia. of generator pulley, and t = thickness of belt, all measured in terms of the same unit of length, then the effective diameters of the pulleys are $d_m + t$ and $d_g + t$, and if the belt did not slip, the ratio of the speed of the generator to that of the motor would be $\frac{d_m + t}{d_g + t}$. By reason of slipping, however, this is reduced in a certain ratio, say $\frac{1 - b}{1}$, where b may be termed the *belt slip*. Thus—

$$\frac{\text{speed of generator pulley}}{\text{speed of motor pulley}} = \frac{d_m + t}{d_g + t}(1 - b)$$

But this ratio may be measured directly. For if s_m = slip of motor rotor relatively to its stator field, and s_g = slip of generator rotor,† we have—

$$\frac{\text{speed of generator rotor}}{\text{speed of motor rotor}} = \frac{1 + s_g}{1 - s_m}$$

* Having subtracted w_b from the power drawn from the mains, we deal with the machines as if they were connected by means of a perfectly pliable belt which encounters no resistance in its motion through the air

† Numerical values of the slip being considered (*algebraically*, the generator slip is *negative*)

Equating the two expressions for this ratio, we find—

$$1 - b = \frac{1 + s_g}{1 - s_m} \cdot \frac{d_m + t}{d_g + t}$$

Now, if by reason of belt slip the speed of the generator is reduced in the ratio $\frac{1-b}{1}$, it is obvious that the power transmitted to it by the motor is reduced in the same ratio. Thus of the total power w^m * developed by the motor a fraction b is lost in producing heat at the pulleys. If η_m = motor efficiency, approximately calculated as already explained, then in order to balance the loss due to belt slip we have to draw from the mains an amount of power $\frac{bw_m}{\eta_m}$. We have now to recalculate both efficiencies, assuming that no losses whatever are occasioned by the belt drive, and that the total waste of power in the machines is—

$$w - w_b - \frac{bw_m}{\eta_m}$$

Although we have explained in detail how to apply the correction due to belt slip, we may state that in most cases this correction is extremely small, and need only be taken account of at heavy loads and where the highest possible degree of accuracy is required.

§ 130. Alexanderson's Method

The following approximate method of testing induction motors has been devised by Mr. E. Alexanderson,† and is used by the General Electric Co. of America. The principle of the method is based on the fact that for ordinary loads the slip is practically proportional to the torque. The load consists of a separately excited continuous-current generator belted to the motor. The useful output of the generator is measured by means of a voltmeter and ammeter, and is equal to Vi (V being the p.d. and i the current); its total output is V$i + ri^2$, r being the armature resistance. The slip s of the motor corresponding to this load is measured (by the stroboscopic method of painting a number of white sectors on the pulley equal to the number of poles, and illuminating the pulley by an alternating arc supplied from the same circuit as the motor). Let w = total

* $w_m = w_g + w - w_b - \frac{w - w_b}{s_m + s_g} s_m$

† *Electrical World and Engineer*, vol. xliv. p. 212; or *The Electrician*, vol. liii. p. 869 (1904).

mechanical power of motor, and w_1 = power required to balance losses due to rotor bearing friction and windage, belt losses, and frictional and core losses of generator; and let T and T_1 be the torques corresponding to the total power w and the losses w_1 respectively. Then obviously—

$$\frac{w}{w_1} = \frac{T}{T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The ratio $\frac{T}{T_1}$ is determined by switching off the load from the continuous-current generator, but maintaining its excitation, and finding the new value s_1 of the slip—which corresponds to the torque T_1. We then have $\frac{T}{T_1} = \frac{s}{s_1}$ approximately, and (1) becomes $\frac{w}{w_1} = \frac{s}{s_1}$. Subtracting unity from each side, we get $\frac{w - w_1}{w_1} = \frac{s - s_1}{s_1}$ or—

$$\frac{w - w_1}{s - s_1} = \frac{w_1}{s_1} = \frac{w}{s}, \text{ so that } w = (w - w_1)\frac{s}{s - s_1}$$

But since $w - w_1$ = total output of generator = $Vi + ri^2$, we have—

$$w = \frac{s}{s - s_1}(Vi + ri^2) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus w, the *total* mechanical power of the motor, becomes known. In order to find its useful or brake-power, we have to determine the power w_f corresponding to the rotor frictional loss at the speed for which the total mechanical power of the motor is w. Let T_f be the corresponding torque. Then we have $\frac{w}{w_f} = \frac{T}{T_f}$. To find the latter ratio, we remove the belt, allow the motor to run light, and measure its slip s_f. Then $\frac{T}{T_f} = \frac{s}{s_f}$, so that $\frac{w}{w_f} = \frac{s}{s}$. Proceeding as before, we find—

$$\text{brake-power of motor} = w - w_f = (s - s_f)\frac{w}{s_f} = (s - s_f)\frac{w}{s}$$

or, using (2)—

$$\text{brake-power} = \frac{s - s_f}{s - s_1}(Vi + ri^2)$$

The brake-power being known, the efficiency is at once obtained

by dividing it by the total power supplied to the motor, as measured by the two-wattmeter method.

Now, in rough measurements, it would not be necessary to measure the slip corresponding to any other value of the load in order to find the brake-power. For, using the same method as before, we find—

$$\text{losses of generator with belt} = w_1 - w_f = \frac{s_1 - s_f}{s - s_1}(\mathrm{V}i + ri^2) \quad . \quad (3)$$

and hence—

brake-power = total output of generator + losses in belt and generator.

Thus the brake-power is obtained by simply adding to the total generator output the value of the losses as given by (3), the assumption being made (which is obviously not quite correct) that these losses are constant within the working range of the motor speed.

§ 131. Induction Motors with Large Starting Torque

In an induction motor having permanently short-circuited windings, high efficiency (which involves low rotor resistance) is incompatible with large starting torque. Such motors are, therefore, incapable of starting against a heavy load, and it is usual to provide a fast and loose pulley, the motor being first allowed to run up to speed on the loose pulley, and the driving belt being then shifted to the fast pulley. Instead of this arrangement, various forms of friction clutch may be used. In a device patented by De Lignières,* the stator as well as the rotor is capable of rotating, and the stator is provided with a suitable brake, by means of which it may be gradually reduced to rest. In starting, the stator is allowed to run up to full speed, and the brake is then gradually applied to it, the torque increasing as the speed decreases, until the rotor is started; the stator being then gradually reduced to rest, and the rotor gaining speed.

The following arrangement for obtaining high starting torque in squirrel-cage motors is used by the Oerlikon Co.† The rotor conductors consist of steel tubes, connected on one side to a short-circuiting ring of copper. On the other side of the rotor is arranged a sleeve capable of sliding along the shaft. This sleeve carries the second short-circuiting ring, from which there project, in a direction

* *Western Electrician*, vol. xxxiii. p. 228 (1903). This arrangement has also been independently invented by Messrs. Mavor and Coulson.

† *Elektrotechnische Zeitschrift*, vol. xxxi. p. 699 (1910).

parallel to the shaft, a number of split copper rods whose ends enter the steel tubes. At starting, the high resistance of the steel tubes gives a powerful torque; when the motor has run up to speed, the sleeve is slid along the shaft, and the copper rods are pushed more and more deeply into the steel tubes, until they completely fill them, thereby making the rotor winding of low resistance.

M. Déri has devised an extremely ingenious method of securing large starting torque with high efficiency in motors having permanently short-circuited rotors.* This consists essentially in halving the number of poles at starting, and in using a special form of short-circuited rotor winding, such that while under normal running conditions, with the full number of poles in use, the resistance of the rotor winding is low, its resistance when only half the number of poles is used is considerable, thereby enabling the rotor to develop a

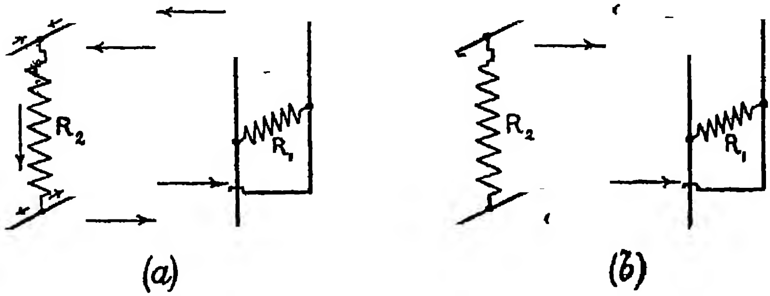


FIG. 157.—Diagram of Déri Winding for Rotors.

powerful starting torque. The change in the number of stator poles is effected by means of a switch. The arrangement of the rotor winding is shown in Fig. 157. The four conductors shown arranged on the rotor periphery form one element of the winding. The zig-zag lines R_1 and R_2 are high resistances bridged across the end-connections of the winding as shown. It will be easily seen that with a rotating two-pole field (starting position of switch) the total e.m.f. induced around the closed circuit formed by the four conductors always vanishes (Fig. 157 (*a*)), and that the only paths open to the currents are those through R_1 and R_2 (which come into play alternately, the currents flowing through them being in quadrature with each other), *i.e.* the circuit of the winding is of high resistance, enabling the rotor to exert a large starting torque. But as soon as the switch is thrown over into the running position, corresponding to a four-pole field—Fig. 157 (*b*)—the e.m.f.s around the circuit formed by the four conductors are added together, the currents flowing

* *Zeitschrift für Elektrotechnik*, vol. xvi. p. 285 (1898), also *Elektrotechnische Rundschau*, vol. xxi. p. 24 (1903).

around a low-resistance circuit (corresponding to high efficiency) without traversing R_1 and R_2.

The most commonly adopted method for limiting the starting current and securing a large starting torque consists in using wound rotors provided with slip-rings, by means of which starting resistances may be introduced into the rotor circuits. In some cases, these resistances are mounted inside the rotor itself, and an automatic switch, controlled by a centrifugal governor, gradually cuts them out as the motor gains speed.

Note on Use of Electric Valve in connection with Stroboscopic Methods of Measuring Slip.

When the slip is large, the ordinary stroboscopic methods become rather troublesome. A great improvement may then, as pointed out by A. Brückmann,* be effected by including an *electric valve* (§ 144) in the circuit of the arc lamp used for illuminating the pulley or stroboscopic disc. Such a valve may consist of electrodes of aluminium and lead in a saturated solution of sodium bicarbonate. The number of flashes of light coming from the arc is thereby halved, and the difference between the maxima and minima of illumination is rendered more pronounced. The effects of these two changes are (1) a reduction of the number of radii or sectors in the stroboscopic figure to half its original amount, and (2) greater contrast and consequent clearness in the stroboscopic pattern.

Note on Induction Type Measuring Instruments.

The principles underlying the action of induction motors are also applied in the construction of certain classes of measuring instruments. An account of the more important of these is given in Appendix V.

* *Elektrotechnische Zeitschrift*, vol. xxxii. p. 219 (1911).

CHAPTER XIV

§ 132. Generator action of induction motor at hypersynchronous speed. Phase relation of stator and rotor currents—§ 133. Vector diagram of e.m.f s and its transformation—§ 134. Generalized circle diagram—§ 135. Characteristic features of induction generator—§ 136. Speed control of induction motors Rheostatic control—§ 137. Speed control by change in number of poles—§ 138. Tandem control. Multiple motor method—§ 139. Scherbius method of speed control—§ 140 Single-phase induction motors. Theory of motor at rest—§ 141. Torque exerted by single-phase induction motor when running—142. § Effect of varying resistance of rotor circuits—§ 143. Starting of single-phase induction motors. Efficiency and power-factor.

§ 132. Generator Action of Induction Motor at Hypersynchronous Speed. Phase Relation of Stator and Rotor Currents

A POLYPHASE induction motor in many respects resembles an ordinary shunt-wound, continuous-current motor. Each runs at a speed which decreases but little with increase of load, so that practically we may regard each type as a constant-speed motor. Each develops a torque which increases, for ordinary load conditions, practically in proportion to the decrease of speed (slip) from no-load speed. Further, each, if *driven* mechanically at a sufficient speed above the no-load speed, is capable of acting as a *generator*. We are thus led to consider the generator action of an induction motor when its rotor is mechanically driven above the speed of synchronism. An induction motor when so used forms an *induction generator*.

In order to bring out as clearly as possible the relation connecting the phase differences between the primary and secondary currents in the two cases when the speed is a certain amount below and above that of synchronism, we may consider the four diagrams (a), (b), (c), and (d) of Fig. 158. The stator field is supposed to move from left to right in each case. The arrows drawn at right angles to the face of the stator core indicate the polarity of the stator field, and are drawn at the points where the induction due to the stator current *alone* reaches a maximum value. Diagrams (a) and (c) refer to the case of a motor running with *positive slip*, *i.e.* below the speed of synchronism. The sine wave of magnetic flux due to the stator current in sweeping past the rotor conductors induces e.m.f.s in them,

so that there will be a sine wave of e.m.f. travelling along the rotor conductors, and this sine wave of e.m.f. will give rise to a sine wave of current. Now, if the rotor current were exactly in phase with the hypothetical rotor e.m.f., a simple application of the rule for finding the direction of an induced current would show that the field due to the rotor currents would be as shown by the arrows drawn at intervals along the surface of the rotor core in diagram (*a*). (This would be approximately the case for very small values of the slip, when the frequency of the rotor currents is so low that they lag by a relatively small amount behind the hypothetical e.m.f. producing them.) Owing, however, to the lag of the rotor currents behind their e.m.f.s, the hypothetical field due to the rotor currents will be as shown in diagram (*c*), where the current is supposed to lag by an angle α behind the e.m.f. Similar reasoning will show that if the motor be

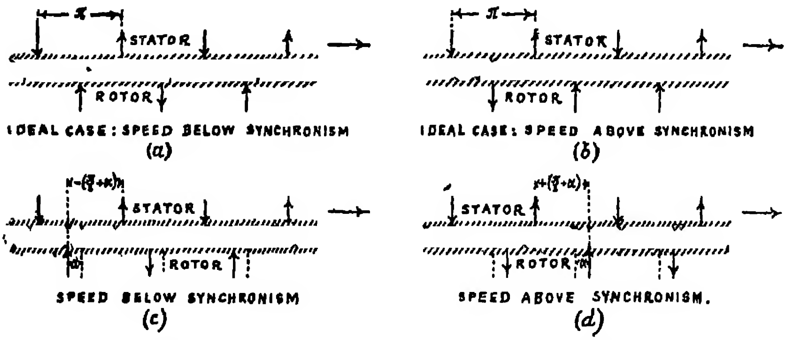


FIG. 158.—To illustrate Phase Relation of Stator and Rotor Currents in Induction Motor and Generator.

driven above the speed of synchronism, so as to have a *negative slip*, in which case the sine wave of the stator flux is reversed in its motion relatively to the conductors, the distribution of the hypothetical rotor field in the ideal case of no lag is as in (*b*), while the actual distribution is as in (*d*). The amount of lag, it is to be noted, depends solely on the slip,* and will have the same numerical value α whether the slip be positive or negative, provided the numerical value of the slip is the same.

On comparing diagrams (*c*) and (*d*), we notice that in passing from any positive value of the slip to an equal negative value, we change the phase difference between the wave of hypothetical rotor flux and that of hypothetical stator flux by an amount $\pi + 2\alpha$.

* Which determines the *frequency* of the rator currents, and hence the angle of lag.

§ 133. Vector Diagram of e.m.f.s and its Transformation

We may next, in order to reduce the stator and rotor currents to the same frequency (§ 115), imagine the negative slip increased, and at the same time resistances introduced into the rotor circuits so as to maintain the current and phase difference unaltered, until the slip becomes numerically equal to unity, *i.e.* until the speed is double that of synchronism. If the original slip was numerically equal to s, then, r_2 denoting the original resistance of one phase of the rotor circuit—*i.e.* the resistance of the short-circuited phase — and R_2 the required equivalent resistance for a negative slip of unity, we must have $R_2 = \frac{r_2}{s}$. For, by increasing the slip in the ratio $1 : s$, we have increased the hypothetical rotor e.m.f. and the rotor reactance in this ratio, so that in order to have the current and its phase relatively to the e.m.f. unaltered, the total resistance must also be increased in the same ratio. If, as before, r denote the external resistance added to each rotor phase, $r = R_2 - r_2 = \frac{1-s}{s} r_2$ (*cf.* § 115).

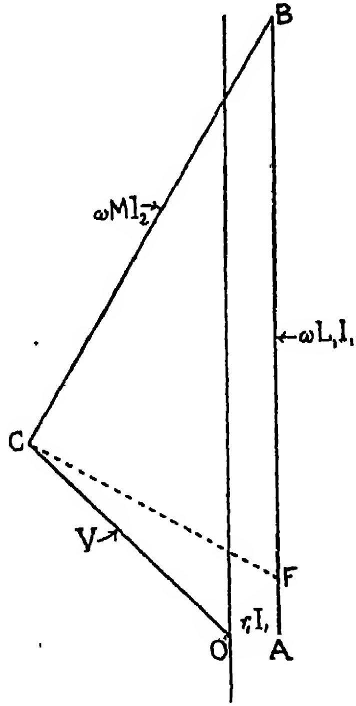


Fig. 159.—Vector Diagram of Induction Generator.

The currents in the two windings having by this artifice been reduced to the same frequency, we may proceed to construct the vector diagram of e.m.f.s similar to the motor diagram shown in Fig. 146. The lines $OA = r_1 I_1$ and $AB = \omega L_1 I_1$ remain unaltered in phase. But the vector $\omega M I_2$ of the secondary drop must, in accordance with diagrams (*c*) and (*d*) of Fig. 158, be shifted forward (*i.e.* in the counter-clockwise direction) by an amount $\pi + 2a$, so as to occupy the position BC shown in Fig. 159. We may now complete the

primary vector diagram by joining O and C. The vector OC corresponds to the constant stator phase p.d. V.

Having thus obtained our vector diagram, we proceed to transform it just as we did in dealing with a motor (§ 118), and so establish the *circle diagram* for an *induction generator*.

§ 134. Generalized Circle Diagram

The circle representing the locus of the extremity of the primary

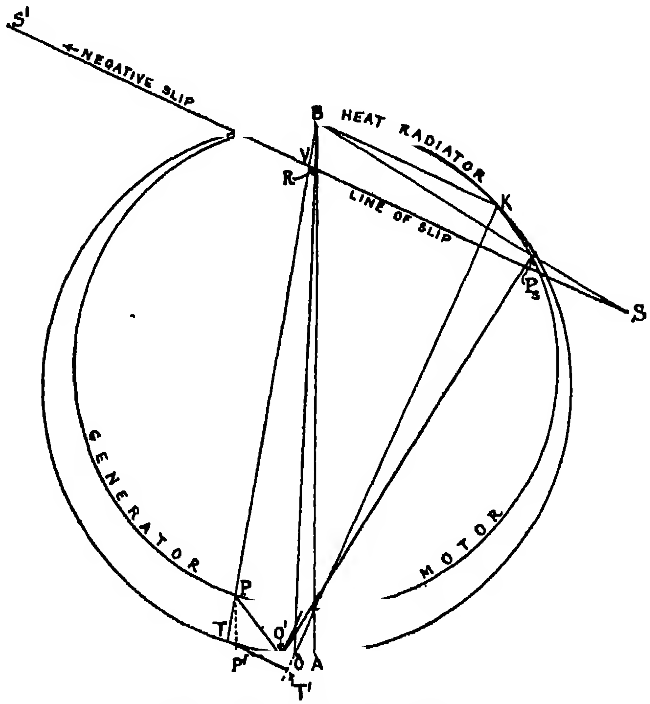


FIG 160.—Generalized Circle Diagram.

current vector is identical with the circle obtained for the machine when used as a motor; and the constructions for electrical power slip, and torque remain unaltered (§ 119).

Such a complete circle diagram, embracing every possible condition of operation of the machine, whether as generator or motor, is shown

in Fig. 160.* Thus, O′P represents a certain value of the stator current. Its projection O′P′ on the line of phase p.d. is proportional to the *useful* output of the generator outside its terminals.† If we draw BT, the intercept RV on the line of slip represents the negative slip of the generator to the same scale as that to which RS represents a slip of unity for the motor. Again, the perpendicular TT′ gives the value of the torque which is actually instrumental in generating

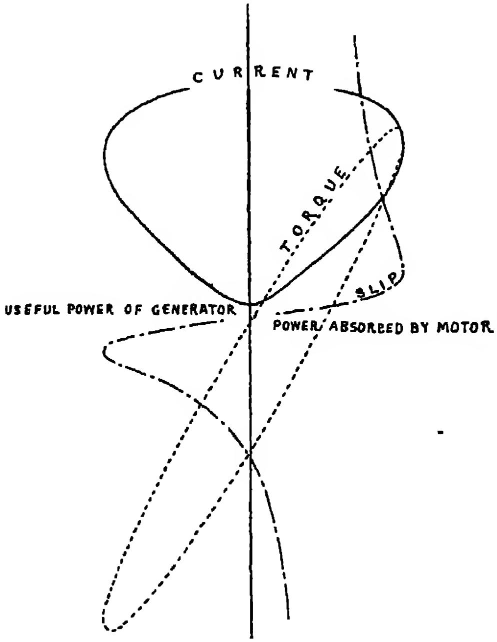


FIG. 161.—Curves Derived from Circle Diagram.

electrical energy, *excluding* the additional torque which must be provided to compensate for rotor friction. The power factor is given by the cosine of the angle P′O′P.

If, using such a complete circle diagram, we determine the relations connecting power supplied to terminals of motor, or power delivered at terminals of generator, with stator current, slip, and

* The experimental data required for the construction of the diagram have already been fully considered in Chapter XIII.

† Total useful output = O′P′ × phase p d. × number of stator phases.

torque, we obtain the elegant graphical representation shown in Fig. 161. The ordinates of the three curves drawn correspond to stator current, slip, and torque respectively, the abscissæ representing power absorbed from, or supplied to, the mains.

Between the regions of generator and motor action—the generator action extending, in the circle diagram of Fig. 160, to a point a little to the left of B (vertically over O′), and the motor action to P_s—we have a region, corresponding roughly to the arc P_sB, but extending slightly beyond B, within which the machine is giving out neither electrical nor mechanical energy, but is absorbing both, acting as a sink of energy pure and simple. This region, it need hardly be pointed out, is of purely theoretical interest, not only because for obvious reasons it would never be used in practice, but because, even if we did attempt to reach it,* the insulation of the coils would be rapidly destroyed by the excessive currents. This portion of the diagram is marked "heat radiator," as the only function then performed by the machine is to absorb energy and convert it into heat.

§ 135. Characteristic Features of Induction Generator

It must be carefully noted that the generator action of an induction motor depends wholly on the fact of its stator terminals being in connection with a polyphase generator which supplies the necessary exciting current required to maintain the rotating field. An induction generator is thus not a self-exciting machine; it can only be run in parallel with a polyphase generator of ordinary construction. A remarkable feature of the induction generator is the fact that its frequency is independent of the speed, and that for small values of the slip its output is roughly proportional to the slip—as is immediately evident from the "slip" curve of Fig. 161.

Since an induction generator does not run at synchronous speed, it is frequently termed an *asynchronous generator*.

If we suppose the induction generator to be working on a non-inductive load, then the current supplied to the load will be represented by the component O′P′ in the diagram of Fig. 160. The

* The point K is, in any case, inaccessible for purely mechanical reasons, corresponding, as it does, to an infinite speed.

wattless component of the current, obtained by taking the projection of O′P′ on the vertical, must be supplied by some external source, which may consist of an ordinary synchronous generator working in parallel with the asynchronous machine, or of a synchronous motor running light, or of a rotary converter (Chap. XV). A generating plant consisting of *large* induction or asynchronous generators deriving their exciting currents from high-speed synchronous motors running light (a high speed being adopted to cheapen the cost) would be lower in cost than a plant of synchronous machines, and would possess some important advantages, such as greater simplicity of switch-gear and absence of synchronizing or hunting troubles. On account of these advantages, induction generators have come into use to a limited extent.*

The fact that an induction motor when driven above synchronous speed is capable of acting as a generator is turned to useful account in connection with railways employing such motors, the motors in descending a steep gradient being made to return power to the line by their generator action; the power is thereby saved, instead of being dissipated by the use of ordinary brakes.

§ 136. Speed Control of Induction Motors. Rheostatic Control

The induction motor is, as we have seen, practically a *constant speed* motor, its slip at full load amounting to not more than 5 per cent. even in the case of small motors, and to much less in larger motors. Now, for many practical purposes—such as traction or crane work—it is essential to have a motor whose speed may be varied within much wider limits. We shall now consider some of the methods of speed control which are used in practice.

The method most commonly employed consists in introducing resistances into the rotor windings. We have already seen (§ 121) that the introduction of non-inductive resistances into the rotor circuits has the effect of reducing the speed while leaving the torque, the currents, and their phase relations unaltered. The total power

* Leblanc has devised an interesting form of exciter for use with an induction generator (*Éclairage Électrique*, vol xviii. pp. 161, 376 (1899)). McAllister has found that the wattless current may be supplied by condensers (*Electrical World and Engineer*, vol. xli. p 109). For an interesting account of induction generators by W L. Waters, see *Proceedings of the American Institute of Electrical Engineers*, vol. xxvii. p. 169 (1908).

drawn from the mains is obviously also unaltered, and the loss of mechanical power due to speed reduction (at constant torque) is accounted for by the power dissipated in the external resistance. The method, although giving any desired range of speed variation, is obviously very wasteful, especially at the lower speeds. Hence it can only be tolerated when the periods of variable speed are comparatively short, and when during the greater portion of the time the motor is running at its normal speed. It is, however, even in other cases, used in combination with the methods described below.

§ 137. Speed Control by Change in Number of Poles

The speed of synchronism of an induction motor being given, in revs. per sec., by $\frac{f}{P}$, f being the frequency of the stator currents and P the number of pairs of poles, it is evident that doubling the number of poles will halve the speed. By suitably arranging the stator winding,* and providing a pole-changing switch, it is possible to run at two different speeds, the lower of which is about half the higher. This method of speed control has been used by the Oerlikon Co. since 1893. In some motors, two independent windings, corresponding normally to different numbers of poles, each of which may be halved by a pole-changing switch, are provided, so that four different speeds are available. Thus, a motor wound for 12 and 6 poles, and 8 and 4 poles, will give (synchronous) speeds, at a frequency of 50, of 500, 1000, 750, and 1500 revs. per min.

§ 138. Tandem Control. Multiple Motor Method

Another method which has been used is that known as the *cascade* or *tandem* or *concatenation* method of coupling induction motors. In its simplest form, it consists in using two motors

* With a squirrel-cage rotor, no special device is required in the rotor when the number of stator poles is changed. But *wound* rotors must be of special construction—such as that illustrated in Fig. 157, in order to enable them to run when the number of poles is altered.

mechanically coupled (the coupling may be direct, as when the motors are mounted on the same shaft; or indirect, as in the case of the two motors of a tramcar or railway carriage; or some form of mechanical gearing having any desired speed ratio may be used) so as to run at the same speed, the stator of the first motor, which we may call motor I, being connected to the mains, and the rotor of this motor being connected to the stator of the second motor (motor II), whose rotor circuits are closed, either by being short-circuited or through suitable resistances, as at starting. Let us suppose that rotor II has been short-circuited, and that stator I is switched on. The motors start, their torques at first increasing as they gain speed. But as the speed increases, the frequency of the currents in stator II (which is the same as that of the currents in rotor I) steadily decreases, and if we suppose that the speed has nearly reached half the speed of synchronism (for motor I) then motor II will be running at nearly synchronous speed. Hence at this speed the torque exerted by it will be very small, the current being also small and nearly wattless. The current in stator II will, therefore, also be small and nearly wattless, and so will the current in rotor I. The torque exerted by motor I will thus also be small. We therefore find that as half the speed of synchronism is approached the combined torque decreases, so that two motors mounted on the same shaft and coupled in cascade will run at approximately *half the speed of synchronism.** If both motors are connected in parallel across the mains, they will run at nearly synchronous speed, with the result that the speed will be doubled. Thus by changing from the parallel to the cascade grouping we are able to halve the speed.

It may be pointed out that in the cascade arrangement motor I performs two functions; it acts as a motor, and also as a *frequency transformer*, practically reducing the frequency of supply to half its value. The rotor winding of motor I must, of course, be wound for the voltage of the supply mains, if the change from the tandem to the parallel grouping is to be feasible.

Various modifications of this arrangement may be used. Thus, motor II may have a different number of poles from motor I, so as to give a speed greater or less than half the speed of synchronism; and a third speed will be obtainable by fitting motor II with a pole-changing switch.†

* So far as motor II is concerned, this speed is, of course, *its* speed of synchronism.

† A large motor of this type, designed by Mr. E. Danielson, has been installed at the iron-works at Sandviken, Sweden (*Elektrotechnische Zeitschrift*, vol. xxv. p. 43 (1904)).

Closely allied to the cascade method is what we may describe as the multiple motor method.* This is used by Messrs. C. Wüst & Co., of Switzerland, and consists in mounting several rotors (in practice, from 2 to 4) on the same shaft, each rotor being surrounded by a corresponding stator. By choosing suitable numbers of poles for the different stators, as many different speeds as there are motors may be obtained by using each motor singly. Other speeds are obtainable by using two or more motors simultaneously, and the speeds will be different according as the torques developed by the motors all act in the same direction or oppose each other.

§ 139. Scherbius Method of Speed Control

One of the main disadvantages of the cascade method of control is the comparatively poor power-factor, especially in the case of large motors when running at a low speed. This disadvantage is largely overcome in a method of speed control devised by Scherbius and used by Brown, Boveri & Co.†

Consider an induction motor whose rotor is connected across a non-inductive resistance, so that the speed is below the normal. It is evident that the action of the motor will not in any way be disturbed if for the non-inductive resistance we substitute a source of e.m.f., the value of the latter being equal to the drop over the resistance, and its phase being directly opposed to the current. Now, by using a suitable source of e.m.f., the power which was formerly dissipated in the resistance may be usefully employed. In the Scherbius method of control, the source of counter-e.m.f. consists of a three-phase commutator motor (Chap. XVI), which may be either mounted on the same shaft as the main motor, or coupled to a generator by means of which it can return power to the mains. By weakening or strengthening the excitation of the auxiliary motor, its counter-e.m.f. may be varied, and with it the speed of the main motor. Where the auxiliary motor is coupled to a generator, this latter is of the induction type (§ 135).

* *Elektrotechnische Zeitschrift*, vol xxiv. p. 694 (1903).
† *The Electrician*, vol. lxv. p. 512 (1910).

§ 140. Single-phase Induction Motors. Theory of Motor at Rest

Induction motors are also used on single-phase circuits. The stator winding in such motors consists of a single-phase winding, while the rotor has a winding of precisely the same type as in polyphase motors. A two-phase induction motor may be run as a single-phase motor by using only one of the phases in the stator winding.

We shall suppose the stator and rotor windings to be so arranged as to give rise to a sine wave of magnetic induction in the air-gap. The stationary or simple alternating sine wave of induction so produced may be written in the form (§ 20)—

$$b = \mathrm{B} \sin \omega t \, . \sin \frac{\pi}{\tau} x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where b is the magnetic induction at time t at a point distant x from the origin, the latter being taken at a point where the induction is always zero; τ stands for the pole-pitch or half the wave-length (*cf.* § 20).

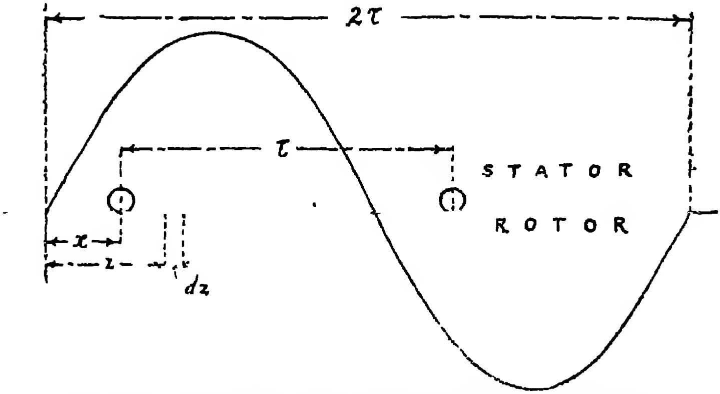


Fig. 162.—To illustrate Theory of Single-phase Induction Motor.

In order to simplify the treatment as much as possible, we shall suppose the rotor winding to consist of isolated coils, each short-circuited on itself, the width of a coil being equal to the pole-pitch τ. One such coil is shown in Fig. 162, the left-hand side of the coil being at a distance x from the origin. The sine curve shows the distribution of the induction at time t, so that the space maximum of the

induction at this instant is $B \sin \omega t$. At a distance z from the origin, the induction has the value $B \sin \omega t . \sin \frac{\pi}{\tau}z$, so that if $l =$ length of coil (in cms.) measured parallel to the shaft, the flux through a narrow strip of width dz is $Bl \sin \omega t . \sin \frac{\pi}{\tau}z . dz$, and hence the total flux through the coil in the position shown is—

$$\phi = \int_x^{x+\tau} Bl \sin \omega t \sin \frac{\pi}{\tau}z . dz$$

$$= Bl \sin \omega t \int_x^{x+\tau} \sin \frac{\pi}{\tau}z . dz$$

$$= \frac{2\tau l}{\pi} B \sin \omega t . \cos \frac{\pi}{\tau}x$$

Assuming the rotor to be at rest, and hence x constant, we may write for the e.m.f. induced in the coil shown in Fig. 162—

$$e = -\frac{d\phi}{dt}^{*} = -\frac{2\omega\tau l}{\pi} B \cos \frac{\pi}{\tau}x . \cos \omega t = -E \cos \frac{\pi}{\tau}x . \cos \omega t$$

where $E = \frac{2\omega\tau l}{\pi}B$. Hence if r, λ stand for the resistance and *leakage* self-inductance respectively of the coil, the current is—

$$i = -\frac{E}{\sqrt{r^2 + \omega^2\lambda^2}} \cos \frac{\pi}{\tau}x . \cos (\omega t - \theta) \dagger$$

where $\tan \theta = \frac{\omega\lambda}{r}$.

This shows (a result which is also otherwise obvious from an inspection of Fig. 162) that, so long as the rotor is at rest (*i.e.* so long as x is not a function of the time), there are different currents circulating in different portions of the rotor winding; the greatest current circulating in the coil which faces a pole-piece, and for which $x = 0$, and $\cos \frac{\pi}{\tau} x = 1$, and zero current in the coil whose sides are in line with the centre lines of the pole-pieces ($x = \frac{1}{2}\tau$, $\cos \frac{\pi}{\tau}x = 0$).

* The negative sign being taken in accordance with Lenz's Law (§ 4). It is to be particularly noted that ϕ here stands for the *resultant actually existing* oscillating flux *common* to the stator and rotor, and due to the combined action of their windings; it is *not* the hypothetical flux due to the stator winding *alone*.

† The current being expressed, for the sake of simplicity, in C.G.S. units.

Thus, when the rotor is at rest, and the stator supplied with current, excessive heating will take place in some of the rotor coils, while others will remain quite cool. This behaviour of a single-phase motor at rest is in striking contrast to that of a polyphase motor, in whose rotor coils uniform heating would take place.

It will be noticed that the coil in which the induced current has a maximum value has its sides placed in a field of zero intensity, and is thus unable to exert any torque, while the coil whose sides are in the region of maximum field intensity conveys no current, and is thus also unable to develop any torque. Any coil in an intermediate position, such as the coil shown in Fig 162, will, however, be subjected to a definite tangential pull, which will obviously be equal to twice the tangential force on one side of it; * this latter force is given by the product of length of conductor, current, and field intensity. We thus get for the total tangential pull T on the coil in the position shown in Fig. 162—

$$T = 2lib$$

or, using the values for b and i obtained above—

$$T = -T_0 . \sin\frac{2\pi}{\tau}x . 2\sin\omega t . \cos(\omega t - \theta)$$

where—

$$T_0 = \frac{lEB}{2\sqrt{r^2 + \omega^2\lambda^2}}$$

Since $2\sin\omega t . \cos(\omega t - \theta) = \sin(2\omega t - \theta) + \sin\theta$, the above may be written—

$$T = -T_0 \sin\frac{2\pi}{\tau}x\{\sin(2\omega t - \theta) + \sin\theta\}$$

Now, since the mean value of $\sin(2\omega t - \theta)$ over any complete number of periods vanishes, we see that the mean value of the tangential pull on the coil is given by—

$$T_m = -T_0 \sin\theta . \sin\frac{2\pi}{\tau}x$$

In order to find out the direction of the pull, let us imagine a *positive* current sent round the coil in Fig. 162, *i.e.* a current tending to produce a magnetic field in the same (upward) direction as the given field. Since both field and current are positive, their product will be positive; but an application of Dr. Fleming's rule easily

* We are assuming the coils to be placed on the surface of the rotor core, so that the pull is exerted on the coils; in reality, the coils would be embedded in the core, and the pull would come mainly on the *core*. Since, however, the total pull would remain unaltered, it is immaterial, so far as calculation goes, whether we assume the coils to be placed on the surface or in slots.

shows us that in this case the coil will tend to move to the *left*. Thus a negative value of T_m corresponds to a force acting from left to right, or in the positive direction.

From this it will be at once seen that any coil for which $x < \frac{1}{2}\tau$ experiences a pull in the *positive* direction (from left to right), while any coil for which $\tau > x > \frac{1}{2}\tau$ experiences a pull in the *negative* direction (from right to left). And since corresponding to every coil at a distance x from the origin there is another at a distance $x + \frac{1}{2}\tau$, it follows that the tangential pulls acting on the various coils will balance each other, so that *the resultant torque on the rotor of a single-phase motor which is at rest vanishes.* A single-phase induction motor is, therefore, not self-starting.

§ 141. Torque Exerted by Single-phase Induction Motor when Running

Let us next investigate the relations which obtain when the motor is running. Considering any one coil, we must now regard x as variable, and we may write $x = vt$, v being the peripheral velocity of the rotor. The positive direction of x in Fig. 162 being from left to right, this would correspond to a displacement of the coil from left to right. The magnetic flux ϕ at any time t is now given by (§ 140)—

$$\phi = \frac{2\tau l}{\pi}B \sin \omega t \cos \frac{\pi v}{\tau}t$$

$$= \frac{\tau l B}{\pi}\{\sin (\omega + \omega_1)t + \sin (\omega - \omega_1)t\}$$

ω_1 standing for $\frac{\pi v}{\tau}$. The e.m.f. induced by this flux is—

$$e = -\frac{d\phi}{dt} = -\frac{\tau l B}{\pi}\{(\omega + \omega_1)\cos(\omega + \omega_1)t + (\omega - \omega_1)\cos(\omega - \omega_1)t\}$$

while the current is given by—

$$i = -\frac{\tau l B}{\pi}\left\{\frac{\omega + \omega_1}{\sqrt{r^2 + (\omega + \omega_1)^2\lambda^2}}\cos[(\omega + \omega_1)t - \theta_1]\right.$$
$$\left. + \frac{\omega - \omega_1}{\sqrt{r^2 + (\omega - \omega_1)^2\lambda^2}}\cdot\cos[(\omega - \omega_1)t - \theta_2]\right\}$$

θ_1 and θ_2 being given by the equations—

$$\tan \theta_1 = \frac{(\omega + \omega_1)\lambda}{r};\ \tan \theta_2 = \frac{(\omega - \omega_1)\lambda}{r} \quad . \quad . \quad . \quad (2)$$

Since $\sin\theta_1 = \dfrac{(\omega+\omega_1)\lambda}{\sqrt{r^2+(\omega+\omega_1)^2\lambda^2}}$, and $\sin\theta_2 = \dfrac{(\omega-\omega_1)\lambda}{\sqrt{r^2+(\omega-\omega_1)^2\lambda^2}}$, the expression for the current may be written in the form—

$$i = -\frac{\tau l B}{\pi\lambda}\{\sin\theta_1 . \cos[(\omega+\omega_1)t-\theta_1] + \sin\theta_2 \cos[(\omega-\omega_1)t-\theta_2]\}$$

In order to find the value of the total tangential pull T on the coil, we have, as before, to form the product $T = 2lib$. Now, b may be written in the form —equation (1) of § 140—

$$b = \tfrac{1}{2}B\{\cos(\omega-\omega_1)t - \cos(\omega+\omega_1)t\}$$

In determining the mean value T_m of T, we need only consider the products of terms of the same frequency, since the mean value of the product of two terms of different frequency vanishes.* Now, since—

$$\cos[(\omega+\omega_1)t-\theta_1]\cos(\omega+\omega_1)t = \tfrac{1}{2}\{\cos[2(\omega+\omega_1)t-\theta_1]+\cos\theta_1\}$$

the mean value of which is $\cos\theta_1$, and since similarly the mean value of $\cos[\omega-\omega_1)t-\theta_2]\cos(\omega-\omega_1)t$ is $\cos\theta_2$, it follows that—

$$T_m = -\frac{\tau l^2 B^2}{\pi\lambda}(-\sin\theta_1 . \cos\theta_1 + \sin\theta_2 . \cos\theta_2)$$

$$= -\frac{\tau l^2 B^2}{4\pi\lambda}(\sin 2\theta_2 - \sin 2\theta_1)$$

§ 142. Effect of Varying Resistance of Rotor Circuits

Remembering (§ 140) that when T_m is negative the pull is from left to right, *i.e.* in the direction of motion, we see that the motor will exert a *driving* torque if $\sin 2\theta_2 > \sin 2\theta_1$, and an opposing torque if $\sin 2\theta_2 < \sin 2\theta_1$. The angles θ_1 and θ_2 are given by the equations (2). At standstill, $\theta_1 = \theta_2$, and the torque vanishes—which confirms our previous result. If, as is always the case, $\dfrac{\omega\lambda}{r}$ is a large quantity, then for a small value of ω_1, *i.e.* at a low speed, both θ_1 and θ_2 will be very large angles, and $2\theta_1$, $2\theta_2$ will both exceed 90°, θ_1 being the greater of the two, so that $\sin 2\theta_2 > \sin 2\theta_1$, as is evident from an inspection of Fig. 163 ; the motor will, therefore, exert a driving torque. By plotting $\sin 2\theta_2 - \sin 2\theta_1$ as a function of the speed, we obtain a curve

* Thus, $\cos[(\omega+\omega_1)t-\theta_1]\cos(\omega-\omega_1)t = \frac{1}{2}[(\cos 2\omega t - \theta_1) + \cos(2\omega_1 t - \theta_1)]$, the mean value of which is zero.

which shows the variation of the torque with speed on the supposition that B is constant.* The exact shape of this curve will depend on

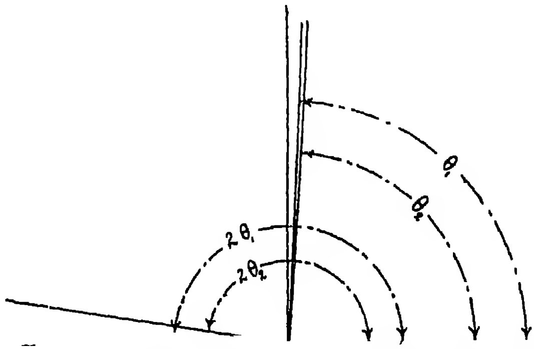


FIG. 163.—To illustrate Theory of Single-phase Induction Motor.

the value of $\frac{\omega\lambda}{r} = \frac{\text{leakage reactance at synchronism}}{\text{resistance}}$. In Fig. 164 are plotted the curves connecting $\sin 2\theta_2 - \sin 2\theta_1$ with the speed, expressed as a fraction of the synchronous speed, for four different values of $\frac{\omega\lambda}{r}$. The curves marked A, B, C, and D correspond to values of $\frac{\omega\lambda}{r}$ equal to 20, 5, 2, and 1 respectively. Curves A and B may be regarded as fairly typical of ordinary single-phase induction motors. Curves C and D correspond to effects which might be produced by introducing additional resistances into the rotor windings. It will be seen that the introduction of resistance has an effect markedly different from that obtained with a polyphase motor. Not only is the maximum value of the torque rapidly reduced, but when $\frac{\omega\lambda}{r}$ is made equal to unity, the motor becomes quite incapable of exerting a driving torque at any speed whatsoever (curve D). Hence the introduction of resistances into the rotor for purposes of speed regulation is attended with the serious disadvantage of reducing the overload capacity of the motor, and cannot be used as freely and advantageously as with polyphase motors.†

It may be shown that for a single-phase induction motor—as for a polyphase one—there exists a circle which represents the locus of

* This condition is, of course, not strictly fulfilled, on account of the existence of the primary or stator leakage flux; as the latter increases, B decreases

† Resistances are introduced at starting to limit the starting current

the extremity of the primary current vector. But since the simple and elegant graphical representation of slip, torque, etc., is in this

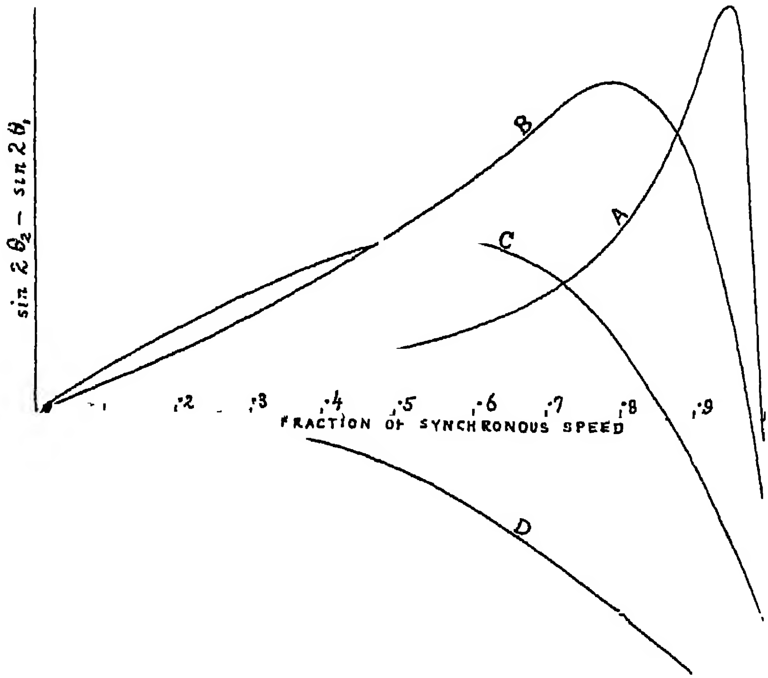


FIG. 161. - Torque-speed Curves of Single-phase Induction Motor.

case no longer possible as with the polyphase motor, the circle diagram for a single-phase induction motor possesses but little practical interest.

§ 143. Starting of Single-phase Induction Motors. Efficiency and Power Factor

The single-phase induction motor not being self-starting, special starting devices have to be provided for it The arrangement most commonly in use is one consisting of an auxiliary starting winding, which occupies the same position relatively to the running winding as either of the two windings of a two-phase induction motor does relatively to the other. The motor will start if there is sufficient

phase difference between the currents in the two windings. The necessary phase difference may be obtained (1) by connecting the starting winding in series with an external reactance coil or condenser (both methods having been used in practice); (2) by arranging the starting winding so that the magnetic leakage corresponding to it is much greater than that corresponding to the running winding—a method very successfully employed by Heyland, and not involving the use of any reactance coils or condensers external to the motor; (3) by using a rotor provided with a winding resembling that of an ordinary continuous-current machine, and having a commutator and brushes, by means of which the motor may be started as a "repulsion" motor; commutator motors of this type are considered in Chapter XVII.

Single-phase induction motors are in every way—as regards size, efficiency, starting torque, power factor and cost—greatly inferior to polyphase motors. The output of an induction motor of given dimensions, weight and price when wound for use as a single-phase motor is only some 68 per cent. of its output when wound as a polyphase motor. As regards efficiency and power-factor, the following table * may be taken as giving average values :—

Horse-power	5	15	25	50
Efficiency at full load	0 78	0·80	0·81	0·82
„ „ half „	0·72	0·75	0·76	0·79
Full-load power-factor	0·78	0·80	0·82	0·85

* A. Still, *Electrical World*, vol 48, p. 1204 (1906).

CHAPTER XV

§ 144. Methods of transforming alternating into continuous current. Mechanical rectifiers and electric valves—§ 145 Rotary converters and their uses—§ 146. Voltage ratio in converters—§ 147. Ratio of currents—§ 148. Relative outputs of armature when generating continuous, single-, two-, and three-phase currents—§ 149. Heating of converter armature—§ 150. Effect of number of slip-rings on output of converter—§ 151. Six-phase converter supplied from three-phase mains—§ 152. Hunting of rotary converters—§ 153. Voltage regulation of rotary converters—§ 154. Converters with variable voltage ratio—§ 155. Starting of rotary converters—§ 156. Racing of inverted rotaries—§ 157. Converters *v.* motor-generators—§ 158. Motor converters or cascade converters.

§ 144. Methods of Transforming Alternating into Continuous Current. Mechanical Rectifiers and Electric Valves

CASES frequently arise in practice in which it becomes necessary to effect the transformation of an alternating into a continuous current. We shall consider the various methods available for effecting this transformation.

When the amount of power to be transformed is large, recourse would invariably be had to some form of *rotary converter* or *motor-generator* (§ 145 *et seq.*). But when dealing with small amounts of power, it is generally more convenient to make use of other methods, rendered preferable on the scores of lower first cost and higher efficiency.

A method which has been used to a limited extent is that of a commutator driven synchronously—either by a small motor or by being mounted directly on the alternator shaft. The main difficulty in this arrangement is the prevention of sparking.

The same principle—that of rectifying the current (*i e.* of reversing every alternate half-wave) by *purely mechanical* means—is embodied in the *mechanical rectifier* with a synchronously vibrating polarized armature. A diagram of this form of apparatus is given in Fig. 165 One of the terminals of the continuous-current circuit

(marked " – ") is connected to the middle point of the secondary of the transformer, while the other (marked " + ") is synchronously alternated from the one extreme terminal to the other of the secondary winding. The vibrating armature by means of which this synchronous change of connections is effected must have a sufficiently rapid natural period of vibration, and this may be controlled by means of the adjusting springs which bear on the ends of two screws. The armature is polarized by means of two small coils supplied with current from a secondary cell. In order to reduce the fluctuations

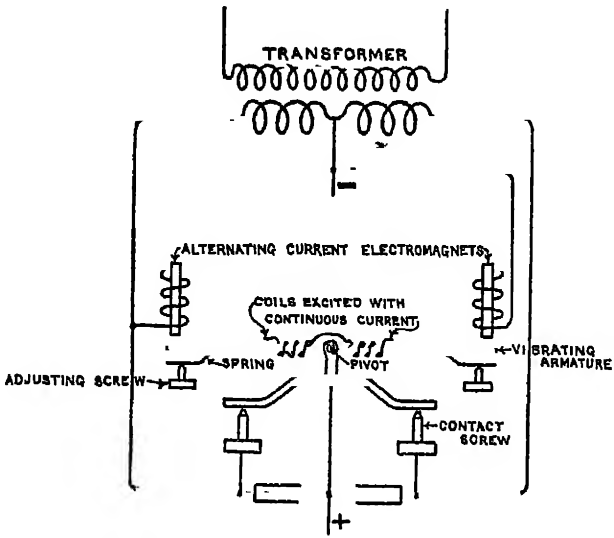


FIG. 165.—Mechanical Rectifier

in the continuous current, a choking coil is included in the continuous-current circuit. Sparking at the contacts is largely prevented by the aid of condensers connected up as shown in Fig. 165.

An interesting type of apparatus is that in which the peculiar behaviour of electrodes of aluminium and its alloys, when immersed in a suitable electrolyte, is made use of for effecting the rectification of an alternating current. It is found that if such an electrode be placed in a solution of ammonium phosphate or borate, the other electrode consisting of some other metal, and if the two electrodes be connected to a source of alternating p.d., only alternate half-waves of current, viz. those for which the aluminium electrode serves as *cathode*, can pass through the electrolytic cell. For this

reason, such a cell has been termed an *electric valve;* it is sometimes spoken of as an *electrolytic rectifier.* It consists of two electrodes, one of which, made of an aluminium alloy, forms the cathode, while the other, which is made of lead, serves as anode. Both electrodes are immersed in a concentrated solution of neutral ammonium phosphate.

A single valve only makes use of alternate half-waves. In order to utilize each half-wave, Grätz has devised the arrangement shown in Fig. 166. Four valves are connected so as to form a Wheatstone's bridge arrangement of connections. The electrodes marked A represent anodes, those marked C cathodes. During one half-wave, the flow of current takes place as shown

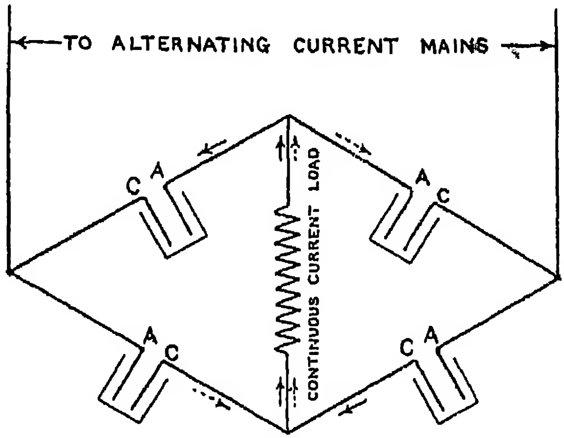


FIG. 166.—Grätz's Arrangement of Electric Valves.

by the full-line arrows; during the next as shown by the dotted arrows.

Electric valves are made for currents up to 20 amperes, and the voltage limit is about 200 volts (alternating).

A form of rectifier which is gaining in importance, and which has recently undergone various improvements, is the Cooper Hewitt *mercury vapour converter.* The form shown in Fig. 167, consists of an exhausted glass vessel of the shape shown and having four platinum wires fused into it. Two of these, corresponding to the cathode and the starting electrode, make contact with mercury cups, while the other two, which represent the anodes, terminate in graphite electrodes. In order to render the passage of a current possible, there must be a sufficient number of mercury ions present

in the vessel. Now the production of such ions takes place at the surface of the mercury if used as a *cathode;* but if an attempt be made to use the mercury as an anode, no mercury ions can be produced, and no current will pass. The arc is first started by means of the starting electrode used as a temporary anode. The two main anodes are across the secondary of a transformer, while the middle point of the secondary is connected, through the continuous current load and a choking coil, to the cathode. The form of rectifier shown in Fig. 167 is made for currents up to 40 amperes,

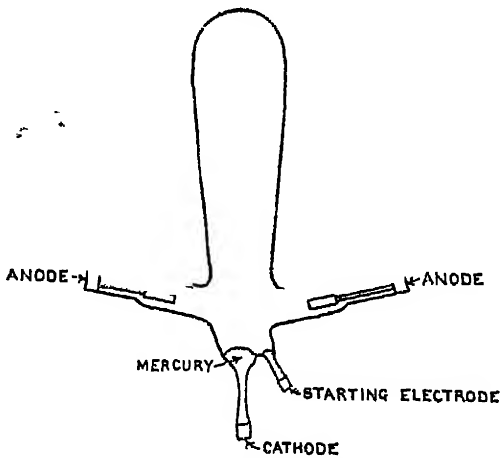


FIG. 167.—Mercury Vapour Rectifier.

and voltages up to 220, although it may be used for very much higher voltages.

B. B. Schäfer greatly improved the construction of the mercury vapour rectifier, so as to render it capable of dealing with currents up to 300 amperes.* He introduced the use of a containing vessel of steel. The joints are rendered air-tight by means of asbestos and mercury packing, and in order to maintain a sufficiently good vacuum a rotary high-vacuum pump is provided which is run for a few hours at intervals of about a month. A small auxiliary electrode (in addition to the starting electrode) is provided, and an arc taking a small continuous current is steadily maintained in each vessel.

* *Elektrotechnische Zeitschrift*, vol. xxxii. p 2 (1911).

The main loss in the mercury vapour rectifier is the constant drop of about 15 volts in the mercury arc. This drop is independent of the current (it varies from 13 to 20, according to the distance between the electrodes), so that the efficiency remains practically constant down to very light loads.

Mercury vapour rectifiers are now in use having outputs up to 600 kw., for currents up to 500 amps. at voltages up to 1200.

§ 145. Rotary Converters and their Uses

A rotary converter is a machine by means of which may be effected the transformation of alternating (single- or polyphase) currents into continuous current, or the transformation of continuous into alternating current. In most cases, the machine is employed in connection with the first-mentioned purpose, and its chief practical importance is due to the fact that while the transmission of power over long distances is most readily and economically accomplished by means of alternating currents, its actual distribution to the motors of an electric railway or tramway system is in a large number of cases effected by means of continuous current. The rotary converter supplies one of the necessary links between the high-voltage alternating-current transmission system and the low-voltage (500 to 600 volts) continuous-current distribution system of an electric railway or tramway. Were it not for its importance in electric railway work, the rotary converter would probably have received much less attention than has actually been bestowed on it.

In some few cases, converters have been used for effecting the opposite kind of transformation, viz. from continuous to alternating current. When so used, they have sometimes been termed *inverted rotaries*. Cases of this kind may occasionally arise in connection with a large continuous-current generating station, when it is desired to transmit a limited amount of power to a greater distance than could economically be accomplished by the use of continuous current. Continuous current is in such cases supplied to an inverted rotary at the generating station, the polyphase currents from the rotary being raised to a high voltage by means of a step-up transformer, and at the far end transformed down and fed into another rotary converter.

The general principles underlying the construction of a rotary converter have already been briefly considered in § 43. We shall

now study it somewhat more in detail, and shall, in the first place, investigate the voltage relations on the alternating- and continuous-current sides.

§ 146. Voltage Ratio in Converters

If we assume the space distribution of the magnetic flux to follow the sine law, then the e.m.f. induced in each coil of the armature winding will be a sine function of the time. The sine wave e.m.f.s induced in the consecutive coils will differ in phase by a constant amount, represented by the phase angle $\frac{\pi}{c}$, c being the number of coils per pole-pitch. These e.m.f.s may, in a vector diagram, be represented, as in Fig. 168, by a series of radial vectors of equal length and spaced at equal angular intervals apart, each angular interval amounting to $\frac{\pi}{c}$. In order to find the resultant alternating e.m.f. between any two points of the winding, we have to add vectorially the e.m.f.s of all the coils included between those two points, using the construction known as the polygon of vectors. Thus, taking two points which correspond to a span equal to the pole-pitch, and thus embrace all the coils whose e.m.f. vectors in Fig. 168 are included between A and B, we get the open polygon OQP of Fig. 169, the closing side OP of which is the resultant e.m.f. of the group of coils considered. If each of the vectors be taken to represent the maximum value of the alternating e.m.f., then the vector OP in Fig. 169 will represent the maximum value of the e.m.f. in a group of coils included within a pole-pitch; but this is evidently equal to the continuous e.m.f. between two brushes. For any smaller group of coils, such as that represented by the vectors included between A and C in Fig. 168, we get for the maximum value of the e.m.f. a vector OQ (Fig. 169).

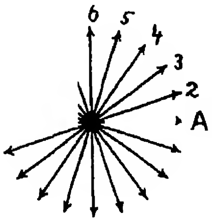


FIG. 168.—Vector Diagram of e m f s in Consecutive Coils.

Now if—as is generally the case—the number of coils between two brushes is considerable, the regular polygon (OQP in Fig. 169), which results from the vectorial addition of the e.m.f.s of consecutive coils, becomes indistinguishable from a circular arc. Hence, in order to find the voltage relations on the continuous- and alternating-

current sides, we may adopt the following construction. On a line OP (Fig. 170) as diameter—the length of OP representing, to a suitable scale, the voltage between two consecutive brushes—describe the semicircle OSQP. If the distance between the points of connection of two given slip-rings is $\frac{1}{d}$th of the pole-pitch, then, in order to find the maximum value of the alternating voltage between the

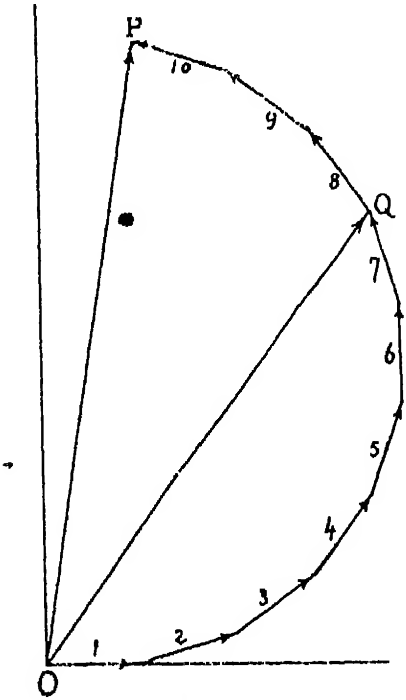


FIG. 169. Polygon Construction for Resultant e.m.f.

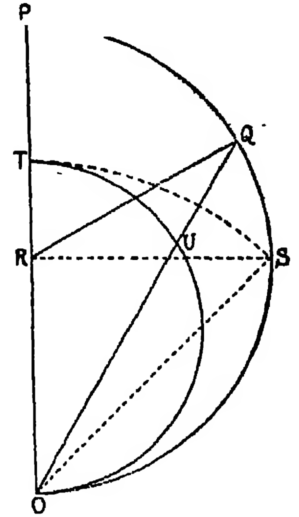


FIG. 170.—Construction for Voltage Ratio in Rotary Converter.

slip rings, lay off from the centre R of the semicircle a line RQ such that the angle ORQ $= \frac{\pi}{d}$ in circular measure, or $\frac{180}{d}$ in degree measure. Join OQ; then OQ gives the required maximum value of the alternating voltage. The r.m.s. value of this voltage is given by $\frac{OQ}{\sqrt{2}}$, but may be immediately obtained by an additional very simple graphical construction, which is as follows. At R erect the perpendicular RS,

and join OS With O as centre and radius OS describe the arc ST, intersecting OP at T, and on OT as diameter describe the semicircle OUT. Then, since OT = OS = $\sqrt{2}$ OR, we have $\frac{OT}{OP} = \frac{\sqrt{2}\,OR}{2OR} = \frac{1}{\sqrt{2}}$, or $OT = \frac{OP}{\sqrt{2}}$. But the semicircles being similar and similarly situated with respect to O, it follows that the ratio of any two secants drawn from O and making the same angle with OP will also be $\frac{1}{\sqrt{2}}$. Thus $OU = \frac{OQ}{\sqrt{2}}$, *i.e.* OU gives the r.m.s. value of the alternating voltage between the slip-rings.*

A simple formula, immediately evident by a reference to the diagram, may also be used for calculating the voltage ratio. For we have—

$$\text{r.m.s. value of voltage} = OU = \frac{OQ}{\sqrt{2}} = \frac{OP \cos POQ}{\sqrt{2}} = \frac{OP}{\sqrt{2}} \sin \tfrac{1}{2}ORQ$$

and since ORQ in degree measure is equal to $\frac{180}{d}$,

$$\text{r.m.s. value of slip-ring voltage} = \frac{\text{continuous voltage}}{\sqrt{2}} \cdot \sin \frac{90^\circ}{d} \quad . \quad (1)$$

the value of d being given by—

$$\text{distance between points of connection of slip-rings} = \frac{\text{pole-pitch}}{d}$$

Using this formula, or the graphical construction just explained, we have, calling the continuous voltage 100, the following values of the alternating (r.m.s.) voltage for a single-, two-, and three-phase converter $\left(\frac{1}{d} = 1, 1 \text{ and } \frac{2}{3} \text{ respectively}\right)$—

Continuous voltage.	Single-phase.	Two-phase.	Three-phase.
100	70·7	70·7	61·2

These values, it must be remembered, are only *approximate*, not only because we have taken no account of the resistance drop in the converter, but because the assumed sine law of the magnetic flux distribution around the armature periphery is more or less departed from in practice.

* This elegant construction is due to Mr. O. J. Ferguson (*Electrical World and Engineer*, vol. xliv. p. 733 (1904)).

§ 147. Ratio of Currents

We shall next consider the ratio of the continuous current to the line currents on the alternating-current side. An approximate value for this ratio may be easily obtained by assuming that the power factor is unity, and that the losses in the converter are negligible; the latter supposition requiring equality of power on the two sides of the converter.

Let V, I stand for the voltage and current respectively on the continuous-current side; and V_1, I_1; V_2, I_2; V_3, I_3 for the voltages and currents on the alternating-current side of a single-, two-, and three-phase converter respectively. The power factor being unity, and the losses being neglected, we must have—

$$VI = V_1I_1 = 2V_2I_2 = \sqrt{3} \cdot V_3I_3.^{*}$$

Now since, as we have just seen, $V_1 = 0{\cdot}707V$, $V_2 = 0{\cdot}707V$, and $V_3 = 0{\cdot}612V$, we get—

$$I_1 = 1{\cdot}414I; \; I_2 = 0{\cdot}707I; \; I_3 = 0{\cdot}943I$$

Exhibited in tabular form, these results are—

Continuous current.	Alternating Line Currents. Single-phase.	Two-phase.	Three-phase.
100	141·4	70·7	94·3

§ 148. Relative Outputs of Armature when generating Continuous, Single-, Two-, and Three-phase Currents

One of the most important problems in connection with converters is the question of *heating*. Before proceeding with this problem, however, we shall investigate the relative output of a continuous-current armature when used to generate continuous, single-, two-, and three-phase currents, the limit of output being determined by the greatest permissible rise of temperature. As a basis of our comparison we therefore adopt the same temperature rise in each case, assuming also that the speed and the effective flux per pole are maintained constant.

It is evident that to obtain the same temperature rise the current

* The three-phase power $= 3V_3I_3'$, where I_3' is the phase current; now $I_3' = \frac{1}{\sqrt{3}}I_3$ (§ 17), so that the power $= \sqrt{3} \cdot V_3I_3$ (see Note II. at end of Chapter IV.).

traversing the armature winding must be the same in each case. Let I, I′, I″, and I‴ be the line currents corresponding to maximum safe temperature rise of the armature when generating continuous, single-, two-, and three-phase currents respectively. If V, V_1, V_2, and V_3 denote the corresponding line voltages, then the relative outputs are—

Continuous current.	Single-phase.	Two-phase.	Three-phase.
VI	$V_1I' = 0{\cdot}707VI'$	$2V_2I'' = 1{\cdot}414VI''$	$V_3I'''\sqrt{3} = 0{\cdot}612VI'''\sqrt{3}$

For the sake of simplicity, we may assume the machine to be a two-pole one. This will not in any way affect the generality of our conclusions, for an ordinary lap-wound multipolar armature is equivalent to a number of simple two-pole armatures connected in parallel, and any result deduced for a two-pole machine will apply to a section of the multipolar winding under cover of two neighbouring pole-pieces, and hence will also apply to the entire multipolar armature.

Assuming, then, a two-pole armature, we have, for the currents in the armature windings, in the four cases under consideration—

$$\tfrac{1}{2}I \qquad \tfrac{1}{2}I' \qquad \frac{\sqrt{2}}{2}I'' \qquad \frac{1}{\sqrt{3}}I'''$$

For equal heating, these currents must be equal. Hence—

$$I' = I;\ I'' = \frac{1}{\sqrt{2}}I;\ I''' = \frac{\sqrt{3}}{2}I$$

Substituting these values in the expressions for the output in the various cases, we obtain—

Continuous current.	Single-phase.	Two-phase.	Three-phase.
VI	0·707VI	VI	0·918VI

§ 149. Heating of Converter Armature

The heating of a converter armature is a very complicated problem, due mainly to the fact that the rate of heat generation is not uniform throughout the armature winding (as it was in the cases just considered), but varies according to the position of a coil relatively to the point of connection to a slip-ring, the greatest rate of heat production occurring in the coils close to the slip-ring

connections. For the sake of simplicity, we shall, as before, assume the armature to be a two-pole one, the power factor to be unity, and we shall determine the output, for a given mean temperature rise, when the armature is used as a converter from an N-phase to a continuous current.

The points of connection to the slip-rings being fixed relatively to the armature winding, while the brushes are constantly changing their position with respect to it, it is evident that the distribution of currents in the armature at any instant will depend (among other things) on the position of the brushes relatively to the slip-ring attachments. Since the effect is one depending on relative position, we shall find it convenient to suppose that the armature is stationary, while the brushes are carried round by the revolving field.

Let there be N slip-rings, dividing the armature winding into N phases. The angle ROS (Fig. 171) subtended by each phase at O in the case of a two-pole armature is $\frac{2\pi}{N}$. If V = continuous voltage between brushes, then, by applying the method explained in § 146 for determining the slip-ring voltage, we have, putting $d = \frac{N}{2}$, and using formula (1) of § 146—

$$\text{maximum voltage of each phase} = V \sin \frac{\pi}{N}$$

Hence if I_N = maximum current in each phase, then, assuming a power factor of unity, the power supplied to each phase of the converter is $\frac{1}{2}VI_N \sin \frac{\pi}{N}$, and the total power = number of phases $\times$ power per phase $= \frac{N}{2}VI_N \sin \frac{\pi}{N}$. Let I be the current on the continuous-current side, so that VI = output on continuous-current side. Neglecting losses in the converter, and equating the input to the output, we get $\frac{N}{2}VI_N \sin \frac{\pi}{N} = VI$, or—

$$I_N = \frac{2I}{N \sin \frac{\pi}{N}} \quad \dots \dots \dots \quad (2)$$

In Fig. 171, R and S are the points of connection of two neighbouring slip-rings, and B_1B_2 is the line of brushes, making, at time t, an angle $\omega t = \theta$ with OR. If i_N = instantaneous alternating current supplied to the phase RS (through the slip-rings at R and S), we may write—

$$i_N = I_N \sin (\theta + a)$$

where the angle a is as yet unknown. In order to determine a, we notice that the phase voltage and current reach their maxima values when T, the middle point of RS, is in line with the middle of the pole-piece, *i.e.* when the line of brushes is at right angles (as shown

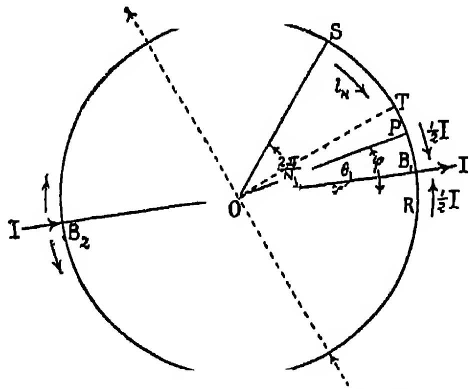


FIG. 171.—To illustrate Heating of Converter with N Slip-rings.

by dotted line) to OT. We must then have $i_N = I_N$, or sin $(\theta + a) = 1$, *i.e.* $\theta + a = \frac{\pi}{2}$; but since, referring to the figure, we at this instant have $\theta = \frac{\pi}{N} + \frac{\pi}{2}$, it follows that $a = -\frac{\pi}{N}$. We thus have—

$$i_N = I_N \sin\left(\theta - \frac{\pi}{N}\right)$$

Consider now any portion of the winding between R and S, such as that at P, and let ROP $= \phi$. Then, so long as $\theta < \phi$, the current at P is given by—

$$i_N + \tfrac{1}{2}I = \tfrac{1}{2}I + I_N \sin\left(\theta - \frac{\pi}{N}\right)$$

while for $\theta > \phi$ the current at P is—

$$i_N - \tfrac{1}{2}I = -\tfrac{1}{2}I + I_N \sin\left(\theta - \frac{\pi}{N}\right)$$

The square of the current is given by—

$$\tfrac{1}{4}I^2 + I_N{}^2 \sin^2\left(\theta - \frac{\pi}{N}\right) \pm II_N \sin\left(\theta - \frac{\pi}{N}\right)$$

or—

$$\tfrac{1}{4}I^2 + \tfrac{1}{2}I_N^2 - \tfrac{1}{2}I_N^2 \cos\left(2\theta - \frac{2\pi}{N}\right) \pm II_N \sin\left(\theta - \frac{\pi}{N}\right) \quad . \quad (3)$$

since $\sin^2\left(\theta - \frac{\pi}{N}\right) = \frac{1}{2}\left[1 - \cos 2\left(\theta - \frac{\pi}{N}\right)\right]$. The upper sign in the last term corresponds to $\theta < \phi$, and the lower sign to $\theta > \phi$.

To obtain the mean rate of heat production at P, we must find the mean value of (3) during a half-period, that is, between $\theta = 0$ and $\theta = \pi$. Now, we notice that the first two terms in (3), $\frac{1}{4}I^2 + \frac{1}{2}I_N^2$, are constant; their mean value is thus $\frac{1}{4}I^2 + \frac{1}{2}I_N^2$. The third term contains the factor $\cos\left(2\theta - \frac{2\pi}{N}\right)$, which is of frequency double that of the supply current; thus half a period of the supply current —from $\theta = 0$ to $\theta = \pi$—will correspond to a whole period of the term $\cos\left(2\theta - \frac{2\pi}{N}\right)$, and since the mean value of the cosine over a whole period vanishes, this term drops out in the mean value of i_N^2. Lastly, we have the term $\pm II_N \sin\left(\theta - \frac{\pi}{N}\right)$, the *plus* sign to be taken so long as $\theta < \phi$, and the *minus* sign when $\theta > \phi$. If we plot this term as a func ion of θ, the area enclosed by its graph is—

$$II_N\left\{\int_0^\phi \sin\left(\theta - \frac{\pi}{N}\right)d\theta - \int_\phi^\pi \sin\left(\theta - \frac{\pi}{N}\right)d\theta\right\} = -2II_N \cos\left(\phi - \frac{\pi}{N}\right)$$

and the mean ordinate of the curve, obtained by dividing the area by the base, is—

$$-\frac{2}{\pi}II_N \cos\left(\phi - \frac{\pi}{N}\right)$$

We thus obtain for the mean value of the square of the current at P—

$$\tfrac{1}{4}I^2 + \tfrac{1}{2}I_N^2 - \frac{2}{\pi}II_N \cos\left(\phi - \frac{\pi}{N}\right) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This expression shows that the heating is greatest at R and S, the points of slip-ring connection, and least at T, the point halfway between them.

If, using (2), we plot the expression (4) as a function of ϕ for $N = 2, 3, 4, 6$, and 12, the extreme values of ϕ being always 0 and $\frac{2\pi}{N}$, so as to correspond to the distance between two neighbouring slip-rings, we obtain the curves of Fig. 172. The value of I—the

continuous current—has been assumed to be the same for all the curves. The vertical scale is the same for all, but the horizontal scale is different, having been so chosen that the length of the base line in each case represents $\frac{2\pi}{N}$, or the distance between two neighbouring slip-rings. The rapid decrease in the total rate of

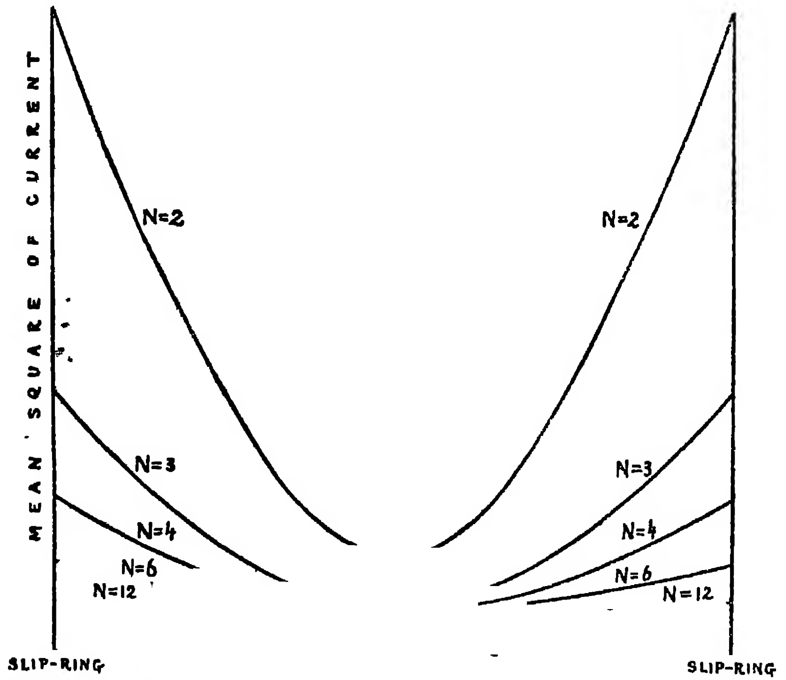


FIG. 172.—Showing Relative Rate of Heat Production in Converter with Varying Number of Slip-rings.

heat production, and the increase in the uniformity of heat generation in the various coils, as the number of slip-rings is increased, are exhibited in a striking manner by these curves.

§ 150. Effect of Number of Slip-rings on Output of Converter

In order to find the mean rate of heat generation in the various coils lying between two slip-rings, we have to find the mean ordinate of the corresponding curve of Fig. 172. A simple integration

enables us to obtain an expression for the mean value of the mean square of the current in the coils included between two slip-rings. Returning to expression (4), we notice that the first two terms are constant, while the mean value of the last term is given by—

$$-\frac{2}{\pi}\mathrm{II_N} \times \text{mean value of } \cos\left(\phi - \frac{\pi}{\mathrm{N}}\right) \text{ between } \phi = 0 \text{ and } \phi = \frac{2\pi}{\mathrm{N}}$$

$$= -\frac{2}{\pi}\mathrm{II_N} \times \frac{\mathrm{N}}{2\pi}\int_0^{\frac{2\pi}{\mathrm{N}}} \cos\left(\phi - \frac{\pi}{\mathrm{N}}\right)d\phi$$

$$= -\frac{2}{\pi^2}\mathrm{II_N N} \sin\frac{\pi}{\mathrm{N}}$$

Hence the mean value of the mean square of the current in a group of coils between two slip-rings is—

$$\tfrac{1}{4}\mathrm{I}^2 + \tfrac{1}{2}\mathrm{I_N}^2 - \frac{2}{\pi^2}\mathrm{II_N} \,.\, \mathrm{N}\sin\frac{\pi}{\mathrm{N}}$$

or, using (2)—

$$\left(\frac{1}{4} + \frac{2}{\mathrm{N}^2\sin^2\frac{\pi}{\mathrm{N}}} - \frac{4}{\pi^2}\right)\mathrm{I}^2 = \left(\frac{2}{\mathrm{N}^2\sin^2\frac{\pi}{\mathrm{N}}} - 0{\cdot}1553\right)\mathrm{I}^2$$

Hence the mean rate of heat production in the armature is the same as if each of its coils were traversed by a continuous current of amount $\sqrt{\mathrm{K}}\,.\,\mathrm{I}$, where—

$$\mathrm{K} = \frac{2}{\mathrm{N}^2\sin^2\frac{\pi}{\mathrm{N}}} - 0{\cdot}1553 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or as if the armature were generating a continuous current of amount $2\sqrt{\mathrm{K}}\,.\,\mathrm{I}$.

If we assume, as before, that the limit of output is fixed by the permissible rise of temperature, and if VI = maximum output of armature when acting as a converter, then its output as a continuous-current generator is equal to $2\sqrt{\mathrm{K}}\,.\,\mathrm{VI}$, or—

$$\text{converter output} = \frac{1}{2\sqrt{\mathrm{K}}} \times \text{continuous-current generator output} \quad (6)$$

Using (5) and (6), we get the following relative outputs, that of a continuous-current generator being taken as 100:—

N ... ... =	2	3	4	6	12
Relative output =	85·2	133	162	193	219·5

The output, it will be seen, increases with the number of slip-rings, rapidly at first, then more slowly, tending ultimately to the limit 230 as the number of slip-rings is increased indefinitely.

As a matter of fact, the output would increase even more rapidly with the number of slip-rings than would appear from the above figures. For the practical limit of output should be determined, not by the *average* temperature rise of the coils, but by the highest temperature to which any one coil is raised. Now, although, owing to the partial equalization brought about by the conduction of heat along the coils and core, and the more powerful radiation of heat from the hotter parts of the armature, the curves of temperature rise in the various cases will not be quite so steep as the curves of Fig. 172 (the end portions becoming depressed, and the middle portion raised, by the combined effect of conduction and radiation), yet—especially in the case of the 2-ring (single-phase) converter—the coils close to the slip-rings will attain a considerably higher temperature than those midway between the slip-rings. It might be thought that a simple method of surmounting this difficulty of excessive local temperature rise would be to increase the cross-section of the conductors close to the slip-rings; but this would introduce a fresh difficulty—excessive sparking in certain positions of the armature, due to the destruction of the symmetry of the winding.

Single-phase converters are of no practical interest; on account of their relatively small output, the enormous concentration of the heating on the coils close to the slip-rings, and the further difficulty due to their tendency to spark by reason of the fluctuating armature reaction, they are never employed in practice, motor-generators being invariably used instead. Most of the large power transmission plants making use of three-phase transmission lines, the favourite type of converter is the six-phase converter, which, as will be explained in § 151, may be connected to the secondaries of transformers whose primaries are across three-phase mains. As compared with the three-phase converter, the six-phase converter suffers from the disadvantage of requiring six instead of three slip-rings. This disadvantage is, however, more than compensated for by the more favourable conditions regarding heating and hence output.

§ 151. Six-phase Converter supplied from Three-phase Mains

Let the circle in Fig. 173 (*a*) represent diagrammatically the winding of a 2-pole armature, fitted with six slip-rings connected to the points A, B, C, A′, B′, and C′ of the winding. Remembering the rule (§ 146) for finding the magnitude and phase of the e.m.f. in any section of the winding, we see that the e.m.f.s between the slip-rings A, B, C form a three-phase system, as also do those between the slip-rings A′, B′, and C′. We thus get a double Δ of voltages· and since AB is in phase with B′A′, BC in phase with C′B′, and CA in phase with A′C′, it is evident that both Δs may be obtained from the same set of three-phase transformers. For this purpose, the

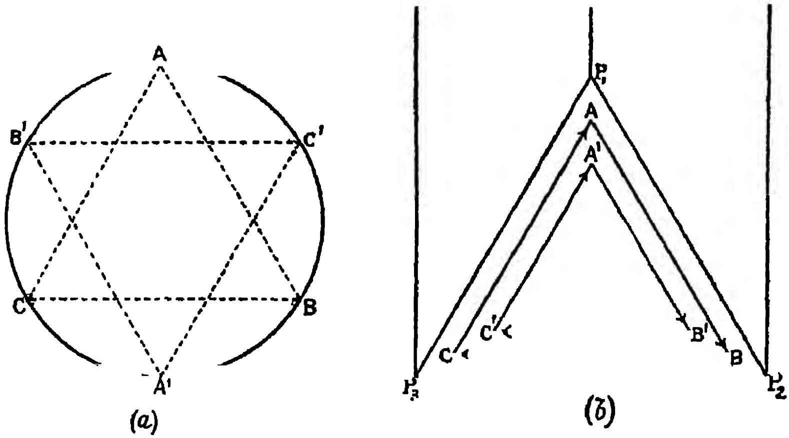


FIG. 173.—Connections of Six-phase Converter.

transformers are wound as shown in Fig. 173 (*b*), a mesh connection for both primaries and secondaries being used by preference.* P_1P_2, P_2P_3 and P_3P_1 are the high-voltage primaries. Each transformer is provided with two equal secondaries, which are connected to form the Δs, ABC, and A′B′C′. The junctions A, B, C; A′, B′, and C′ of the two sets of secondary windings are connected to the points A, B, C; A′, B′, C′ of the armature winding respectively.

Instead of using a Δ-connection of the double secondaries, a Y-connection is occasionally employed. The two methods are sometimes described as the *double-delta* and *double*-Y methods of

* With a mesh connection of transformers, continuity of supply will be maintained even if one of the transformers should break down (§ 64).

connection. The favourite modern method of connecting a six-phase converter to three-phase mains is, however, that known as the *diametral method* of connection. This involves the use of only one set of secondaries. The secondaries are not directly interconnected, but the ends of each are connected to two points in the converter armature spaced a pole-pitch (180 electrical degrees) apart, and the three sets of points are separated by intervals corresponding to two-thirds of the pole-pitch (120 electrical degrees). Thus, referring to the diagram of Fig. 173 (*a*), the three secondaries would be connected to the three pairs of points marked AA′, BB′, and CC′.

§ 152. Hunting of Rotary Converters

The diminished heating of the armature of a converter as compared with that of a continuous-current generator supplying an equal current is due to the partial neutralization of the continuous by the alternating current in the armature windings, and a further result of this neutralization is reduced field distortion. For this reason, a converter is less liable to spark, and is capable of standing a much heavier momentary overload without sparking than a continuous-current generator, *provided no hunting takes place.* The hunting or phase-swinging trouble has in some cases assumed a very acute form, and has at times rendered the running of converters impossible. The polyphase currents circulating in the armature give rise to a rotating field, which, owing to the synchronous rotation of the armature, is stationary in space, *i.e.* stationary with respect to the field-magnets. If hunting takes place, however, this field begins to oscillate, and its oscillations may cause vicious sparking at the brushes. A converter which is likely to hunt is thus peculiarly liable to the sparking trouble.

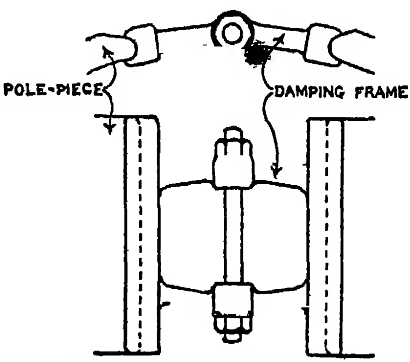


FIG. 174.—Damping Device for Rotary Converter.

Difficulties due to hunting have been experienced mainly in cases where the angular velocity of the generators was not sufficiently uniform, or where the converters were supplied through a very long

line of comparatively high resistance. Hunting may be to a large extent prevented by the use of copper damping bridges placed so as to connect neighbouring poles (the equivalent of a squirrel-cage being thereby obtained), the poles themselves being of solid metal, not laminated. Any oscillation of the field is rapidly damped out by this equivalent of a squirrel-cage winding. Such a damping arrangement is shown in Fig. 174.

Practical experience has shown that where a number of converters have to be run in parallel, it is not advisable to run them from a common bank of transformers; each rotary should be supplied from a separate transformer or bank of transformers.

§ 153. Voltage Regulation of Rotary Converters

A very important problem in connection with converters is that of voltage regulation. The voltage ratio is approximately constant, and is but little affected by altering the exciting current. Hence the voltage of a rotary converter of ordinary construction cannot be controlled (except within extremely narrow limits) by the method commonly in use with continuous-current generators —that of varying the field excitation. A marked change in the voltage on the continuous-current side of such a converter can only be obtained by changing the alternating voltage. The two methods commonly in use in practice for effecting this change are (1) the method involving the use of an *induction regulator*, and (2) the method depending on the use of choking coils and a compound winding on the converter field.

The induction regulator method is mostly in use where the converter supplies a lighting load, in which case very perfect voltage control is desirable. An induction regulator is essentially a polyphase transformer with movable primary. In construction, it is identical with a polyphase induction motor whose wound rotor represents the primary, the stator carrying the secondary winding. The rotor is normally held fast, and can only be slowly rotated in either direction by means of worm gearing driven by a small auxiliary induction motor mounted on the top of the regulator case. In one of the extreme positions of the primary, the rotating magnetic fields due to primary and secondary are in exact coincidence of phase, while in the other extreme position they are in direct opposition of phase.

The diagram of connections corresponding to one phase is given

in Fig. 175. The primary of the induction regulator is connected across the corresponding phase of the converter armature, while the secondary is in the main circuit. Let us suppose that with a certain current passing into the converter the primary of the induction regulator is rotated in such a direction as to increase the separation of phase between the primary and secondary hypothetical fields. Now, so long as the secondary current remains unaltered, and the voltage across the primary is maintained approximately constant, it is evident that both the secondary hypothetical field and the resultant or actually existing field must remain approximately constant. Hence, if the separation of the phases of the two hypothetical fields is increased, the primary field must necessarily increase in order to maintain the resultant constant, and as a consequence the resultant will swing round so as to be nearer (as regards phase) the more powerful primary component. The e.m.f. induced in the secondary, whose magnitude and phase depend on those of the resultant field, will thus remain approximately unaltered in magnitude, but will be brought more nearly into coincidence of phase with the secondary voltage of the step-down transformer, so that their vectorial resultant will be increased. The two extreme positions of the induction regulator are those in which the regulator secondary voltage is arithmetically subtracted from, and added to, the voltage of the step-down transformer.*

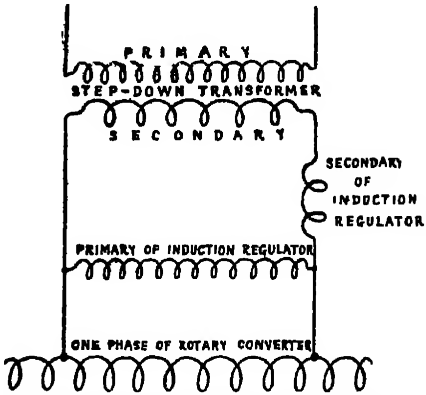


Fig. 175.—Connections of Induction Regulator.

In the second method of regulation, the field of the converter is compound wound, so that with increasing load the field is automatically strengthened. Now, a mere strengthening of the field would, as already pointed out, produce no appreciable rise of voltage on the continuous-current side unless there is a rise on the alternating side. In order to effect this latter rise, suitable choking or reactance coils are inserted in series with each phase of the converter. Let us assume that the voltage at which the step-down transformer is

* The action of the induction regulator may be rendered automatic by the use of a relay which controls the auxiliary motor by means of which the displacement of the movable member of the regulator is effected. The range of voltage variation obtainable with this method is about 30 per cent.

supplied is constant, and let its secondary voltage, also approximately constant, be denoted by V. The excitation due to the shunt is such that at light loads the converter takes a lagging current, represented by I_a in Fig. 176 (*a*). In order to obtain the slip-ring voltage V_s, we have to subtract (vectorially) from V the drop due to the reactance coil—VV_s in Fig. 176 (*a*).* As the load increases, the field is

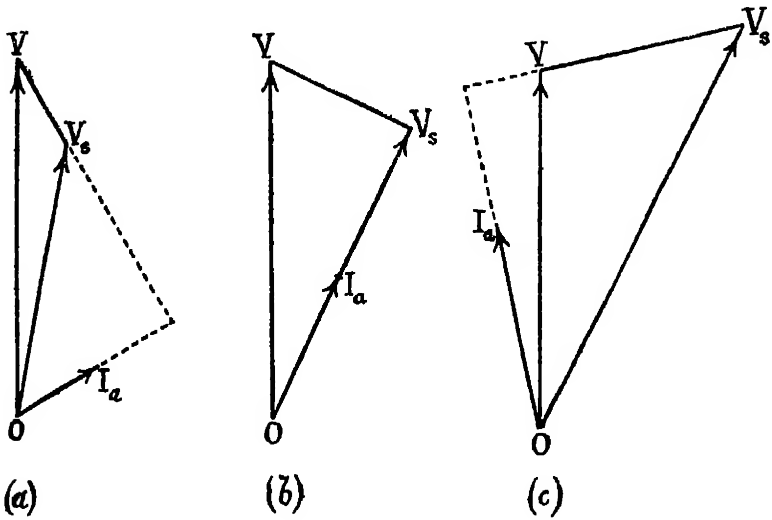


Fig. 176.—Automatic Regulation of Converter.

strengthened, and the current comes more nearly into phase with V_s (§ 82), so that when, as in Fig. 176 (*b*), coincidence of phase is reached, V_s may, in spite of the increase in VV_s, remain the same or even increase in value. This constancy or increase is maintained when, as in Fig. 176 (*c*), the current has become a leading one. By this method, an amount of over-compounding up to 15 per cent. may be obtained.

§ 154. Converters with Variable Voltage Ratio

The methods just considered of varying the ratio of the continuous to the alternating p d. involve the use of special apparatus outside the converter. There are, however, two types of *self-contained* converters of special design which allow of a fairly wide control of the

* VV_s being, of course, perpendicular to the current vector

voltage ratio. These two types are the *synchronous booster* converter and the *split-pole* converter.

The *booster converter* consists of an ordinary converter whose shaft carries the rotor of an alternator whose armature windings are connected in series with those of the converter. By altering the excitation of this auxiliary generator or booster, the voltage on the continuous-current side of the converter may be easily controlled. As the booster is required to furnish an e.m.f. of the same frequency as that supplied to the converter, its field must have a number of poles equal to that of the converter. In order to obtain a compact form of construction the rotor of the booster generally forms its armature, and is mounted between the converter armature and the slip-rings, no additional bearings being provided. The field of the booster in some cases rests on the bedplate, in others is supported from the converter yoke-ring by a strong end-shield. Booster converters of large output (1000 kw.) have been built to give a change in the voltage ratio up to about 30 per cent.*

The action of the *split-pole converter* is based on the fact—explained in § 50—that the total flux per pole may be varied without altering the r.m.s. value of the slip-ring voltage, the latter depending on the flux distribution in the gap as well as on the total flux, whereas the brush voltage depends only on the total flux, and not on the way in which the flux is distributed. Hence if, by suitable means, we alter the flux distribution in such a manner as to keep the slip-ring voltage constant while the total flux (which determines the brush voltage) is varied, we obtain a variable ratio of the alternating to the continuous voltage. In practice, the total flux and its distribution are altered by having a deep radial groove in each pole which divides it into two parts, one of which is of much larger size, and forms the main pole, while the other forms the regulating pole. Each part of the split pole is provided with an exciting coil, the main pole being, in the simplest type of this kind of converter, excited with a constant current, while the excitation of the regulating pole is varied. For normal brush voltage, the regulating pole is not excited; for a lower brush voltage, it is excited in a direction opposed to that of the main pole; and for a higher, in the same direction as the main pole. This method has so far been only used in the case of 25-cycle converters, and is suitable for a voltage range of some 25 per cent.

§ 155. Starting of Rotary Converters

There are various methods of starting rotary converters. Where possible they are run up to synchronous speed from the continuous-

* *The Electric Journal*, vol. v. p. 616 (1908).

current side, synchronized, and switched on to the alternating-current mains like ordinary alternators. But in some cases a supply of continuous current may not be available. The best method is, then, to provide a small auxiliary induction motor coupled direct to the converter shaft, and having a somewhat smaller number of poles, so as to enable the converter to be run up to a speed slightly above that of synchronism. The induction motor is then switched off, and as the speed of the converter slowly diminishes it is synchronized and thrown into circuit. A third method, which does away with the auxiliary starting motor, consists in opening the field circuit—preferably in several places—and connecting the armature, through suitable resistances or reactance coils, to the alternating current bus bars. By the action of eddy currents and hysteresis, the armature will run up to practically synchronous speed. Since, however, it may have run up with the wrong polarity on the continuous-current side, a double-throw reversing switch is provided in the exciting circuit, by means of which the field magnetism may be momentarily wiped out, and the armature made to "slip a pole," thus coming up with the right polarity. The field switch is then thrown into its normal position.*

§ 156. Racing of Inverted Rotaries

When a converter is used as an "inverted rotary," to transform continuous into alternating currents, difficulties may arise on account of the excessive weakening of the field by lagging currents on the alternating side (which is now the generator side), and the dangerously high speed to which the converter armature may run up. In order to prevent this "running away" of an inverted rotary, it is provided with a separate exciter coupled direct to the converter shaft, and designed so that under normal running conditions its field is comparatively weak, and hence very sensitive to an increase in the exciting current. Any tendency on the part of the converter armature to race produces a very rapid increase in the exciting current, which immediately checks any further increase of speed. This arrangement is used by the Westinghouse Co.

§ 157. Converters *v.* Motor-generators

The relative merits and demerits of rotary converters as compared with motor-generator sets—that is, alternating-current motors coupled to continuous-current generators—have formed the subject of much

* See Appendix XII.

controversy. Considered by itself, a rotary converter is considerably cheaper and more efficient than a motor generator. But while the latter is self-contained and requires no auxiliary apparatus (such as step-down transformers, for the alternating-current motor may be wound for the high voltage of the line—unless this is exceptionally high), the rotary converter requires various accessories, which have the double effect of increasing the total cost of the transforming plant and reducing its efficiency. When the additional cost of the step-down transformers, the induction regulator, or the reactance coils and compound winding, or other devices required for voltage control on the continuous-current side, is taken into account, the difference in first cost between a rotary converter and a motor-generator plant is very slight—not exceeding some 5 per cent.—the rotary converter plant being the cheaper of the two. Similarly, as regards efficiency there appears to be but little to choose between them, the rotary converter plant being again slightly more efficient. On the other hand, the motor-generator plant allows of a simpler mode of voltage control over a wider range.

Two types of motor-generators are in use, depending on the type of motor—the synchronous and the asynchronous or induction motor type. For capacities up to 300 k.w., the induction motor-generator set would appear to be slightly cheaper, while for outputs exceeding 500 k.w., it would appear to be slightly more costly than the synchronous set. The induction set is self-starting; on the other hand, the losses with the synchronous set may be made somewhat smaller by reason of the higher power factor.

§ 158. Motor Converters or Cascade Converters

An interesting type of converting machine, known as the *motor converter* or *cascade converter*, and partaking partly of the characteristics of a motor-generator, partly of those of a converter, was patented in 1902 by J. L. la Cour and O. S. Bragstad.

The motor converter consists essentially of an induction motor coupled, mechanically and electrically, to a rotary converter. The arrangement of connections is shown in Fig. 177. In the simplest case, the induction motor has the same number of poles as the converter. The alternating current to be converted is supplied to the stator of the induction motor, whose rotor windings are on one side connected to a starting resistance (through slip-rings), and on the other to the armature of the converter. The machine set is started from the induction motor side, by connecting the stator to the alternating current mains, and gradually cutting out the starting

resistances. At the moment of starting, the frequency of the e.m.f.s in the rotor of the induction motor is equal to the frequency of the supply, and there are no e.m.f.s induced (by rotation) in the armature winding. As the speed rises, the frequency of the rotor e.m.f.s steadily decreases, while the frequency of the e.m.f.s induced in the armature winding steadily increases. Each set of e.m.f.s gives rise to a corresponding set of currents, and these become superposed. When the rotor has nearly run up to half its synchronous speed, the frequencies of the rotor and armature e.m.f.s become nearly equal, and as the corresponding currents circulate in the same circuits, the phenomenon of "beats" or slow alternate increase and decrease in

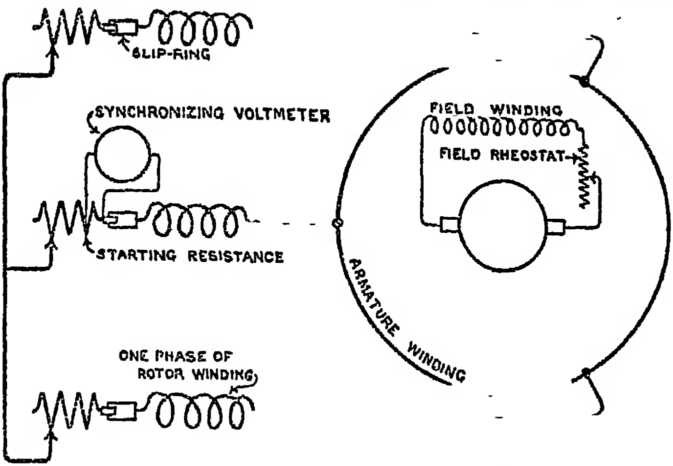


FIG. 177.—Connections of Motor Converter.

the r.m.s. value of the current becomes apparent. This is indicated by a low-reading voltmeter connected across the last section of one phase of the starting resistance. The beats gradually become slower, and finally disappear altogether when the armature is pulled into synchronism by the field. The slip-rings are then short-circuited, and the set will go on running at half the speed of synchronism, and is ready to supply a continuous current.

When running up to speed, a suitable amount of resistance must be introduced into the field circuit of the converter, as otherwise the currents due to the e.m.f.s induced in its armature would be excessive.

The action of the motor converter when loaded is briefly as follows:—Since the rotor is running at half synchronous speed, only half the power transmitted by the stator to the rotor is transformed

into mechanical power, and this is transmitted by the torsion of the shaft to the armature, which by acting as a generator develops an equivalent amount of power in the shape of continuous current. The remaining half of the power transmitted by the stator to the rotor is transformed into electrical power, as in an ordinary transformer, with the exception that owing to the rotation of the rotor the frequency is halved. This electrical power is passed into the armature, which acts as a converter and transforms it into the continuous-current form.

Thus the induction motor part of the motor converter acts partly as an induction motor driving the armature, partly as a frequency and — in general — voltage transformer. The continuous-current machine acts partly as a generator, partly as a converter.

In practice, instead of a three-phase winding on the rotor as shown in Fig. 177, a twelve-phase winding is employed, connected to twelve points in the armature winding, so that the converter is twelve-phase, and an increased output is thereby obtained (§ 150). In order, however, to do away with the cost of 12 slip-rings and a twelve-phase resistance for starting purposes, only 3 phases are used at starting, the remaining 9 being open-circuited at one end during the starting period, and only connected to a common neutral point when the last section of the starting resistance is cut out after synchronization.

Among the advantages claimed for the motor converter as compared with the motor-generator may be mentioned lower first cost, smaller amount of floor space and somewhat higher efficiency. As compared with the rotary converter, the motor converter allows of a greater range (up to 30 per cent.) of voltage regulation without the use of a booster or other auxiliary appliances.*

* For further details regarding motor converters, the reader may consult a paper by Mr. H. S. Hallo in the *Journal of the Institution of Electrical Engineers*, vol. 43, p. 374 (1909).

CHAPTER XVI

§ 159. Polyphase commutator motors—§ 160. Representation of rotating waves—§ 161. Approximate theory of series polyphase commutator motor—§ 162. Shunt polyphase commutator motor—§ 163. Commutator motor used as plain induction motor—§ 164. Effect of brush displacement when a p.d. is impressed on brushes—§ 165 Effect of impressing on brushes a p d. which is in phase with the rotor current—§ 166. Effect of impressing on the brushes a p.d. which is in quadrature with the rotor current—§ 167. Methods of controlling speed and power factor of shunt polyphase commutator motor—§ 168. Cascade connection of induction and commutator motor—§ 169. Phase advancers.

§ 159. Polyphase Commutator Motors

THE two main disadvantages of the simple type of induction motor may be said to be its low power factor at light loads and the absence of a simple and efficient method of speed regulation. Again, cases may arise where a motor is required to possess a *series characteristic*—*i.e.* where its speed is required to decrease *automatically* with increase of load—and not a *shunt characteristic* like that of the induction motor, whose speed changes but little with fluctuations in the load.

The requirements of a high power factor, economical speed regulation, and either a shunt or a series speed characteristic may be met by the adoption of a type of motor whose rotor in every respect resembles a continuous current armature and therefore has a commutator. Such motors are accordingly termed *polyphase commutator motors.*

The use of a continuous-current armature for the rotor of an alternating current motor is an old device, having been patented as far back as 1888 by E. Wilson in England, and in 1891 by Görges in Germany. Although Görges gave a sketch of the theory of such motors in 1891, there is no doubt that the possibilities presented by the use of a commutator rotor were by no means fully realized at that time, and it was only by the later labours of Heyland and Latour in 1900 that this type of motor was once more brought into prominence. The name given by these inventors to the polyphase commutator motor was that of *compensated induction motor.*

Motors of this type would be much more generally used were it not for the sparking troubles which are inseparable from the use of

a commutator. The output of such motors is restricted to some 100 h.p., although in special cases outputs up to 300 h.p. might be obtained. The motors work best on circuits of low frequency, and for this reason are suitable for use in the rotor circuits of ordinary induction motors whose speed it is desired to control economically (§ 139).

When a commutator rotor is used in a three-phase motor, it is provided with three brush sets spaced 120 electrical degrees apart. The continuous current winding then forms a delta-connected three-phase winding.

It is important to bear in mind the following points regarding the behaviour of a commutator rotor when supplied with three-phase currents.

(1) The speed of rotation *in space* of the rotating field due to the three-phase rotor currents is *always* that of synchronism, and is entirely independent of the speed of the rotor.

(2) The frequency of the e.m.f.s and currents in the rotor windings is determined by the *relative* speed of the rotating field and the rotor, and is thus always equal to the *slip* frequency. Hence the rotor acts as a *frequency changer*, transforming the frequency of the supply to slip frequency.

§ 160. Representation of Rotating Waves

In considering the approximate theory of polyphase commutator motors, it is convenient to make certain simplifying assumptions. We shall suppose the reluctance of the iron part of the magnetic circuit and the core losses to be negligible, so that the flux is proportional to and in time phase with the ampere-turns producing it. We shall suppose the stator and rotor cores to have continuous smooth surfaces at the air-gap, so that the reluctance of the latter is uniform along the entire circumference. Lastly, we shall suppose that the windings of each phase of both stator and rotor are distributed continuously according to the simple sine law, so that the number of conductors per unit length of air-gap circumference is given by the ordinate of a sine curve. If an alternating current be sent through any phase, the ampere-conductors per unit length of circumference will be represented by an oscillating sine curve (§ 20). The magnetic induction in the gap will also be represented by an oscillating sine curve, but this curve will be displaced by 90° relatively to the ampere-conductor or ampere-turn curve producing it. Hence we may say that the ampere-turn curve is in *space quadrature* relatively to the induction curve, although the ampere-turns and the induction are *co-phasal in time*.

Let us now suppose that polyphase currents are sent through the windings of, say, the stator. The oscillating waves of ampere-conductors or ampere-turns give rise (§ 22) to a rotating or travelling ampere-turn wave of constant amplitude, and similarly the oscillating waves of magnetic induction become fused into a pure rotating wave. It is convenient to adopt a simple graphical method of representing such waves. Let (Fig. 178) an e.m.f. or current be considered to have a positive value if directed away from the observer; and let the induction or field be considered positive if directed outwards. Then if we take distances along the air-gap circumference as abscissae, and ampere-turns per unit length and values of the induction as ordinates, we shall obtain the two sine waves shown in Fig. 179. It will be noticed that the induction wave is displaced through 90° to the left of the ampere-turn wave.

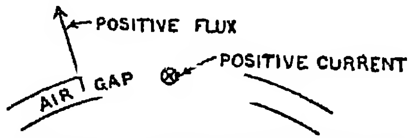


Fig. 178.—Showing Directions assumed as positive for e.m.f., Current and Flux.

But there is a much simpler mode of representing these waves. Since a distance along the air-gap circumference equal to twice the pole-pitch represents 360 electrical degrees, any distance of one point in the air-gap from another may be expressed as so many *electrical degrees.* Hence a travelling or rotating sine wave may be conventionally represented by a rotating vector whose length corresponds to the amplitude of the sine wave, and whose angular distance from a fixed line of reference represents the distance, in electrical degrees, of the maximum ordinate of the sine wave from a fixed point of reference on the air-gap circumference. Hence the ampere-turn and

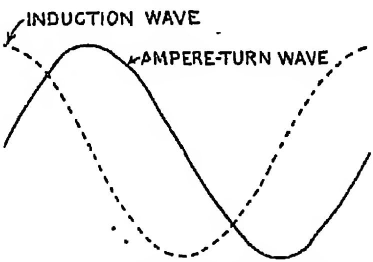


Fig. 179.—Ampere-turn and Induction Waves.

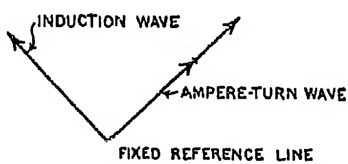


Fig. 180.—Representation of Rotating Sine Waves by means of Vectors.

induction waves of Fig. 179 may be represented by rotating vectors as shown in Fig. 180. Such a vector diagram may be termed a *space diagram,* as it represents the magnitudes and relative *positions in space* of two rotating sine waves. It must be carefully distinguished

from an ordinary *time* diagram, in which the *projections* of rotating vectors on a fixed straight line represent instantaneous values of quantities which vary *in time* according to the sine law.

If we now consider the winding of any one phase, then, in accordance with the result established in § 22, p. 47, the oscillating ampere-turn wave due to that phase will reach its maximum amplitude at the instant when the rotating ampere-turn wave is coincident with it in position. Again, it is easy to see that the e.m.f. induced in any one phase reaches its maximum value when the rotating induction wave becomes coincident in position with the oscillating ampere-turn wave of that phase. Now, since the rotating induction wave is displaced by 90 electrical degrees *in space* relatively to the rotating ampere-turn wave (Fig. 180), the e.m.f. induced by the rotating induction wave in the winding of any one phase will be in *time-quadrature* with the ampere-turns or current in that phase, and will lag behind it.

§ 161. Approximate Theory of Series Polyphase Commutator Motor

We are now in a position to consider the approximate theory of the *series* polyphase commutator motor * represented in Fig. 181. The mesh-connected armature (or rotor) windings may be supposed to be replaced by an equivalent star winding (number of turns in each ray of the star $= \frac{1}{\sqrt{3}}$ that in each side of the delta). Then each stator phase is in series with a rotor phase, and the stator and rotor currents are necessarily cophasal. Both stator and rotor windings give rise to rotating waves of ampere-turns and induction. Let us take as the *standard position* of the brushes that position for which the rotating flux or induction waves of stator and rotor are coincident in position, so that the stator and rotor ampere-turns are added arithmetically at every instant. There being no displacement of the stator and rotor fields, no torque is exerted on the armature, and the machine will remain at rest, acting as a simple choking coil.

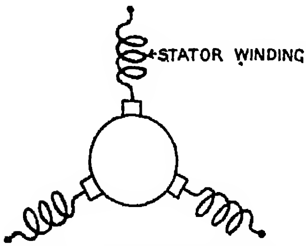


FIG. 181.—Diagram of Polyphase Series Motor.

In order to enable the machine to exert *torque*, the rotor field must be displaced relatively to the stator field, and this can only be

* R. Rüdenberg, *Elektrotechnische Zeitschrift*, vol. xxxiv. p. 807 (1913).

done by displacing the brushes from the standard position. The direction of the torque will depend on the direction of brush displacement. Let us suppose that the brushes are displaced *backwards*, or against the direction of rotation of the field. Then the torque will be a *forward* one, the motor tending to run in the direction of rotation of the field. If θ be the angle of displacement, and A_s, A_r the stator and rotor ampere-turns respectively, then the torque is proportional to $A_sA_r \sin\theta$; or, if $m = A_r/A_s$, torque $\propto mA_s^2 \sin\theta$.*

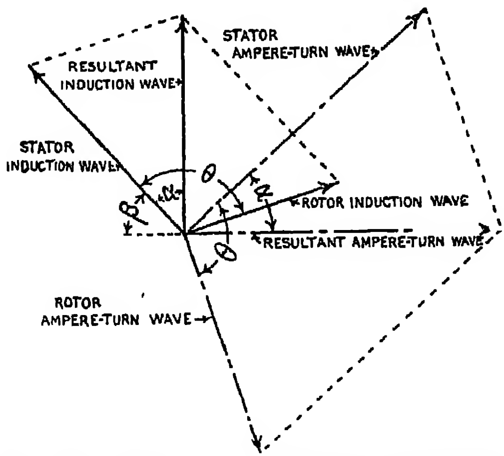


FIG. 182.—Space Diagram of Stator and Rotor Waves.

The stator and rotor currents being in time-phase with each other, and the brushes having been displaced through an angle θ from the standard position, the stator and rotor ampere-turn and induction waves are also displaced through the same angle relatively to each other. These waves are shown in the space diagram of Fig. 182. It will be noticed that the resultant induction wave is

* The resultant ampere-turns, which are proportional to the resultant induction are given by $\sqrt{A_r^2 + A_s^2 + 2A_rA_s \cos\theta} = A_s\sqrt{m^2 + 1 + 2m\cos\theta}$. For a given current—*i.e.* a given value of A_s, the torque is a maximum when $\theta = \frac{\pi}{2}$. Again, for a given value of A_s, the ratio $\frac{\text{torque}}{\text{resultant induction}}$, which is proportional to $\frac{m\sin\theta}{\sqrt{m^2+1+2m\cos\theta}}$, has, for a given value of m, a maximum value when $\cos\theta = -m$. We then have—

$$\text{torque} \propto mA_s^2 \sin\theta$$
$$\propto A_s^2 m\sqrt{1-m^2}$$

which is a maximum when $m = \frac{1}{\sqrt{2}}$, $\cos\theta$ being then equal to $-\frac{1}{\sqrt{2}}$, and $\theta = \frac{3}{4}\pi$ or 135°.

ahead of the stator ampere-turn wave by the angle $\frac{\pi}{2}-a$.* Hence the time-interval which elapses between the coincidence of the resultant induction wave with the oscillating ampere-turn wave of one phase of the stator and the coincidence of the stator ampere-turn wave with the same oscillating stator wave corresponds to a time-lag $\frac{\pi}{2}+a$ of the e.m.f. induced in any stator phase behind the current in the same phase (Fig. 183; this is due to the fact that when the

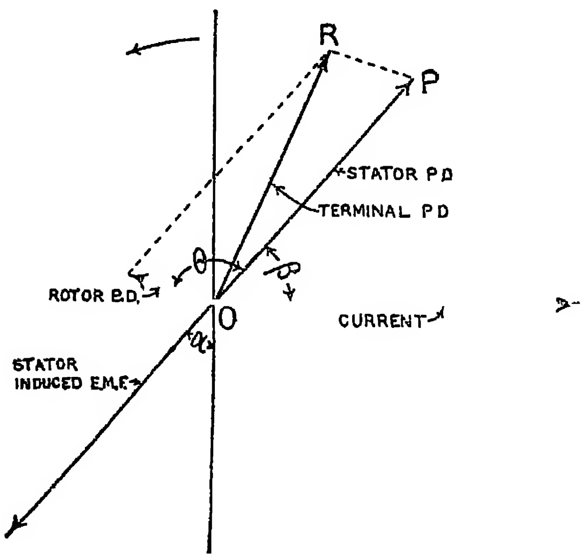


FIG. 183.—Time Diagram of P.D.s and Current.

induction at any stator conductor is at its maximum *positive* value, the e.m.f. induced in that conductor is at its *negative* maximum).

In addition to the assumptions already made regarding our ideal polyphase commutator motor, we shall now make the further assumptions of negligible magnetic leakage and negligible resistance of windings. Then the stator p.d. must be equal and in direct phase opposition to the induced e.m.f. Hence, as will be seen by reference to Fig. 183, the stator current lags behind the stator p.d. by an

* If A_s = stator ampere-turns, and A_r = rotor ampere-turns, then the resultant ampere-turns A are given by $\sqrt{A_s^2+A_r^2+2A_sA_r\cos\theta}$, and $\sin a=\frac{A_r}{A_s}\sin\theta$; or, putting $m=A_r/A_s$, $\sin a=\frac{m\sin\theta}{\sqrt{m^2+1+2m\cos\theta}}$.

angle $\beta = \frac{\pi}{2} - \alpha$, so that the power factor of the stator is $\cos \beta = \sin \alpha$, and remains constant as long as θ is constant—*i.e.* so long as the brushes remain in a fixed position. Again, since the stator induced e.m.f. is proportional to the resultant induction, and since this latter is, for a fixed value of θ, proportional to the current, we see that the stator p.d., which is numerically equal to the e.m.f., is proportional to the current. Hence the stator windings behave as if they had a constant impedance of power factor $\sin \alpha$; this impedance being a function of θ. It will be noticed (Fig. 182) that the stator power-factor steadily improves with increasing θ.

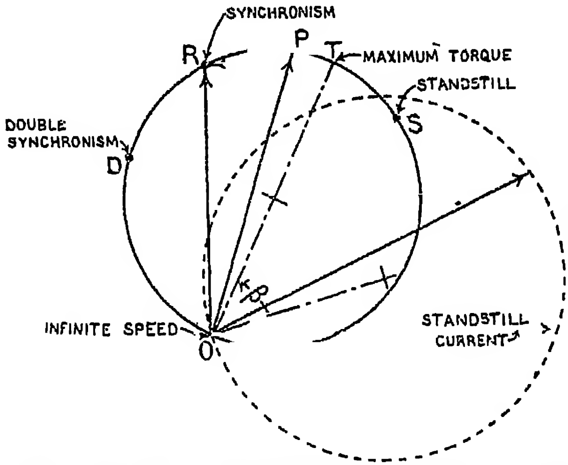


Fig. 184.—Circle Diagram of Series Polyphase Commutator Motor.

The rotor windings being displaced *backwards* in space relatively to the stator windings, the rotor induced e.m.f. will be ahead of the stator e.m.f. in time-phase by an angle θ, and the same applies to the p.d.s. Hence the vector time-diagram of p.d.s and current will be as shown in Fig. 183.

It is important to observe that all the conclusions we have so far arrived at are quite independent of the *rotation of the armature.* We have now to consider the effect which arises from such rotation. Since we have assumed the brushes to be displaced *backwards,* the torque and direction of rotation will be *forwards*, so that the relative speed of the rotating field and the armature windings will decrease with increasing speed of rotation until synchronism is reached; beyond this point the relative speed will increase, but the conductors will now be cut by the flux in the opposite direction. Thus the magnitude of the e.m.f. induced in the rotor windings will at first

decrease and reach a zero value at synchronism (when there is no relative motion of the field and conductors, both having the same velocity in space), and will then begin increasing after undergoing phase reversal.

If we now suppose the terminal p.d. OR in Fig. 183 to be kept constant, then since the angle OPR $= \pi - \theta$ is also constant so long as the brushes remain in a fixed position, it follows that with changing load or current the point P moves along a circle of which OR is a chord. This circle is shown in Fig. 184 as a full-line circle. When the motor is at rest, we have $\frac{\text{stator p.d.}}{\text{rotor p.d.}} = \frac{A_s}{A_r} = \frac{1}{m}$, and the point S on the circle for which this condition is satisfied corresponds to standstill. Since with increasing speed the rotor p.d. at first steadily decreases, it follows that we have to proceed along the circle in a counter-clockwise direction from S. The rotor p.d. vanishes and undergoes phase reversal at the point R, which corresponds to synchronous speed. Beyond R the rotor p.d. increases and the stator p.d. decreases, and at the point D, which is such that OD/DR = OS/SR,* the speed is twice that of synchronism. The point O, at which the whole of the p.d. is concentrated on the rotor, corresponds to infinite speed.

The remaining arc of the circle, which is traversed by proceeding from S to O in a clockwise direction, corresponds to rotation of the armature in a direction opposed to that of the rotating field. Since the torque now opposes the rotation, it is obvious that this region corresponds to a *generator action* of the machine.

We have already seen that the current is proportional to the stator p.d., and lags behind it by the constant angle β (where $\cos \beta = \sin \alpha = m \sin \theta / \sqrt{m^2 + 1 + 2m \cos \theta}$). Since the extremity of the stator p.d. vector traces out a circle, that of the current vector also traces out a circle, and the angle between the diameters of the two circles is β. In Fig. 184, the current circle is shown dotted. At standstill the motor takes no power (since we have assumed the losses to be negligible), and the current is in quadrature with the p.d.

A number of interesting results may at once be deduced from the circle diagram. Thus, we may determine the shape of the torque-speed curve. The torque is proportional to the square of the current. Since we have rotor p.d. $\propto$ flux $\times$ slip, it follows that slip $\propto \frac{\text{rotor p.d.}}{\text{flux}}$, or $\propto \frac{\text{rotor p.d.}}{\text{current}}$, and speed $\propto 1 - s$, where s is the slip $\left(s = \frac{\text{speed of synchronism} - \text{rotor speed}}{\text{speed of synchronism}}\right)$. This enables us to plot the relation connecting torque and speed.

* The straight lines corresponding to OD, DR, OS and SR are not shown in Fig. 185.

An examination of Fig. 184 shows that up to the speed of synchronism the stator absorbs power from the mains, while the rotor returns power to them. This means that the amount of power absorbed by the stator is in excess of that represented by the brake-power of the motor, and the balance of power is returned by the rotor to the mains. Beyond synchronism, both stator and rotor absorb power.

In the special case to which Fig. 184 refers, the torque (which is proportional to the square of the current, and hence to the square of the *stator* p.d.) increases with increasing speed between S and T, and the arc ST corresponds to a region of *instability*. Stable running can only be obtained beyond the point T. But by suitably choosing m ($= A_r/A_s$), stability may be secured at all speeds when the machine is running as a motor.

From Fig. 184 it can be easily seen, even without plotting the torque-speed curve, that over the region of stability the general shape of this curve is similar to that for an ordinary continuous current series motor. A series polyphase commutator motor is thus essentially a variable-speed motor, and may be said to possess a series characteristic. It is thus well adapted for use in all cases where an automatic decrease of speed with increase of load is required.

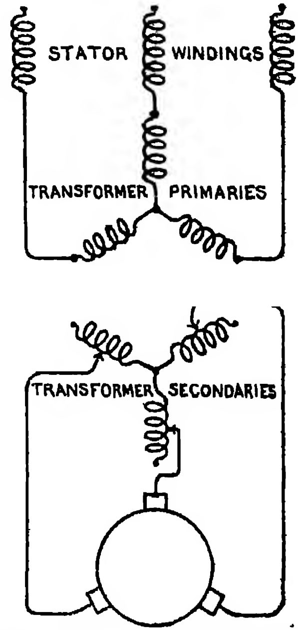


FIG. 185.—Connections of Series Polyphase Commutator Motor with Speed Regulating Transformer.

Speed regulation is easily effected by varying m. The simplest method of doing so is to use a variable-ratio current transformer, as shown in Fig. 185. It is also possible to regulate the speed by brush displacement.

Commutation troubles are serious with this type of motor. The most favourable conditions as regards commutation occur at synchronism. In order to facilitate sparkless running, the armature winding is subdivided as much as possible, and the brushes are narrow, covering only one, or at most two segments.

Motors of this type are made up to about 100 h.p., and are used for driving mine fans, pumps, etc.

§ 162. Shunt Polyphase Commutator Motor

In the shunt polyphase commutator motor, the stator and rotor windings are arranged to form two parallel circuits across the supply mains. A motor of this type possesses, as will be seen presently, a *shunt characteristic*—*i.e.* its speed will remain approximately constant at all loads. If the motor were required to run at one speed only, the use of a commutator motor would hardly be justifiable, as it would be inferior in nearly every respect to an ordinary induction motor. Its use is justified only in cases where economical *speed control* is essential, and such control is obtained by varying the p.d. impressed on the brushes. Hence in this type the rotor is connected to the mains through either an ordinary transformer or an auto-transformer with a variable number of turns, the stator winding itself being at times utilized as an auto-transformer winding and provided with a suitable number of taps.

Imagine the stator and rotor windings traversed by currents which are in phase with each other (as would be the case if the stator windings were connected in series with the rotor windings—as in the series motor), and let the brushes be adjusted so as to bring about coincidence of the axes of the magnetic fields due to the stator and rotor windings. We shall refer to this position as the *standard* or *neutral* brush position.

On rocking the brushes through 180 electrical degrees from the neutral position, we obtain (still assuming the stator and rotor currents to be in phase with each other) direct opposition of the stator and rotor fields. This second brush position we shall term the *short-circuit* position.

§ 163. Commutator Motor used as Plain Induction Motor

The action of the motor will be followed most readily if we first consider its behaviour when the brushes are simply short-circuited by connecting them to a common or neutral point instead of having them across the mains. Suppose the brushes to be in the *neutral* position; then, owing to the coincidence of the magnetic axes of the corresponding stator and rotor phases, the stator and rotor e.m.f.s induced by the main rotating flux will be in phase with each other. The space-waves of rotor ampere-turns will, with respect to the direction of rotation, lag behind the space-waves of stator ampere-turns. Now suppose that the brushes are displaced either way, say backwards, from their neutral position; then if the phase of the rotor currents had remained unaltered, the rotor ampere-turns would

have been displaced backwards relatively to the stator ampere-turns. But the backward displacement of the brushes will cause the maximum rate of cutting of the rotor conductors by the main rotating field to occur at an *earlier* instant than with the brushes in their neutral position, and will thus *accelerate the phase* of the rotor e.m.f. and current, the time-angle of phase advance being equal to the electrical space-angle of brush displacement. Now a phase advance of the rotor current will result in an equal space-advance of the rotor ampere-turns relatively to the stator ampere-turns. Hence the backward displacement of the rotor ampere-turns arising from the brush shift is exactly compensated for by the forward displacement due to the phase advance of the rotor current. The angle between stator and rotor ampere-turns remains unaltered, and hence the motor will exert the original torque with the same currents in stator and rotor.

We thus see that when the brushes are short-circuited their position is immaterial, and the working of the motor will be totally unaffected by brush displacement.

§ 164. Effect of Brush Displacement when a P.D. is impressed on Brushes

The case is otherwise, however, when instead of short-circuiting the brushes we impress a certain p.d. of fixed magnitude and phase upon them. Assume the motor to be running under those conditions, with the brushes in the neutral position. Let now the brushes be displaced backwards. As we have seen, this will accelerate the phase of the rotor e.m.f. Let us provisionally assume that the phase of the p.d. impressed on the brushes is accelerated by an equal amount. Then the phase of the resultant e.m.f. will be accelerated by the same amount, and hence also that of the rotor current. As already explained, the effect of this phase shift will be neutralized by the backward shift of the rotor field due to the brush displacement, and no change will occur in the working of the motor. But since our original assumption was that the phase of the brush p.d. remains unaltered, we must, in order to return to this original assumption, now *retard* the phase of the brush p.d., and this will clearly in general alter both the magnitude and phase of the rotor and hence also of the stator currents. We thus see that with a p.d. impressed on the brushes, *brush displacement through θ electrical degrees is equivalent to a phase change of θ degrees in the impressed brush p.d.*

§ 165. Effect of impressing on Brushes a P.D. which is in Phase with the Rotor Current

Let us next suppose that such a motor is running under load with the brushes short-circuited and in the neutral position, and let us consider the effect of impressing a p.d. on the brushes. The nature of the effect produced will clearly depend on the *phase* of the impressed p.d. Whatever its phase, the impressed p.d. may be resolved into two components, one in phase with the rotor current, and the other in quadrature with it. We shall consider the effects due to these components separately.

First, then, suppose that there is impressed on the brushes a p.d. which is in direct *phase opposition* to the current. Such a p.d. is equivalent in its effects to a non-inductive resistance, and we know that the introduction of a resistance will reduce the speed without changing the rotor and stator currents (§ 115). The source of opposing p.d. will absorb some of the power which was formerly transformed into mechanical power. If the source of p.d. is the secondary of a transformer whose primary is across the mains, then the power absorbed by this secondary will be transmitted (with some loss, of course) to the primary and *returned by it to the supply mains.* The excess power will therefore be returned to the mains instead of being dissipated in a resistance, and a highly efficient method of speed regulation becomes possible. The range of speed variation is from a speed a little below synchronism (with brushes short-circuited) to standstill, and it will be observed that with *increase* of brush p.d. the speed *decreases.* Let us next suppose that the impressed brush p.d. is in *phase coincidence* with (instead of phase opposition to) the rotor current. Imagine the rotor to be originally short-circuited, and suppose that a small p.d. co-phasal with the current is suddenly impressed on it. The immediate effect is to increase the rotor current and torque, causing acceleration. As the speed rises the current decreases, and finally a new steady state of running is reached corresponding to a higher speed. With increase of speed, the magnitude and frequency of the induced e.m.f. decrease and vanish at the speed of synchronism. At this speed, the rotor copper losses are entirely supplied from the source of impressed p.d. A further increase of p.d. will cause the rotor to run above the speed of synchronism, and owing to the phase reversal of the rotor induced e.m.f. as we pass through synchronism this e.m.f. will now *oppose* the motor current; the source of brush p.d. now not only supplies the power corresponding to the rotor copper losses, but also an additional amount of power which undergoes transformation into mechanical power. Speed regulation for speeds *above* synchronism

is thus possible, and it will be noticed that for such speeds an *increase* of brush p.d. causes an *increase* of speed.

It may be pointed out that for speeds *below* the natural speed (corresponding to short-circuited brushes) of the motor the stator is taking power from the mains, while the rotor is returning power to them; and that for speeds *above* the natural speed, both stator and rotor are drawing power from the mains.

It will further be obvious that since owing to the smallness of the rotor resistance drop the rotor induced e.m.f. and the brush p.d. nearly balance each other, a small change of e.m.f. (or speed) will result in a large change of current and torque, and hence the motor will, for a given fixed value of the brush p.d., run at a speed which changes but little with the load—*i.e.* it will have a *shunt characteristic.*

§ 166. Effect of impressing on the Brushes a P.D. which is in Quadrature with the Rotor Current

We shall next consider the effect of impressing on the brushes a p.d. which is *in quadrature* with the rotor current. Two cases may again arise—those of a leading and lagging p.d. A leading p.d. (equivalent to a lagging e.m.f.) produces the same effect as an increase of reactance in the rotor circuit, and will cause the rotor current to be *retarded* in phase; in order to maintain the original torque, a larger current will be necessary (and hence an increased slip), and the power factor of the motor will be reduced. On the other hand, a lagging p.d. (equivalent to a leading e.m.f.) has the same effect as a capacity, *advancing* the phase of the rotor current, initially, assuming that the rotor current was lagging behind the induced motor e.m.f., this will, for a given torque, cause a reduction in the rotor current; but as the magnitude of the p.d. is steadily increased, the rotor current, after passing through a minimum value, will become a leading current, and will numerically increase.

Whether the quadrature p.d. is leading or lagging, it can only produce a phase shift—accompanied, in general, by a change of magnitude—in the motor current, but it cannot become a medium of exchange of *power* between the rotor and the circuit external to it.

§ 167. Methods of controlling Speed and Power Factor of Shunt Polyphase Commutator Motor

We have already seen that the *speed* of a shunt polyphase commutator motor may be controlled by impressing on its brushes a p.d. which is either in phase with, or in phase opposition to, the rotor current, the magnitude of such p.d. being adjusted to give the desired speed; and that the phase of the rotor current, and hence the *power factor* of the motor, may be controlled by impressing on the brushes a p.d. which is in quadrature with the rotor current.

For the simultaneous control of speed and power factor we therefore require a p.d. whose magnitude and phase may both be varied at will. One method of obtaining a p.d. of variable magnitude is to make use of a transformer whose primaries are across the mains, and the *middle* points of whose secondaries are connected to a common point which forms a neutral point, as shown in Fig. 186. If the rotor brushes are connected to this neutral point, the machine runs as a plain induction motor at a speed slightly below synchronism. By moving the points of connection away from the neutral one way (full-line arrows in Fig. 186), we can reduce the speed; and by moving them the other way (dotted arrows in Fig. 186), we can increase it and obtain speeds above synchronism. The passage through the neutral points results in a *phase reversal* of the p.d.s impressed on the rotor.

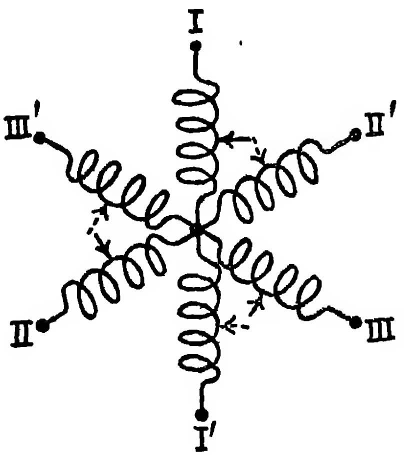


FIG. 186.—Connections of Transformer Secondaries for obtaining Phase Reversal.

In a type of motor manufactured by the *Allmanna Svenska Aktiebolaget*, the variable brush p.d. is obtained without the use of a special variable-ratio transformer, by adopting the following arrangement. The rotor is provided with two windings. One of these is a three-phase winding connected to slip-rings, while the other is a continuous current winding connected to a commutator on the other side of the rotor. The three-phase rotor winding corresponds to the stator winding in the ordinary type of motor, and is connected directly to the supply mains. Three brush sets, spaced 120 electrical

degrees apart, are provided on the commutator. Each brush set consists of two half-sets, which may be moved, by means of suitable gearing, simultaneously in opposite directions, so that by varying the arc spanned by the half-sets the p.d. between them may be controlled, reduced to zero (when the half-sets coincide) and reversed in phase, after passing through zero. The brush sets are entirely disconnected from each other, and the half-sets of each set are connected to one phase of the stator winding, whose phases are kept distinct, and which corresponds to the rotor winding in the ordinary type of motor.*

As regards control of power factor, this is easily obtained with the ordinary type of motor by shifting the brushes, such brush shift being, as already explained, equivalent to a phase shift of the brush p.d.

In the Swedish motor just described, the phase of the p.d. impressed on the secondary winding could be obtained by shifting the brush sets *bodily*—*i.e.* without altering the relative positions of the half-sets).

§ 168. Cascade Connection of Induction and Commutator Motor

An important practical application of the polyphase commutator motor is its use as a cascade-connected motor in conjunction with an ordinary induction motor (§ 139). The development of this method of economical speed control is due mainly to O. Kramer and A. Scherbius.

When so used, the polyphase commutator motor is supplied with currents which are of the *slip frequency* of the main motor. Owing to this low frequency, the commutation troubles are greatly reduced. The commutator motor in such cases is designed to run at speeds greatly exceeding the speed of synchronism (with respect to the rotor currents of the main motor), and its behaviour then closely resembles that of an ordinary continuous current motor.

Such a cascade-connected set may be arranged for a speed range either below synchronism (for the main motor), or both below and above synchronism.

When the speed is below synchronism, the rotor of the main induction motor absorbs a portion of the total power transmitted to it by the stator, and passes on the balance to the auxiliary or

* For a full account of this type of motor, the reader should consult a paper by F. J. Teago in the *Journal of the Institution of Electrical Engineers*, vol. lx p 328 (1922).

commutator motor; this latter then transforms the balance of the power, after allowing for losses, into mechanical power.

Let us now suppose both motors mounted on the same shaft, so that the torque exerted by the auxiliary motor is added to that of the main motor, and the mechanical power of the auxiliary motor is immediately utilized. Let the e.m.f. of the auxiliary motor be reduced to zero, so that the main motor runs as a plain induction motor (with, however, an increased resistance—corresponding to that of the auxiliary motor—in its rotor circuit). Suppose that the safe limit of load for the main motor has been reached. Then any further increase in the resisting torque would cause an increase of current and consequent overheating of the motor. Let us, however, suppose that when such an increase of load has occurred we excite the auxiliary motor so that it generates a counter e.m.f. The current will decrease, and at the same time the auxiliary motor will develop a torque which is added to that of the main motor. Let us suppose the excitation of the auxiliary motor to be so adjusted that when a steady new speed has been reached the currents in the main motor windings are the same as they were originally, *i.e.* let us regulate for constant main motor current, or, what amounts to the same thing, for *constant output*. Since the main motor current is maintained constant, its torque will also remain constant; but with increasing auxiliary motor field the torque of the *auxiliary* motor will steadily increase. At the same time, in order to maintain the main motor rotor current constant with increasing counter-e.m.f. in its circuit, the total rotor e.m.f. must increase—hence the slip must increase, or the speed of the set must decrease. The law connecting the total torque of the set and its speed is, approximately, the hyperbolic law, for since the total output is (owing to the constancy of the main motor current) approximately constant, the product of total torque into speed must be constant, and hence the speed must vary inversely as the torque.

From the above it will be seen that *mechanical coupling of the main and auxiliary motors is suitable in cases where it is desired to regulate the set for constant output with variable torque and speed.*

Instead of being mechanically coupled to the main motor, the auxiliary motor may be coupled to an induction generator which returns power to the mains. Let us suppose that the excitation of the auxiliary motor is steadily increased while the resisting torque due to the load remains unaltered. Then there will be a steady drop in the speed of the main motor, a larger and larger proportion of the total power transmitted to the rotor being passed on to the auxiliary motor-generator set, which (after allowing for losses) returns it to the mains. The main motor in this case continues to draw the same amount of power from the mains, but converts a steadily

decreasing fraction of it into mechanical power, and returns the remainder, through the medium of the motor-generator, to the mains. The *total* power drawn from the mains is thus roughly proportional to the speed of the main motor. The use of a *commutator motor-induction generator set electrically connected to the rotor of the main induction motor* is seen to be suitable in cases where it is desired to regulate for *variable speed with constant torque.*

So long as the speed regulation is confined to speeds below synchronism, the surplus power drawn by the main motor from the mains is transferred to the commutator machine, which acts as a *motor*, and drives the induction machine coupled to it as a *generator.* Suppose the excitation of the induction machine to be gradually reduced to zero; the main motor will now run as a plain induction motor at a speed very little below synchronism Imagine next that from a suitable external source *whose frequency is always kept equal to that of the rotor currents* we excite the commutator machine so as to reverse the phases of the e.m.f.s which it generated before these were reduced to zero. Then these e.m.f.s, instead of being opposed to the rotor e.m.f.s and rotor currents, will be *in phase* with them; the immediate temporary effect will be to increase the rotor current and accelerate the rotor: but with increasing rotor speed the rotor e.m.f and current will decrease until the original torque is reached with the same rotor current but a higher rotor speed. The rotor current is now only partially maintained by the rotor e.m.f., the required balance of e.m.f. being provided by the commutator machine. Since the e.m.f. of this machine is in phase with the rotor current, the machine is acting as a *generator*, and is being *driven* by the induction machine coupled to it. Let the excitation of the commutator machine be steadily increased; then the speed of the main motor will steadily rise, and ultimately reach synchronism. At this stage, since the rotor of the main motor is not generating any e.m.f., the whole of the e.m f. required to maintain the rotor current is supplied by the commutator machine. Further, since both the rotor and its field are now running at synchronous speed, there is no relative motion of rotor and field, and hence the rotor current must be of zero frequency, *i.e.* a continuous current. Let this current be slightly increased: further acceleration of the rotor will result, and its speed will rise above synchronism. At the same time, the rotor will begin generating e.m.f.s, but of *opposite phase sequence* to the phase sequence below synchronism, and of *reversed phase.* Let us assume that as the main motor speed passes through synchronism the phase *sequence* of the source which provides the commutator machine excitation is automatically reversed, but without phase reversal of the e.m.f.s. Then the commutator machine e.m.f.s will have the same sequence as the main motor rotor e.m.f.s, but the

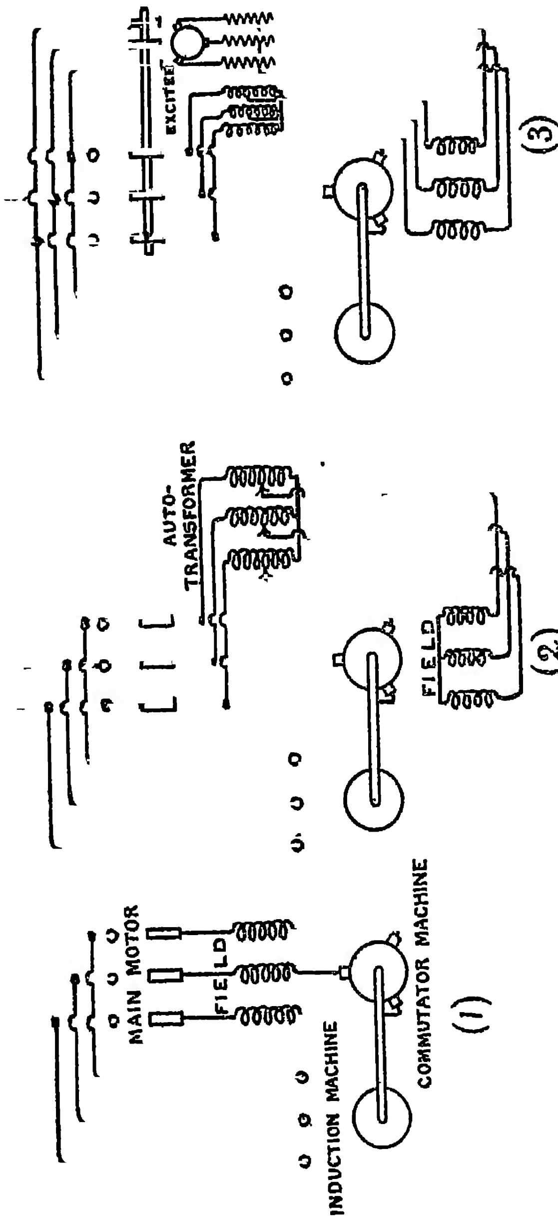


Fig 187.—Cascade Connections of Induction and Commutator Motors.

latter will *oppose* the former. The rotor currents flowing in the direction of the commutator machine e.m.f.s, it is evident that this machine will continue to act as a generator (being driven by the induction machine, which continues to act as a motor), and that it will supply to the rotor not only the power required to overcome the rotor copper losses, but also additional power which becomes converted into mechanical power. The main motor is now *doubly fed*, both stator and rotor absorbing power from external sources. With steadily increasing commutator machine excitation, the speed of the main motor will steadily increase.

Provided, therefore, that we can devise a machine whose frequency always remains equal to the rotor frequency, and whose phase sequence is reversed as the rotor passes through synchronism, the main motor speed can be regulated for values both below and above synchronism. An exciter of this type is described below.

The commutator machines used in such regulating sets are provided with *compensating windings* on their stators, which neutralize the armature field.

The most common arrangements of regulating sets used in practice are the following: (1) a series commutator machine, for *single-range* speed regulation (*i.e.* for speeds below synchronism only); (2) a shunt-wound commutator machine, for single-range regulation; (3) a shunt-wound commutator machine, with special exciter, for *double-range* speed regulation (above and below synchronism).

In each case the commutator machine is mechanically coupled to an induction machine, and in the case of single-range regulating sets the induction machine is driven as a generator by the commutator machine; in double-range sets the induction machine may, as we have seen, act either as a generator or as a motor.

Since in the case of an induction machine a very small percentage change of speed is sufficient to cause the machine to pass from full-load motor to full-load generator action, the speed of the regulating set will change but little with changes of speed of the main motor, so that the regulating set may be regarded as having an approximately constant speed.

The three arrangements of regulating sets mentioned above are diagrammatically represented in Fig. 187, where, however, the compensating windings of the commutator machine are omitted for the sake of simplicity.

Arrangement (1) would be used in cases where the main motor is required to have a *series* characteristic, *i.e.* where an increase of torque is required to result in an *automatic* decrease of speed, as with an ordinary continuous current series motor. That arrangement (1) fulfils this requirement will be evident from the following consideration. The speed of the commutator machine being practically

constant, its e.m.f. will be proportional to the field flux, and, so long as the core is not saturated, the flux may be taken to be proportional to the current. Hence the e.m.f. is approximately proportional to the current, and, so far as the main motor is concerned, the commutator machine might be replaced by a dead resistance. But an induction motor with sufficiently high resistances in its rotor circuits *has* a series characteristic (§ 121). With a constant load, the speed may be regulated by hand as desired if the field windings of the commutator machine are supplied through a suitable transformer with tappings.

Arrangements (2) and (3) are used where a *shunt* characteristic (approximate automatic constancy of speed with varying load) is desired. With a shunt winding, since the reactance of the winding is proportional to the slip frequency of the main motor, and since the rotor e.m.f. is also proportional to the slip, the ratio of the p.d. across the field winding to its impedance will remain approximately constant, and hence also the field current and field flux. But since the speed of the commutator machine is also approximately constant, so will be its e.m.f. Now since the rotor and commutator machine e.m.f.s nearly balance each other, a very small change of rotor speed will result in a large change of rotor current and torque, *i.e.* the main motor will run at an approximately constant speed with varying load. The value of this speed may be regulated by hand, a suitable transformer with tappings being provided for this purpose as shown in Fig. 187.

Arrangement (3), although more complicated than (2), requiring as it does a special exciter, has the advantage that for a given range of speed regulation it requires a regulating set of only half the output of that corresponding to (2). It has the further advantage that in case of breakdown of the regulating set its main motor will run at a speed corresponding to the *middle* of the range, which is likely to be a more suitable emergency speed than that of (2), where the main motor would run at the *maximum* speed of the range.

We have now to explain the construction of the special exciter required for arrangement (3). This consists of a rotor mounted on the shaft of the main motor and provided with a continuous-current winding connected to slip-rings on one side and to a commutator on the other. The rotor is surrounded by a stator without any winding. The slip-rings are connected to the mains in such a manner that the rotating field revolves in a direction *opposed* to the direction of rotation of the rotor. Since the speed of the field relatively to the rotor is always the same—that of synchronism—the brush p.d. will (neglecting the drop in the rotor windings) be practically constant. The *frequency* of the brush p.d. will, however, be determined by the speed of the field *in space*, and will be equal to the *slip frequency*

of the main motor. The machine therefore acts as a *frequency changer* pure and simple, without altering the value of the p.d. It will be noticed that in passing through synchronism the phase *sequence* of the brush p.d.s is automatically reversed. The machine therefore fulfils all the conditions required of an exciter for arrangement (3).

As will be seen by reference to Fig. 187 (3), the special exciter does not provide the whole of the excitation (except at synchronism) for the field of the commutator machine, the rotor voltage being made use of for this purpose for speeds both above and below synchronism. *At* synchronism, however, when the rotor e.m.f.s vanish, the *whole* of the excitation is due to the exciter. The current at synchronism being a continuous current, the voltage drop in the rotor is a pure resistance drop; hence the exciter, which provides this drop, has been termed an *ohmic drop exciter*. At speeds sufficiently far removed from synchronism, either below or above synchronism, the exciter could be dispensed with. But it is evident that without such exciter the speed could never be made to reach, much less to surpass, synchronism; for at synchronism, the rotor e.m.f.s being zero, no excitation at all could be obtained.

In Fig. 187, the ohmic drop exciter is shown connected in series with the commutator machine field windings and the main rotor slip-rings. Instead of this arrangement, two distinct field windings may be used, one set being across the main rotor slip-rings and the other across the brushes of the exciter (through suitable regulating resistances).

§ 169. Phase Advancers

The most important disadvantage of an induction as compared with a synchronous motor is its relatively low power-factor, and from the earliest days of the induction motor attempts have been made to bring about an improvement in this respect. A machine which is intended to improve the power factor of an induction motor is known as a *phase advancer*, since its effect is to cause the current taken by the stator to advance in phase so as to become nearly or entirely coincident in phase with the p.d. Two types of phase advancers have come into commercial use: the rotating and the vibrating type.

The simplest form of *rotating phase advancer* is that originally invented by Leblanc,* but apparently first brought into commercial use by Scherbius, and hence frequently spoken of as the Scherbius phase advancer. It may be described as a three-phase commutator

* *La Lumière Electrique*, vol xxii. p. 291 (1913).

machine without any field winding, the armature being surrounded by an unwound stator whose function is to reduce the magnetic reluctance offered to the flux. The armature is provided with three sets of brushes per pole-pair, and currents are supplied to it from the three slip-rings of the wound rotor belonging to the induction motor whose power factor it is desired to improve. The phase advancer is driven by being either direct-coupled or belted to the induction motor, or by being direct-coupled to a small auxiliary motor, and the direction of rotation is the same as that of the rotating field of the advancer. The frequency of the currents supplied to the advancer is very low, corresponding to the slip of the induction motor, and the advancer runs at a speed which is a large multiple of the speed of synchronism. Under those conditions, the e.m.f.s generated in the three phases of the advancer armature are ahead of their corresponding currents by 90°. This will be readily understood from the space and time diagrams of Fig. 188. The connections are supposed to be such as to give rise to a field rotating counter-clockwise. The armature of the phase advancer is also driven in a counter-clockwise direction. In order to determine the phase relation of the induced e.m.f. to the current in any phase, we may consider phase I at the instant when the current in it is at its maximum positive value. At this instant, the rotating ampere-turn wave must coincide in position with the oscillating wave due to the current in phase I (see § 22, p. 47, and § 160), so that the rotating ampere-turn vector occupies the position shown in Fig. 188 (a). The flux vector is in space quadrature with and lies to the left of it (§ 160). An instant later, the vectors will have advanced, and there will be a flux having a negative or inward direction through the arc occupied by the windings of phase I. With a radially inward flux and a counter clockwise direction of rotation of the conductors relatively to the field, the induced e.m.f. will have a direction towards the observer, or be *negative* (§ 160). Hence the time-diagram must be as shown in Fig. 188 (b), the projection of the e.m.f. vector assuming a *negative* value when the current begins to decrease, and

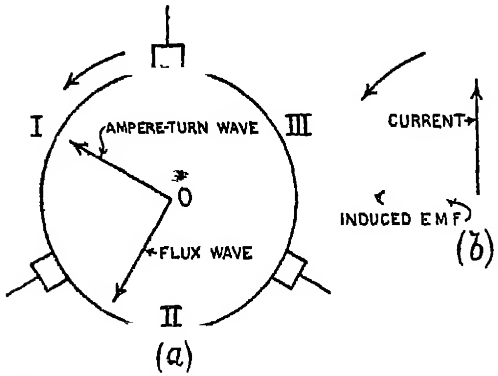


FIG. 188.—Space and Time Vector Diagrams of Phase Advancer.

we see that the induced e.m.f. is 90° *ahead* of the rotor current as regards phase.

The way in which the injection of such an e.m.f. into the rotor circuits is capable of improving the power factor of the stator will be understood by a study of the vector diagrams of Fig. 189. In this figure, diagram (*a*) refers to a motor not provided with a phase advancer. The frequency of the rotor currents may be rendered equal to that of the stator currents by the use of the artifice mentioned in § 115, so that both stator and rotor e.m.f.s, currents, etc., may be

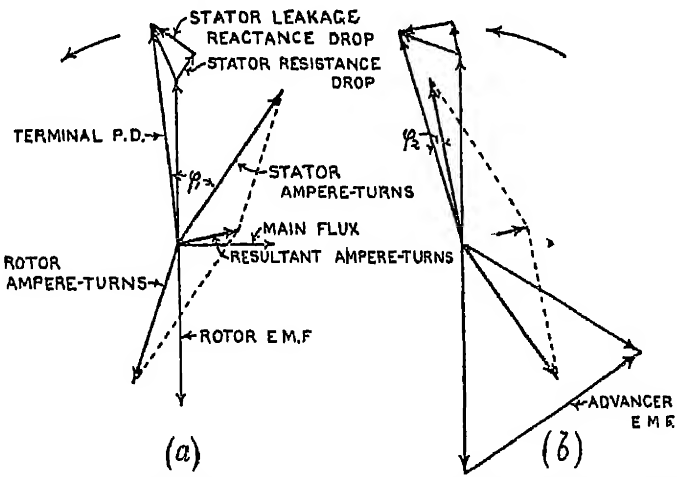


FIG. 189.—Vector Diagrams of Induction Motor without and with Phase Advancer.

represented in the same diagram. Fig. 189 (*b*) shows the effect produced by the phase advancer—*i.e.* by the injection of an additional e.m.f. into each rotor circuit, such e.m.f. being 90° ahead of the current. The current has been advanced in phase relatively to the e.m.f. generated in the rotor of the induction motor. This advance of phase is accompanied by a corresponding advance in the phase of the stator current relatively to the stator p.d., and by suitably adjusting the magnitude of the phase advancer e.m.f., the power factor of the stator may be made unity. If desired, the stator may even be made to take a leading current.

In the type of phase advancer described, the armature of the advancer was supposed to be surrounded by an unwound stator. Since the only function performed by the stator in this case is to reduce the reluctance of the magnetic circuits, there is no reason why the stator core should not undergo fusion with the rotor and revolve with it. We then obtain a type of advancer possessing no

stator, but having its windings embedded in closed slots or tunnels well below the surface of the core. This is the form of construction adopted by Scherbius, and carried out by Messrs. Brown, Boveri & Co.

When a phase advancer is required for a motor of very large output, the type without any stator is unsuitable, owing to commutation troubles. It is then preferable to adopt the type having a

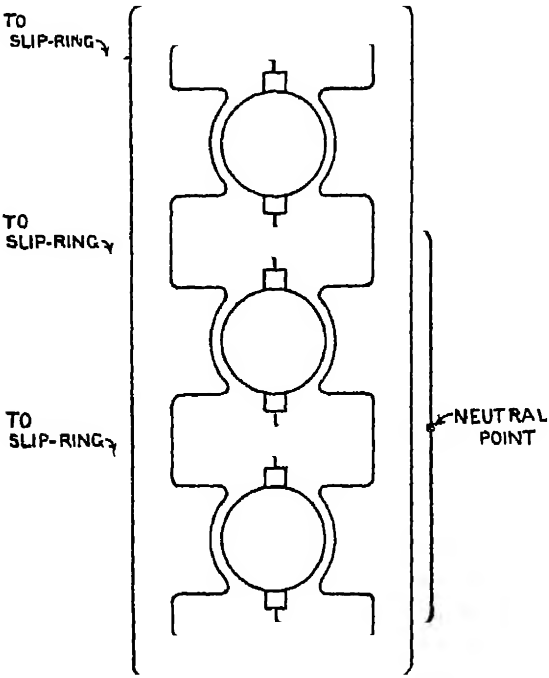


FIG. 190.—General Arrangement of Kapp Vibrator.

ound stator, which has been studied in detail by Miles Walker.* ie stator windings are connected in series with the armature.

An entirely different form of construction is presented by the *vibrating* phase advancer devised by Kapp.† The principle on which this advancer is based was originally pointed out by Swinburne.‡

* *Journal of the Institution of Electrical Engineers*, vol. xlii. p. 599 (1909); and vol. l. p. 329 (1913).

† *The Electrician*, vol. lxix. p 222 (1912); *Journal of the Institution of Electrical Engineers*, vol. li. p. 256 (1913).

‡ *Journal of the Institution of Electrical Engineers*, vol. xxxii. p. 26 (1902).

Kapp's vibrator consists of a continuous current armature placed in a field excited by a *continuous* current. The number of armatures required corresponds to the number of rotor phases, so that a three-phase rotor is shown in Fig. 190. Each armature is on one side connected to a rotor slip-ring, and on the other to a common neutral point. The alternating torque due to the low-frequency current passing through the armature throws the latter into *vibration*, and such vibration generates an e.m.f. It is easy to show that this e.m.f. is in quadrature with the current. For it is obvious that

$$\begin{aligned} \text{induced e.m.f.} &\propto \text{angular velocity} \\ \text{current} &\propto \text{torque} \\ &\propto \text{angular acceleration,} \end{aligned}$$

and since the angular velocity is in quadrature with the angular acceleration (the acceleration being zero when the velocity is at its maximum), it follows that the e.m.f. is in quadrature with the current. In order to find whether the e.m.f. lags or leads with respect to the current, we may consider a particular instant, *e.g.* that at which the current is at its maximum positive value and the e.m.f. at its zero value. Referring to Fig. 191 (*a*), let the field in which

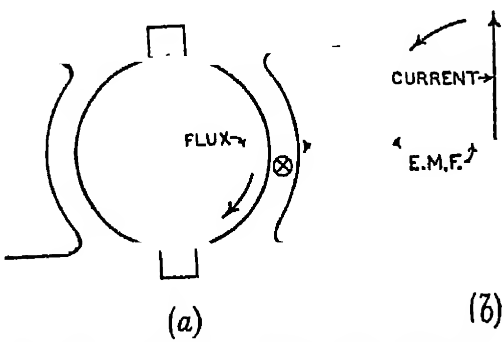


FIG. 191.—To illustrate Theory of Kapp Vibrator.

the armature vibrates have a direction from left to right as shown, and let the direction of the current be considered positive if it flows away from the observer in the right-hand half of the armature. When the current is at its positive maximum, the armature is momentarily at rest, but is on the point of beginning to move *clockwise.* Now a clockwise displacement of the armature will generate in the right-hand half of the armature winding an e.m.f. which is *towards* the observer, *i.e.* which is *negative.* Thus at the instant considered the e.m.f., which is zero, is about to assume a negative value. Hence the vector diagram of current and e.m.f. is as shown

in Fig. 191 (*b*), and we see that the e.m.f. is 90° ahead of the current in phase.

Since the e.m.f. generated by a phase advancer is in quadrature with the rotor current, it is incapable of contributing towards the power in the rotor circuit. But the resistance of the advancer windings, by increasing the total rotor resistance, causes an *increase of slip*, with a corresponding reduction of efficiency.

CHAPTER XVII

§ 170 Single-phase commutator motors—§ 171. Series motor—§ 172. Vector diagram of series motor—§ 173. Circle diagram of series motor—§ 174. Variations in power factor of series motor—§ 175. Experimental determination of phase relations of vectors in vector diagram—§ 176. Conditions determining power factor of series motor—§ 177. Neutralized series motor—§ 178. Analysis of e.m.f.s in short-circuited coil—§ 179. Methods of securing sparkless running—§ 180. Some technical data relating to series single-phase motors—§ 181. Method of starting single-phase series motors.

§ 170. Single-phase Commutator Motors

WITH the rapid growth of electric railway and tramway systems, the necessity of high-voltage transmission on economical grounds soon became apparent. It was realized that beyond a certain distance the usual 500- or 600-volt continuous-current systems could no longer be successfully employed, as the cost of the conductors required to secure reasonable regulation and losses became a very heavy item in the total cost of the system. This led to the development of the rotary converter system, in which we have three-phase generation and transmission to converter sub-stations at a high voltage, combined with low-voltage (500 volts) continuous-current distribution. But the cost of rotary converter sub-stations is still very heavy.* Attempts were then made, in the case of very long lines, to use the three-phase system pure and simple, the current being generated and transmitted at a very high voltage, and then transformed to a lower voltage before being supplied to the three-phase induction motors employed for driving the cars.

Apart from the complication and increased cost of overhead construction necessitated by the use of two overhead conductors, the three-phase system of electric traction suffers from another defect—the unsuitability of induction motors for variable speed work. The induction motor is, as we have seen, essentially a constant-speed motor, and is extremely wasteful during the period of acceleration. Such motors can only be satisfactorily employed for traction work on lines where stoppages are infrequent, and where long runs at constant speed are the rule.

* Within recent years important developments have taken place in high-voltage continuous-current traction, voltages as high as 3000 being in successful use. The high-voltage continuous current system is a serious rival to the single-phase system.

In view of the disadvantages of the three-phase system of traction and the heavy cost of rotary converter sub-stations, the single-phase system of traction, in which both transmission and distribution are effected by the use of single-phase current, has received a great deal of attention, and has led to the development of single-phase *commutator* motors. In the present chapter we shall give an account of the theory and construction of such motors.

The more important types of single-phase commutator motors* are: (1) the series-wound motor, with or without a neutralizing winding; (2) the repulsion motor; (3) the compensated repulsion motor (Latour-Winter-Eichberg).

§ 171. Series Motor

The general arrangement of the plain series-wound single-phase motor closely resembles that of an ordinary continuous-current motor; in fact, such motors may, and in practice actually are, used on both single-phase and continuous-current circuits. The differences between the ordinary continuous-current and the single-phase types are partly constructional, partly differences of design. It is obvious that, in order to prevent excessive eddy-current loss, and to enable the field-magnet to develop its full magnetic flux,† the core of the field-magnet must be *laminated throughout*.

In Fig. 192 is given the diagram of connections of a plain series-wound single-phase motor. A is the armature, FW being the field winding. T_1 and T_2 are the motor terminals. The dotted arrow indicates the direction of the field flux, which, as in a continuous-current motor, is at right angles to the line of the brushes.

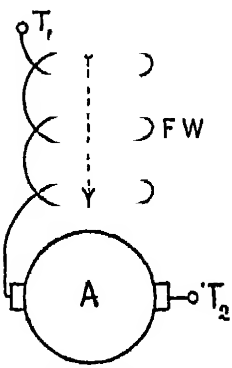


Fig. 192.—Diagram of Connections of Series Motor.

Since in this type of motor the armature is connected in series with the field, it is obvious that the armature and field currents must necessarily be in phase with each other, so that the armature current reaches its maximum value at the instant of maximum field intensity. So long as the magnet is well below saturation—*i.e.* within the range of approximately constant permeability—the torque at any instant is proportional to the square of the current. From this it follows that the motor will exert equal torques when supplied, in the one

* The single-phase induction motor is still less suitable for traction than the three-phase induction motor, and its use for this purpose could not be seriously considered

† Eddy currents exert a demagnetizing or screening effect on the core, so that with a massive core the central portions would not be appreciably magnetized.

case, with continuous current of amount I, and in the other, with single-phase current whose r.m.s. value is I. In the first case, however, the torque is a steady one; in the second, a rapidly fluctuating or pulsating one.

As will be seen presently, it is advisable to have a relatively small number of turns in the field winding of such a motor; this is one of the important differences between an ordinary continuous-current and a single-phase series motor.

§ 172. Vector Diagram of Series Motor

The behaviour of a series-wound single-phase motor will be best understood by a study of its vector diagram. Such a diagram is shown in Fig. 193.

It is convenient to choose for our vector of reference the vector representing the magnetic flux; this is drawn in a horizontal direction in Fig. 193. Owing to the fact that the alternating flux which enters the armature core from the pole-pieces becomes linked with the coils short-circuited by the brushes, an alternating e.m.f. will be induced in these coils, which we may speak of as the *transformer e.m.f.*, since the coils practically form the short-circuited secondary of a transformer whose primary is represented by the field winding. This transformer e.m.f. is represented by the vector OA in the diagram, lagging 90° behind the magnetic flux vector. The current OB resulting from the transformer e.m.f. will lag by a certain angle a behind it, the value of a being determined by the ratio of the leakage self-inductance of the short-circuited coil to its resistance. Now the resultant or exciting field ampere-turns, represented by OD, will, if we take into account the hysteresis and eddy-current losses in the iron, be slightly in advance of the flux, as shown in the figure.* It is evident that the resultant field ampere-turns when added vectorially to the ampere-turns DC of the short-circuited coils (with sign reversed) will give the total field ampere-turns OC.

We may now construct the e.m.f. polygon. The alternating magnetic flux through the field and armature will induce in these an e.m.f. in quadrature with the flux, and to balance this a component OE of the terminal p.d. will be required. A second component, represented by EF, corresponds to the total resistance drop, and is clearly in phase with the current (or the total ampere-turns). A third component, represented by FG, balances that part of the e.m.f.

* By the resultant or exciting ampere-turns are here meant not only the ampere-turns required to produce magnetization (which are wattless, being in phase with the flux), but the total ampere-turns necessary to produce magnetization and cover the losses in the core. See § 14.

induced in the field which is due to the leakage field flux; * FG is thus perpendicular to OC. Lastly, the rotation of the armature gives rise to an e m.f. in phase opposition to the main field flux, and this has to be balanced by a component GH. The resultant OH of all these components clearly gives the impressed p.d. The power factor of the motor is given by cos ϕ.

The angle α being constant, OB and hence also CD will remain parallel to itself for all conditions of load. The angle β, which depends on the iron losses, will not, on the other hand, remain quite constant, but will vary with the current. If we suppose β to remain constant, and make the further assumption that the magnetic flux is proportional to the magnetizing current, then this would be equivalent to the assumption that the iron losses vary as the square of the magnetizing current or flux; for, in order to increase the magnetic flux m-fold, m times the magnetizing current will be required at m times the voltage, and since the phase relation of current to voltage remains unaltered (owing to the assumed constancy of β), the power lost in the core will be increased m^2-fold. This assumption, though not quite correct, is found to be nearly so for all practical purposes.†

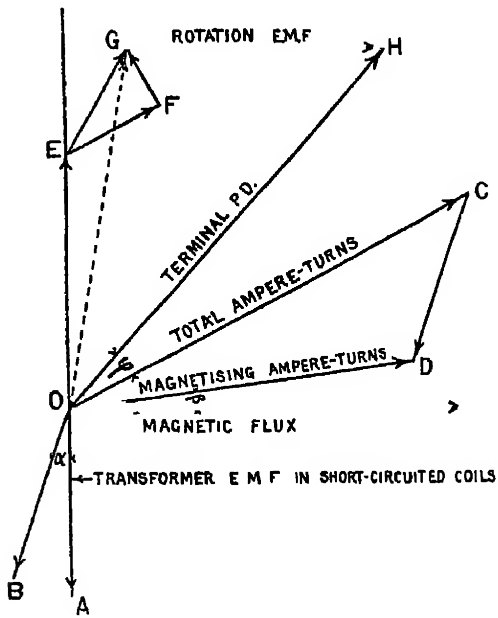


FIG. 198.—Vector Diagram of Series Motor.

Assuming, then, that α and β are both constant, and that the cores are well below saturation, so that the flux is proportional to

* In a motor provided with a neutralizing winding (§ 177), FG would also include the e.m.f. induced in the armature by armature leakage lines.

† In the case of a stationary core, the hysteresis loss would, for a constant frequency, vary approximately as the 1·5th or 1·6th power of the induction, while the eddy-current loss would vary as the square of the induction. Thus the total core loss would vary as some power of the induction between 1·6 and 2 Owing, however, to the rotation of the armature, the total core loss in a series motor presents very complicated conditions.

the current, we see that the directions of OD and DC are fixed relatively to the flux vector. Since, however, CD is proportional to OD, the triangle ODC remains similar to itself and the direction of OC is also fixed. Hence the directions of OE, EF, and FG remain fixed, and since *each* of these vectors is proportional to the current, their resultant OG remains fixed in direction. The direction of GH being also fixed, it follows that the angle OGH remains constant.

§ 173. Circle Diagram of Series Motor

Considering now the voltage triangle OGH (Fig. 193), in which OG may be termed the "total impedance" component of the impressed p.d., it is evident that if the impressed p.d. OH is maintained constant, then, on account of the constancy of the angle OGH, the circle circumscribing the triangle OGH is of constant diameter. Hence, if we now choose OH for our vector of reference (instead of the vector of magnetic flux, as we did originally), *i.e.* if we suppose its direction to remain fixed while the other lines in the diagram swing round it as the load changes, then the point G will move round a circle, of which OH is a chord. Thus, since OG is proportional to, and therefore, to a suitable scale, represents the current in magnitude (but *not in phase*), we see that the changes in the current may be represented by a circle diagram, as shown in Fig. 194. It must be clearly understood that this diagram is not a circle diagram in the ordinary sense of the term, as the angle between OG and OH does *not* represent the phase difference between p.d. and current.

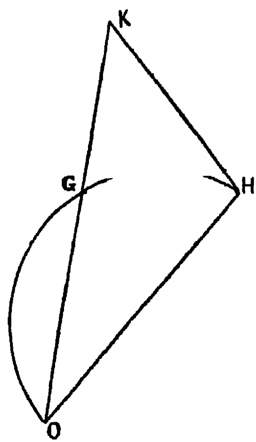


FIG. 194.—Circle Diagram of Series Motor.

A simple construction enables us to represent the relation connecting current and speed. If from H we draw HK, making the angle OHK equal to OGH, and if we produce OG until it meets HK at K, the two triangles OGH and OHK will be similar, so that—

$$\frac{KH}{GH} = \frac{OH}{OG}, \text{ or } KH = OH \cdot \frac{GH}{OG} \quad . \quad . \quad . \quad . \quad (1)$$

But since GH is proportional to current × speed, and OG is proportional to current, $\frac{GH}{OG}$ will be proportional to speed. Hence, OH being constant, we see by (1) that KH will, to a suitable scale, represent the speed.

§ 174. Variations in Power Factor of Series Motor

Returning now to the vector diagram of Fig. 193, we see that the greatest phase difference ϕ, or least power factor cos ϕ, occurs when the motor is at a standstill, in which case the rotation e.m.f. GH vanishes. As the motor speed increases, OG shrinks while GH lengthens (as is at once evident from the circle diagram of Fig. 194), so that OH in Fig. 193 swings back towards OC, the power factor cos ϕ gradually improving. When OH becomes coincident with OC, cos ϕ reaches a value of unity. A still further increase of speed will cause the current to *lead*, cos ϕ decreasing. It will be seen that *the possibility of obtaining a leading current is largely due to the effect of the short circuited coils.* For if OD were very small, OC would become coincident with OD, and OH could not be made to lag behind it unless the speed exceeded the limit of safety.

From what has been said above regarding the effect due to the short-circuited armature coils, it might at first sight appear as if such coils, by improving the power factor, exerted a favourable effect on the performance of the motor. This, however, would be a fallacy. An increase in the power factor *per se* is not necessarily an advantage, unless the efficiency is not decreased to such an extent as to wipe out the effect due to the gain in the power factor. Now, in the case under consideration, the currents in the short-circuited coils, whilst improving the power factor, increase the total current and total power drawn from the mains, the additional power being, however, largely wasted in the coils themselves, and the efficiency of the apparatus lowered. Methods of improving the power factor without a counterbalancing loss of efficiency are considered in § 176.

It may be pointed out that the currents in the short-circuited coils do not directly influence the torque so long as the brushes are not given any lead.

§ 175. Experimental Determination of Phase Relations of Vectors in Vector Diagram

The following experimental method may be used for determining the angles β, DOC, and EOG in Fig. 193. The method is due to

M. Breslauer.* The armature being held fast, so that the rotation e.m.f. vanishes, and therefore the p.d. vector OH becomes coincident with OG, the p.d., current and power are measured, and so ϕ determined. Now ϕ in this case becomes GOC, and hence this angle is known.† Next, the measurement is repeated, but with the ordinary brushes replaced by very narrow ones, so that there are no short-circuited coils. The vector OC now coincides with OD (since CD is zero), and the newly determined power factor enables us to find the angle GOD. The relative directions of OG, OC, and OD thus become known. Finally, using the last measurement, the resistance drop is subtracted vectorially from OG, and so the point E and the direction OE are determined (the leakage reactance drop FG becomes practically merged in OG). Since the magnetic flux vector is at right angles to OE, its direction, and thus also the angle β, become known.

§ 176. Conditions determining Power Factor of Series Motor

A reference to Fig. 193 shows that the power factor ($\cos \phi$) increases with increase of the ratio $\frac{GH}{OG}$. It is therefore desirable, in order to secure a high power factor, to make this ratio as large as possible. Now, the impedance-voltage OG may be reduced by reducing the number of turns on the field—which will reduce that part of the reactance voltage depending on the field winding—and by designing the magnetic circuit so as to interpose a large magnetic reluctance along the path of the flux due to the armature current, and thereby reduce the part of the reactance voltage which corresponds to armature self-inductance.

The first method of reducing the reactance is very generally adopted in practice, and leads to the design of motors in which the armature ampere-turns bear a ratio of from 2 to 5 to the field ampere-turns. It must be noted that, beyond a certain point, an increase of power factor by a reduction of the field ampere-turns can only be obtained at a sacrifice of efficiency; for in order to secure a given torque with a very weak field, a larger current will be required (corresponding to a heavier copper loss) than would otherwise be necessary.‡ It is thus not advisable to reduce the field-turns below a certain limit.

* *Elektrotechnische Zeitschrift*, vol. xxvii p 407 (1906).

† In order to secure accuracy, the three-point wattmeter method (§ 32) may be used for determining $\cos \phi$

‡ This is on the assumption that the field-turns are reduced without any change

In order to throttle the armature flux as much as possible, it is usual to employ stators or field-magnets having well-defined or salient poles, such as are typical of continuous-current motors. The reluctance to the armature flux may be further increased by making the width of the pole small in comparison with the length of the interpolar arc, and by working the poles at a high induction so as to reduce their permeability.

Let the speed at which the motor is required to run when fully loaded be specified. The speed being given, the full-load torque is also known. Now, for a given gap induction, and a given number of ampere-wires per cm. length of armature circumference at full load, the torque is independent of the number of poles. The self-inductance of the field is also independent of this number so long as the gap induction and the ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$ remain constant.* We may, therefore, vary the number of poles without altering the reactance of the field. If we assume the number of poles to be doubled, then for the same speed, with a lap-wound armature, we shall require the same number of conductors per pole-pitch, but each conductor will now carry only half as much current. The polar arc having been reduced to half its original length, there will be only half the original number of ampere-turns producing the cross-field, and only half the flux linked with the conductors lying between two neighbouring sets of brushes. The armature reactance e.m.f. will thus be reduced to half its original value, and the power factor will be increased.

This shows the advantage gained by increasing the number of poles. There is another way of expressing the same result. Since the speed of synchronism, for a given frequency, varies inversely as the number of poles, increasing this number (for a given speed of the motor) is equivalent to making the ratio $\frac{\text{speed of motor}}{\text{speed of synchronism}}$ larger. The result just established is, therefore, frequently stated in the form that it is best to design the motor so that it will run at a high speed as compared with the speed of synchronism. The normal full-load speed is generally about double that of synchronism.

in the armature turns. The same result as regards increased armature loss is, however, also arrived at by supposing that the current remains unaltered, and that the original value of the torque is maintained by increasing the number of armature conductors in the same ratio as that in which the field has been weakened. The current remaining unaltered, and there now being more conductors on the armature, *i e.* a larger armature resistance, it is evident that the copper loss in the armature will have been increased The increase in the armature copper loss is only to a very feeble extent counterbalanced by the decrease in the field copper loss.

* If, *e.g.*, we double the number of poles, the flux per field-turn will be halved, and the number of field-turns doubled, the total flux linked with the field winding thereby remaining unaltered.

It must be clearly understood, however, that the improvement in the power factor is due, not to any effect of increased relative speed *per se*, but to a decrease in the reactance of the motor consequent on increasing the ratio $\frac{\text{motor speed}}{\text{speed of synchronism}}$.

§ 177. Neutralized Series Motor

Besides the methods already described for improving the power factor, another method is frequently employed. This involves the use of a supplementary field winding, known as a *neutralizing winding*. This winding is displaced relatively to the main field winding by half a pole-pitch, so as to counteract the magnetic effect of the armature current, and practically suppress the self-inductance of the armature.

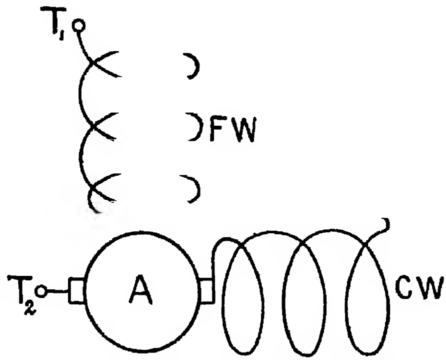


Fig. 195.—Compensated Series Motor.

A series-wound motor fitted with such a neutralizing winding is known as a *neutralized** series-wound motor. There are two methods of connecting the neutralizing winding. In one, shown in Fig. 195, it is included in the main circuit. The neutralizing winding CW supplies a number of ampere-turns roughly equal to those on the armature. In the second method, shown in Fig. 196, the neutralizing winding CW is short-circuited on itself, and acts as the secondary of a transformer, of which the primary is represented by the armature winding. The armature and neutralizing winding constitute, in fact, an arrangement corresponding to a somewhat leaky short-circuited transformer, and the equivalent reactance (due to leakage),

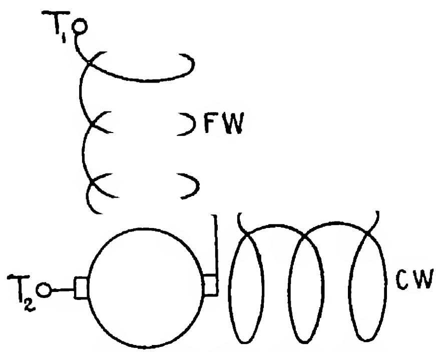


Fig. 196 —Neutralized Series Motor.

* Sometimes termed *compensated*.

is very much less than that of an armature without a neutralizing winding.*

Since a properly designed neutralizing winding will destroy the greater portion of the armature reactance, it follows that very little additional advantage will be gained by the use of further devices having the same object in view. There is, for example, no longer the same need to use salient poles, and the stator may be similar to that of an ordinary induction motor. There is, further, very little advantage gained by increasing the number of poles, *i.e.* by running considerably above the speed of synchronism.

§ 178. Analysis of e.m.f.s in Short-circuited Coil

A short-circuited coil may be regarded as the seat of three distinct e.m f.s, viz. (1) a reactance e.m.f., due to the current reversal; (2) a transformer e.m.f., due to the transformer action of the short-circuited coil; (3) a rotation e.m.f., generated by the rotation of the short-circuited coil in any cross-field which may be present (or in the reversing field due to a reversing pole if such be used). In order that the reversal may take place sparklessly, it is obvious that the resultant of (1), (2), and (3) must either vanish or have a sufficiently small value.

As regards the reactance e.m.f. in a short-circuited coil, it is obvious that its instantaneous value will, for a given speed and hence given frequency of commutation, depend on the value of the current to be reversed. Now, although this current is an alternating one, yet the frequency of commutation is generally so much higher than that of the current, that the instantaneous value of the current taken by the motor may be regarded as approximately constant during the brief period of commutation. In other words, we may assume that the alternating current, instead of varying continuously, does so by little jumps, as shown in Fig. 197, remaining constant over the period of short circuit of a coil by a brush. The reactance e.m.f. will

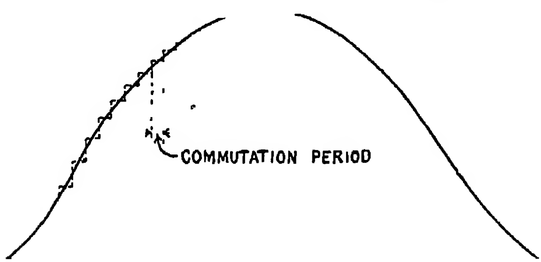


FIG 197.—To show that Reactance e.m.f is in Phase with Current.

* By some writers, the term *compensating winding* is applied to this winding.

then vary with the instantaneous value of the current, reaching a maximum value when the current is at its maximum, and vanishing when the current is passing through its zero value. In this sense, then, we may describe the reactance e.m.f. generated in a short-circuited coil by commutation as being *in phase* with the alternating current.

The transformer e.m.f. (represented by OA in the vector diagram of Fig. 193) lags slightly more than 90° behind the current.

Lastly, the rotation e.m.f. is necessarily in phase opposition to the magnetic flux.

§ 179. Methods of securing Sparkless Running

The problem of obtaining sparkless running resolves itself into that of causing the resultant of the reactance, transformer, and rotation e.m.f.s in the short-circuited coil either to vanish, or, at least, to assume a sufficiently small value. It is obvious that by making each of the three components very small, the resultant will necessarily be also small. This end is practically attained by (1) a very thorough subdivision of the winding, *i.e.* the use of as many commutator segments as possible; (2) the use of narrow carbon brushes, so that only one coil is short-circuited at a time. The first expedient

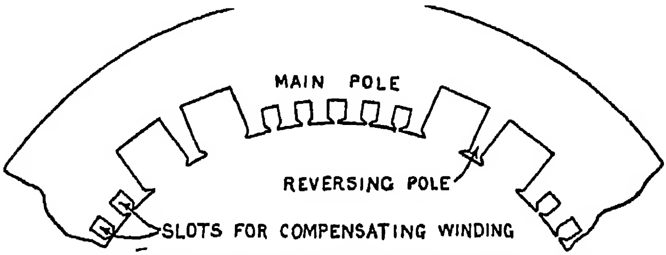


FIG. 198.—Stator Stamping showing Main and Reversing Poles, and Slots for Compensating Winding.

reduces all the e.m.f.s in the coil; the second reduces the reactance e.m.f., but has no effect on the transformer and rotation e.m.f.s.

The use of reversing or inter-poles, which has exerted so marked an effect on the design of continuous-current machines, has also been of immense importance in the problem of sparkless commutation in single-phase motors. In most forms of series-wound motor, a neutralizing winding is provided, so that armature reaction is practically suppressed. In addition to the neutralizing winding, commutating or inter-poles, arranged between the main poles, are

frequently provided, and are wound with series coils carrying the main current. In Fig. 198 is shown a stator stamping with salient main poles. inter-poles, and slots in the main poles to receive the neutralizing winding. Both the reactance and the rotation e.m.f. being directly proportional to the product of current × speed, and being practically in phase opposition, it is evident that by suitably adjusting the turns on the inter-pole, the rotation e.m.f. may be made to balance the reactance e.m.f. at all speeds. That such perfect balance is quite attainable has been amply proved by the remarkable results obtained with continuous-current machines, in which, of course, there is no transformer e m.f.

From the above it will be evident that the main disturbing factor in connection with sparkless commutation is the presence of the transformer e.m.f. in the short-circuited coil. Were it not for this transformer e.m.f., sparkless running at all speeds could be easily secured. The transformer e.m.f. is not directly dependent on the speed, but only on the current. Now the current taken by the motor decreases as the speed increases (as is evident from the circle diagram of Fig. 194), so that *with increasing speed the commutating properties of a series-wound motor fitted with inter-poles steadily improve.* Hence such a motor is well adapted for high speeds. Further, provided the speed exceeds the limit beyond which the transformer e.m.f. ceases to be troublesome, good commutation will be obtained over a wide range of speeds.

Even in the case of a motor not fitted with inter-poles, but provided with a neutralizing winding, and so designed that the reactance e.m.f. in the short-circuited coils is very small, good commutation at all speeds may be obtained within certain limits; for if the transformer e.m.f. ceases to be important beyond a certain speed, the reactance e.m.f. is the only one which need be considered,* and this increases but slowly with the speed, the product current × speed, on which the reactance e.m.f. depends, changing but slowly with the speed.

From the above it will be seen that the series-wound motor is well adapted for a variable speed service.

If, as is generally the case, the motor is required to exert a powerful torque at starting, the starting current must be large, the transformer e.m.f. high, and the sparking difficulty great. There is further the excessive heating of the short-circuited coils by the abnormally large currents induced in them. In order to meet these difficulties the use of high resistance connectors between the armature winding and the commutator segments has become very general

* There being no rotation e m f, since the armature cross-field is wiped out by the compensating coil.

in this type of motor. Such connectors are frequently placed at the bottom of the armature core slots—an arrangement adopted by Messrs. Ganz & Co. and the Westinghouse Co. A device invented by Richter * consists in substituting for the high-resistance connectors coils of thin copper wire arranged in suitable slots, so as to increase the torque of the motor.

With the ordinary type of inter-pole provided with a series winding, it is only possible to compensate the reactance e.m.f. of the short-circuited coil, while the transformer e.m.f., whose phase differs by about 90° from those of the reactance and rotation e.m.f.s, remains practically unaffected. The ideal arrangement would be one in which the rotation e.m.f. is equal in magnitude and opposed in phase to the resultant of the reactance and transformer e.m.f.s, so that the vector diagram of e.m.f.s in the short-circuited coil is as shown in Fig. 199. In order to secure this result, it is clear that the flux due to the reversing pole must have a phase differing from that of the main current, which is in phase with the reactance e.m.f. The requisite phase difference mny be obtained by means of an arrangement due to Richter and Latour,† which consists in providing the inter-pole with a compound winding; this consists of a series coil included in the main circuit, and a shunt coil connected in series with an adjustable reactance or choking coil, and then placed across the motor terminals. The flux due to the series coil will be practically in phase with the main current and reactance e.m.f., while that due to the shunt coil will be nearly in quadrature with the terminal p.d. By suitably adjusting the relative values of the two components, it is clear that the resultant flux, and hence the rotation e.m.f. due to it, may be made to have the required intermediate phase, corresponding to direct phase opposition to the resultant of the reactance and transformer e.m.f.s. ‡ In order to prevent any

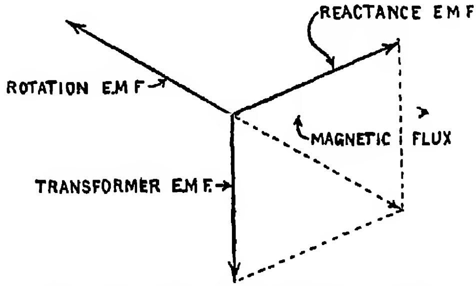


FIG. 199.—Ideal Vector Diagram of e.m.f.s in Short-circuited Coil.

* *Elektrotechnische Zeitschrift*, vol. xxvii. p. 537 (1906).

† *Elektrotechnik und Maschinenbau*, vol xxiv. p. 50 (1806)

‡ Assuming the power factor of the motor to be cos ϕ, the current in the series coil will lag by an angle ϕ behind the p.d., while the current in the shunt coil will lag by nearly 90° If the power factor is very low, compensation becomes difficult owing to the fact that the two components of flux will be nearly in phase with each other.

inductive action between the coils, which would tend to disturb the phase relation of the currents, the series winding is made to surround a single commutating tooth, while the shunt winding surrounds three teeth, as shown in Fig. 200.*

It is evident that although a reversing field influenced by two windings may be made to give excellent results at any particular load—such as full load—yet as soon as the load changes, the existing balance of e.m.f.s in the short-circuited coil will be disturbed, since, as already explained, the reactance and rotation e.m.f.s depend on

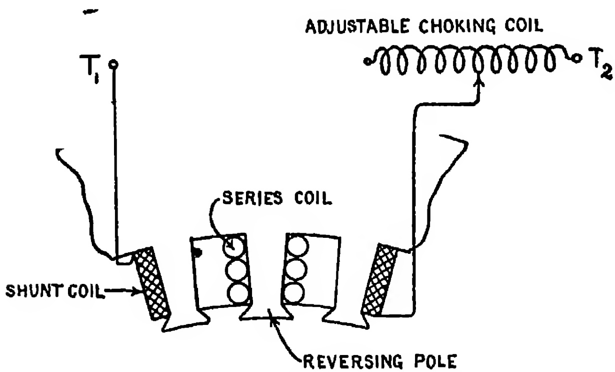


FIG. 200.—Richter's reversing pole with compound winding.

the product current × speed, while the transformer e.m.f. is simply proportional to the current. From this it will be seen that a decrease of load and consequent increase of speed will result in the transformer e.m.f. being over-compensated, while an increase of load and decrease of speed will cause the transformer e.m.f. to be under-compensated.

§ 180. Some Technical Data relating to Series Single-phase Motors

Since the width of carbon brush and commutator segment cannot be reduced below a certain limit, for purely mechanical reasons, it follows that, if the subdivision of the winding be carried as far as possible—*i.e.* with only one turn per coil—the number of armature conductors will be limited by the least possible width of segment. Hence, it will be impossible, for a given speed, to construct a motor beyond a certain voltage without overstepping the limitations imposed

* *Elektrotechnische Zeitschrift*, vol. xxvii. p. 543 (1906).

by considerations of sparkless running. For this reason, the limit of voltage for motors of this type is 300. From a constructional point of view, the armatures of series motors are characterized by the relatively large size of the commutator, whose diameter approaches that of the armature.

Since the power-factor and sparkless running are greatly improved by lowering the frequency, series-wound commutator motors have invariably been used on circuits of frequency not exceeding 25, and in some cases much lower frequencies have been used.*

The ampere conductors or ampere wires per cm. length of armature circumference vary from about 150 to 300. The ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$ lies between 0·5 and 0·85. The length of armature core is generally about equal to the pole-arc. The maximum value of the air-gap induction ranges from 4000 to 6000.

The output per pole-pair of large neutralized series motors is about 100 h.p.

The full-load power factor of a series motor is generally of the order 0·85, while over the usual working range it varies from about 0·8 to 0·95, increasing with decrease of load (*i.e.* with increase of speed), as explained in § 174.

The efficiency of single-phase series motors is appreciably less than that of continuous-current motors. The efficiency is of the order 0·8 for about a 40 h.p. motor, rising to 0·85 for a motor of 150 h.p.

If a series-wound motor is intended to operate on both continuous- and alternating-current circuits, then since, when using continuous current, it is desirable to work with a stronger field, the field winding may be divided into two sections, which are connected in series with each other for continuous-current, and in parallel for alternating-current operation.

The plain series type of motor is made in very small sizes ($\frac{1}{200}$ h.p. to $\frac{1}{15}$ h.p.) for driving fans, etc.

§ 181. Method of starting Single-phase Series Motors

The usual arrangement adopted for starting single-phase series motors consists in making use of an auto-transformer (§ 65). At starting, only a sufficient number of turns is included to give the required starting current. As the motor gains speed, the number of turns is gradually increased. A difficulty arises in this connection

* The Oerlikon Co. has used a frequency as low as 15. See *Elektrische Bahnen und Betriebe*, vol. iv. p. 16 (1906).

if it is desired to vary the number of turns without interrupting the supply. This difficulty is similar to that connected with an accumulator switch, and is overcome in a similar manner. Instead of connecting the movable motor cable directly to the controller contact, it is connected to the middle point of a choking coil, technically termed a *preventive coil.* The actual arrangement of connections is shown in Fig. 201, which represents the state of affairs when the controller handle is in its first position. In order to understand the action of the preventive coil, we may first suppose that the coil is bridged across a certain number of the transformer turns, as shown, but that

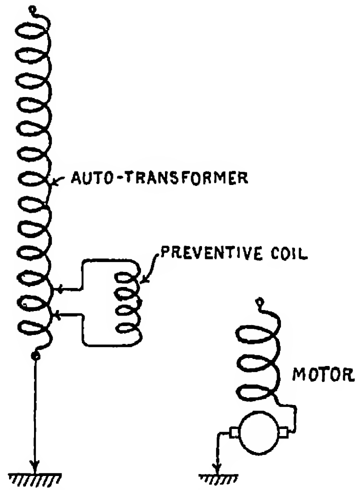


Fig. 201.—Connections of Auto-transformer and Preventive Coil.

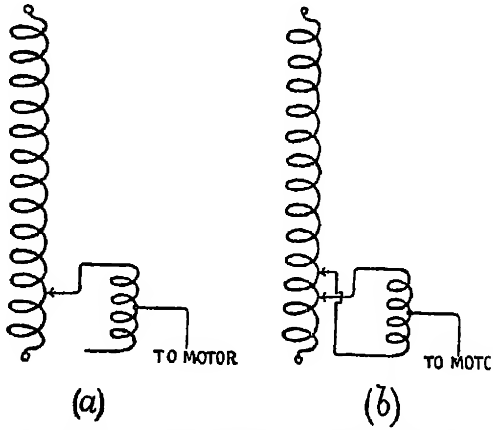


Fig. 202.—Showing method of using Preventive Coil.

the motor cable has been disconnected from its middle point. Owing to the high reactance of the preventive coil, it takes only a very small current—behaving like an ordinary transformer on open circuit—so that there is practically no load on the turns across which it is bridged. Let, now, the motor cable be connected to the middle of the coil. A current flows, from the lower point of contact, through the lower half of the coil to the motor. Now, this current will, by its inductive action, cause a practically equal current to flow through the upper part of the coil, the upper and lower halves behaving towards each other like the primary and secondary of a transformer. The preventive coil is, in fact, a small auxiliary auto-transformer bridged across a section of the main transformer. The p.d. impressed on the motor will be the mean of the p.d.s corresponding to the

points of connection of the preventive coil. In increasing the number of turns, the operations shown in Fig. 202 are gone through. The lower end of the coil is first disconnected momentarily, so that the entire current supplied to the motor flows through the upper part of the preventive coil (as shown in Fig. 202 (*a*)). During this period the upper half of the coil acts as a powerful choking coil, and there is a slight drop in the current. Immediately afterwards, however, a new connection is established between the lower end of the preventive coil and a point higher up in the auto-transformer winding, as shown in Fig. 202 (*b*), a rise of voltage being thereby obtained. By means of this hand-over-hand process, the preventive coil, as it were, gradually climbs up the main transformer winding, the voltage increasing by a definite amount at each step.

CHAPTER XVIII

§ 182. Mechanical forces exerted between mutually inductive circuits—§ 183. Repulsion motor—§ 184. Vector diagram of repulsion motor—§ 185. Ratio of two components of armature flux as depending on speed—§ 186. Conditions for sparkless commutation in repulsion motor—§ 187. Atkinson's repulsion motors—§ 188. Compensated repulsion motor—§ 189. Theory of Latour-Winter-Eichberg motor—§ 190. Vector diagrams of Latour-Winter-Eichberg motor—§ 191. Series-repulsion motor—§ 192. Mixed-action commutator motors.

§ 182. Mechanical Forces exerted between Mutually Inductive Circuits

WHEN dealing with mutual inductance (§ 11), we investigated the electromagnetic effect which the secondary had on the primary (§ 12), and we found that when the secondary contained no impressed e.m.f., it reduced the apparent self-inductance and increased the apparent resistance of the primary. In addition to the purely electromagnetic effect due to the secondary, there is an important mechanical or dynamical effect produced by it, which was first studied in detail by Professor Elihu Thomson, of the United States, and which led to the invention by him of the *repulsion type* of single-phase commutator motor.

Consider two mutually inductive circuits, one of which (the primary) is supplied with alternating current from some suitable source, while the other (the secondary) is simply closed on itself. It is evident that since there are currents circulating in both circuits, there will in general be dynamical forces exerted between them. We proceed to investigate the nature of these forces. In doing so, we shall find it convenient to employ the hypothetical flux method (§ 11).

The primary hypothetical flux induces in the secondary circuit an e.m.f. which is in quadrature with the flux, lagging 90° behind it. This e.m.f. gives rise to a current in the secondary which—owing to the self-inductance of the secondary—lags behind the e.m.f. by an angle θ, such that $\tan\theta = \dfrac{\text{secondary reactance}}{\text{secondary resistance}}$. Hence the phase difference between the primary and secondary currents is $90° + \theta$,

i.e. it exceeds 90° (*cf.* Fig. 19). We may suppose the secondary current resolved into two components, one of which is in quadrature with the primary current, while the other is in direct phase opposition to it. Now the dynamical stress between the two circuits is at any instant proportional to the product of the currents, and the mean stress is, for a given relative position of the circuits, proportional to the mean value of the product. If we consider the stresses arising from the two components of the secondary current, then it is clear that the quadrature component contributes nothing towards the stress (since the mean product of this component into the primary current vanishes), so that the stress is due entirely to the component in direct phase opposition to the primary current. Owing to this phase opposition, the circuits will *repel* each other. If the secondary circuit is prevented from having a motion of translation, but is mounted in such a manner as to be free to rotate, it will tend to set itself so as to make the magnetic axes of the two circuits perpendicular to each other; supposing the motion to be allowed to take place, the torque acting on the moving circuit will gradually decrease and vanish when the circuit reaches a position in which no magnetic flux passes through it—*i.e.* a position in which the magnetic axes of the circuits are perpendicular to each other. The position is one of equilibrium, for if we suppose the circuit still further displaced in the same direction, the induced current will undergo a phase change of 180°, and the resulting torque will oppose the displacement.

It thus appears that *a closed secondary circuit placed near a primary conveying an alternating current will tend to move so as to reduce the magnetic flux passing through it to a minimum.* The motion if allowed to take place may be a motion of translation, or of rotation, or a combination of both.

Numerous striking instances of such *electromagnetic repulsion* have been studied by Professor Elihu Thomson and others. If a ring of conducting material be placed around the upper end of the core of an alternating current electromagnet held in a vertical position, the ring is projected upwards as soon as the current is turned on. Again, if an alternating current electromagnet be placed with its axis horizontal, and a disc of copper be suspended in front of it by a string attached to a point near its edge (so that the disc hangs with its plane vertical), then on switching on the current the disc will both swing away from the magnet and turn so as to make its plane parallel to the magnetic field.

The effects we have just been considering receive an important application in the type of single-phase motor known as the *repulsion motor*.

§ 183. Repulsion Motor

The repulsion type of single-phase commutator motor was originally invented by Professor Elihu Thomson in 1887. In its simplest form, it is shown in Fig. 203. T_1 and T_2 are the terminals of the field winding, which is entirely disconnected from the armature. The brushes are displaced from the neutral position through a certain angle, and are short-circuited. The two parallel halves of the armature winding, together with the short-circuiting cable connecting the brushes, form a closed conducting circuit. If an alternating current be sent through the field winding, the closed circuit of the armature, being placed in an alternating field, will, in accordance with the principles just considered, tend to move so as to render the alternating flux through it a minimum. Now the algebraical sum of the alternating fluxes through the various turns of the armature winding will clearly be zero if the brushes be placed in the neutral position. Hence, with the brushes displaced as shown in Fig. 203, it is evident that, if for a moment we suppose the brushes to be rigidly attached to the commutator and free to revolve with it, the armature and brushes would move bodily in a *clockwise* direction (this motion resulting in a decrease of the algebraical sum of the fluxes through the various turns of the winding) until the neutral position was reached, when the algebraical sum of the fluxes through the armature coils would vanish, and with it also the torque. If, however, we fix the brushes in their original displaced position, the armature will move, and, since the brushes retain their position, the torque will be maintained, and *continuous rotation* will result.

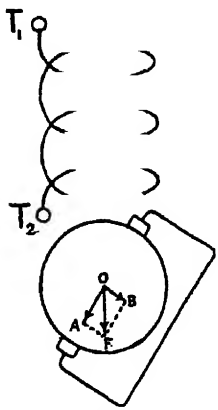


FIG. 203.—Repulsion Motor.

We may here at once note one of the results due to the short-circuiting of the coils as they pass under the brushes. It will be seen that whereas the magnetic axis of the armature winding *as a whole* is along the line joining the brushes, the magnetic axis of a coil undergoing short-circuit by a brush is at right angles to the line of brushes, the plane of the coil being parallel to this line. Hence regarding the short-circuited coil as an independent closed circuit placed in an alternating magnetic field, it follows by the principle dealt with in § 182 that this coil will tend to move so as to bring its plane into parallelism with the field (or its magnetic axis at right angles

to the field). From this we see that the coils short-circuited by the brushes exert an *opposing torque.* We have, in fact, to deal with two independently short-circuited circuits in the case of the armature of a repulsion motor; one of the circuits, which produces the driving torque, consists of all the armature coils not under cover of the brushes, and short-circuited *en masse* by the cable connecting the brushes; while the other, as already explained, consists of the coils independently short-circuited by the brushes, and giving rise to a torque opposed to the torque developed by the winding as a whole.*

We here come across one of the differences between the series and repulsion types of commutator motor. In the series motor the currents due to the transformer e.m.f. in the short-circuited coil do not directly affect the torque (§ 174), since the brushes are in the neutral position. But in the repulsion motor, the transformer e.m.f. gives rise to currents producing an opposing torque.

Since the armature is entirely disconnected from the field, the latter may be wound for a very high voltage—up to 6000 volts—a step-down transformer being thereby dispensed with. This is an advantage which the repulsion motor possesses over the series motor, as the latter cannot be wound for voltages above 300.

The value of the torque will clearly depend on the angle through which the brushes have been displaced from the neutral line. It is easy to see that there are two extreme positions for which the torque vanishes. One of these is along the neutral line, and the other at right angles to it. Between these two, there is a certain position corresponding to maximum value of the torque. This position is found in practice to correspond to a displacement of the brushes from the neutral line through about 70°.

§ 184. Vector Diagram of Repulsion Motor

The total flux OF (Fig. 203) passing through the armature may be resolved into two components, one of which, OA, is along the line of brushes, while the other, OB, is at right angles to it. The "transformer e.m.f." (or e.m.f. due to alternations of flux) in the armature is due entirely to the component OA; while the "rotation e.m.f." arising from the rotation of the armature in the field is due

* If the cable connecting the brushes be removed, the torque due to the winding as a whole vanishes, and we are left merely with the torque due to the independently short-circuited coils under the brushes. The motor will, therefore, now rotate in the *opposite* direction Various types of motor constructed on this principle have been experimented upon, but none appears to have come into general use.

entirely to OB. Again, the ampere-turns due to the armature can only affect the component OA, which will therefore be nearly in phase with the resultant of the field (or primary) and the armature (or secondary) ampere-turns. But since the armature current is nearly in opposition of phase to the field current, the resultant ampere-turns will be nearly in quadrature with both armature and field currents. Thus the flux component OA will be nearly in quadrature with the field current. The component OB, on the other hand, which is uninfluenced by the armature current, will be nearly in phase with the primary current. Hence the two components OA and OB will be nearly in quadrature, as represented in Fig. 204.

In the armature we have two e.m.f.s. One of these, denoted by E_t in Fig. 204, is due to transformer action, and lags 90° behind the flux OA; while the other, E_r, is due to rotation of the armature in the flux OB, and is therefore in phase with OB. The vectorial addition of E_t and E_r gives us the resultant armature e.m.f. The armature current will lag behind this resultant e.m.f. by a certain angle β, whose value depends on the armature leakage flux. In order to maintain the flux OA, a given number of resultant ampere-turns will be required, which will be slightly in advance (owing to hysteresis and eddy-current losses) of OA. By adding to the resultant ampere-turns, the armature ampere-turns with the sign reversed, we obtain the primary or field ampere-turns. The flux OB will lag slightly behind the field current.

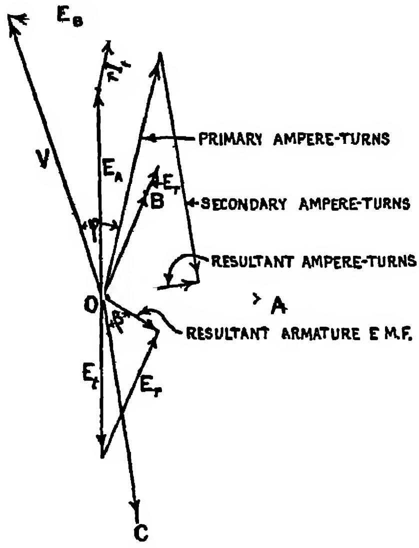


FIG. 204.—Vector Diagram of Repulsion Motor.

The e.m.f. polygon for the field winding is now easily constructed. The flux OA induces an e.m.f. in the field winding, which lags 90° behind OA, and which has to be balanced by a component E_A of the impressed p.d. To balance the resistance drop, we require a component rI_1 in phase with the field current, r being the field resistance. Lastly, to balance the e.m.f. induced by the flux OB, and by the field leakage flux, a component E_B in quadrature with this flux

will be required. The resultant of E_A, rI_1, and E_B, gives us the total impressed p.d., V. The power factor is given by $\cos\phi$.

When the motor is at rest, the rotation e.m.f. E_r vanishes, so that the resultant armature e.m.f. becomes identical with E_t, and the armature current vector will lie in the third quadrant in Fig. 205, to the left of E_t. As a result, ϕ will be large, and $\cos\phi$ small. With increasing speed, the resultant e.m.f. vector will rapidly swing forward in a counter-clockwise direction, and the power factor will rapidly increase.

§ 185. Ratio of Two Components of Armature Flux as depending on Speed

We shall now calculate approximately the ratio of the two components, OA and OB (Fig. 203), of the armature flux, assuming that these are in quadrature with each other, and that they are distributed around the armature circumference according to the sine law.

Considering the flux OA, let the origin be taken at one of the brushes. Then the instantaneous value of the air-gap field at time t and at a point distant x from the origin may be written as—

$$B_A \sin\frac{2\pi}{T}t \cdot \cos\frac{2\pi}{\lambda}x$$

where B_A is the amplitude of the alternating flux wave (§ 20), and λ its wave-length (so that $\frac{1}{2}\lambda$ is the pole-pitch). Using this expression, we can easily calculate the average instantaneous e.m.f. e_t per cm. length of coil due to the transformer action of the flux OA. Consider the coil, one of whose sides is at a distance x_1 from the origin. The flux linked with this coil, per cm. length of it, is given by *—

$$B_A \sin\frac{2\pi}{T}t\int_{x_1}^{x_1+\frac{\lambda}{2}}\cos\frac{2\pi}{\lambda}x \,.\, dx = -\frac{\lambda}{\pi}B_A\sin\frac{2\pi}{T}t \,.\, \sin\frac{2\pi}{\lambda}x_1$$

and hence the average instantaneous flux per cm. length of coil is †—

$$-\frac{2\lambda}{\pi^2}B_A\sin\frac{2\pi}{T}t$$

since the average value of $\sin\frac{2\pi}{\lambda}x_1$ over a pole-pitch is $\frac{2}{\pi}$ (§ 3). The

* The span or width of coil being assumed equal to $\frac{1}{2}\lambda$.

† The average being taken over a pole-pitch, or over a set of coils included between two neighbouring brushes.

average instantaneous transformer e.m.f. per cm. length of coil is thus—

$$e_t = -\frac{d}{dt}\left(-\frac{2\lambda}{\pi^2}\mathrm{B_A}\sin\frac{2\pi}{\mathrm{T}}t\right) = \frac{4\lambda}{\pi\mathrm{T}}\,.\,\mathrm{B_A}\cos\frac{2\pi}{\mathrm{T}}t \quad . \quad . \quad (1)$$

Taking next the flux OB, choosing as before our origin for x at one of the brushes, and remembering that OB is 90° ahead of OA as regards phase, we may write the air-gap induction at a point x from the origin as—

$$\mathrm{B_B}\cos\frac{2\pi}{\mathrm{T}}t\,.\,\sin\frac{2\pi}{\lambda}x$$

$\mathrm{B_B}$ denoting the amplitude of the flux wave. The coil, one side of which is at the point x from the origin, is moving across a field having the above intensity. The average field intensity for all the coils within a pole-pitch is given by—

$$\frac{2}{\pi}\mathrm{B_B}\cos\frac{2\pi}{\mathrm{T}}t$$

since the average value of $\sin\frac{2\pi}{\lambda}x$ over a pole-pitch is $\frac{2}{\pi}$. Now if v denote the peripheral velocity of the armature, then the average instantaneous rotation e.m.f. e_r per cm. length of coil is—

$$e_r = \frac{4v}{\pi}\mathrm{B_B}\cos\frac{2\pi}{\mathrm{T}}t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

since each coil consists of two conductors, each of which is cutting lines at the same rate.

Now, under normal working conditions, the drop due to armature resistance and leakage reactance is small in comparison with the transformer e.m.f. Hence the transformer and rotation e.m.f. s in the armature will be nearly equal at all speeds within the working range. For the sake of simplicity, we shall take the ideal case in which the resistance and reactance drops are quite negligible, and in which, therefore, e_t must exactly balance e_r. Then it follows, from equations (1) and (2), that—

$$\frac{\lambda}{\mathrm{T}}\mathrm{B_A} = v\mathrm{B_B}$$

Now $\frac{\lambda}{\mathrm{T}}$, the quotient of the wave-length (or twice the pole-pitch) by the period, represents the speed of synchronous rotation. Let this be denoted by v_s. Then our equation becomes—

$$\mathrm{B} = \frac{v}{\ }\mathrm{B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or the ratio of the component OA of the flux (Fig. 203) to the component OB is equal to the ratio of the actual speed to that of synchronism. With increasing speed, therefore, the component OA steadily increases relatively to the component OB.

The case of synchronous speed is of particular importance. Since, in this case, v becomes equal to v_s, we see from (3) that $B_A = B_B$, or the two flux components become equal. Now, two alternating magnetic flux waves of the same period and amplitude, and displaced relatively to each other by a quarter wave-length, give rise, as shown in § 22, to a pure rotating wave of magnetic flux. We thus see that *at synchronous speed the armature flux of the repulsion motor is represented by a pure rotating wave.* The great importance of this result will be understood when we consider the commutation in a repulsion motor (§ 186).

The results just established are based on approximate assumptions only, the armature resistance and leakage reactance drops having been neglected. Since these are not entirely negligible, the transformer e.m.f. in the armature will always be in excess of the rotation e.m.f., but with decreasing current and increasing speed the difference becomes less, owing to the decreased impedance drop in the armature.

In the vector diagram of Fig. 204 we have, for the sake of simplicity, neglected the effect of any currents which may be induced by transformer action in the short-circuited coils under the brushes. It is clear that if the conditions of operation are such as to give rise to such currents, these latter will tend to reduce the flux component OB (Fig. 203), to which they are due, and, in order to balance their effect, an additional component will be required in the primary current. As in the case of the series motor (§ 172), the effect of such currents is to increase the power factor and lower the efficiency of the motor; but in the repulsion motor a further effect is produced—viz. a lowering of the driving torque (§ 183).

§ 186. Conditions for Sparkless Commutation in Repulsion Motor

In the short-circuited coils of a repulsion motor, as in those of a series motor, we have three distinct e.m.f.s, viz. the reactance, the transformer, and the rotation e.m.f. The reactance e.m.f. is in phase with the armature current, and may be rendered small by the methods explained in connection with the series-wound motor (§ 179)—thorough subdivision of the winding, and the use of narrow brushes. The transformer e.m.f. is due solely to the component OB of the armature flux, as is at once evident by a reference to Fig. 204;

his e.m.f. is, therefore, in quadrature with OB, and lags 90° behind it. Since, however, OA lags nearly 90° behind OB (Fig. 204), the transformer e.m.f. in the short-circuited coil will be approximately coincident in phase with OA. On the other hand the rotation e.m.f. in the short-circuited coil is due entirely to the component OA of the flux (Fig. 203), and, by Lenz's law, is in direct phase opposition to it. From this follows the highly important result that *in a repulsion motor the transformer and rotation e.m.f.s in a short-circuited coil are nearly in direct phase opposition*, and hence will more or less completely neutralize each other.

It will be noted, therefore, that the phase relation of the transformer and rotation e.m.f.s in the short-circuited coil of a repulsion motor is entirely different from that which occurs in a series motor (§ 179), where these two e.m.f.s are nearly in quadrature, rendering it impossible, without the use of special devices (such as poles with a compound winding) to obtain any neutralization of the one e.m.f. by the other.

If the conditions are such that the rotation e.m.f. completely neutralizes the transformer e.m.f., the commutation will be as good as in a continuous-current motor. Now, this state is practically reached at synchronism. For at synchronous speed a rotating wave of flux is produced (§ 183), and it is evident that, since the armature is *stationary* with respect to this rotating flux wave (both travelling at synchronous speed), the only e.m.f. in the coils undergoing commutation is the reactance e.m.f.

The speed of synchronism is, however, the only speed at which exact neutralization of the transformer and rotation e.m.f.s in the short-circuited coil takes place. At speeds below synchronism, the component OB of the armature flux predominates (equation (3) of § 185), and the transformer e.m.f. overpowers the rotation e.m.f.; while at speeds above synchronism, the opposite effect takes place. The range of practically sparkless commutation in a repulsion motor is thus limited to a comparatively narrow range of speed on either side of the speed of synchronism. Hence, a repulsion motor is best adapted for running at a fairly constant speed in the immediate neighbourhood of synchronism. In this respect it differs from the series motor, which will run well over a wide range of speeds, provided the speed exceeds a certain limit (§ 179). Another respect in which the repulsion motor differs from the series is that it may be constructed for much higher frequencies—up to 60—owing to the possibility of neutralizing, more or less perfectly, the transformer e.m.f. in the short-circuited coils by a rotation e.m.f.

Plain repulsion motors made by the General Electric Co. of the United States are used to drive exhaust fans or as crane motors.*

* *General Electric Review*, vol. xx. p. 493 (1917).

§ 187. Atkinson's Repulsion Motors

A form of repulsion motor, due to Atkinson, is represented in Fig. 205. So far as the principle of its action is concerned, this motor is identical with the simple form shown in Fig. 203, the only difference being that, whereas in the simple form the two components OA and OB of the impressed field (Fig. 203) are produced by a single winding, in Atkinson's motor they are produced by two independent windings—the transformer winding TW and the exciting winding (or field winding proper) EW. The behaviour of this motor is in every way identical with that of the simple type of Fig. 203; it, however, possesses the advantage that the direction of rotation is easily reversed, by a simple reversal of the connections of one of the stator windings. In order to reverse the simple form of repulsion motor, it becomes necessary to rock the brushes to the other side of the line OF of the impressed field (Fig. 203).

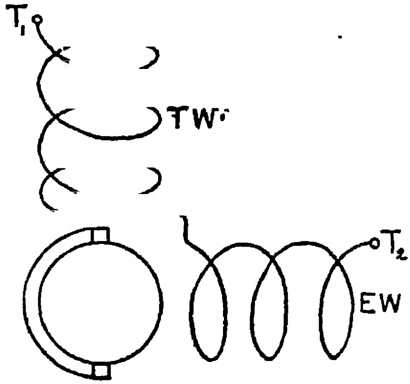


FIG. 205.—Connections of Atkinson's Motor.

A variety of Atkinson's motor is one in which only the transformer winding TW is connected across the mains, the exciting winding EW being across the brushes. The exciting winding then forms a load for the armature winding, regarded as the secondary of a transformer, of which TW is the primary.

§ 188. Compensated Repulsion Motor

The so-called "compensated repulsion motor" was invented simultaneously and independently by Latour in France and by Eichberg and Winter in Germany.* In its simplest form it is

* By some writers, this motor has been termed the compensated series motor. Mr. Fynn has proposed the term "series induction motor" as being more appropriate. See *Journal of the Institution of Electrical Engineers*, vol. xxxvi p. 328 (1906).

shown in Fig. 206. At first sight it would appear as if the motor, differed but little from the simple series-wound motor of Fig. 192. In fact, the removal of the brushes B_1, B_2 would transform it into such a simple series motor. But the presence of these brushes considerably modifies the action of the motor. One effect is to wipe out the self-inductance of the stator winding to a very large extent, since the current flowing between B_1 and B_2 causes the armature winding to act as the short-circuited secondary of a transformer, of which the stator winding is the primary. The compensated repulsion motor is characterized by a very high power factor for all speeds above synchronism, and by the absence of sparking troubles.

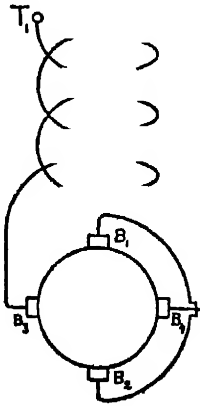


FIG. 206.—Diagram of Compensated Repulsion Motor.

In the simple form of compensated repulsion motor just considered, the stator could not be wound for a high voltage, as it is connected in series with the armature. By a slight modification, however, indicated in Fig. 207, the use of a high voltage stator winding becomes possible. This is the arrangement actually adopted in the Latour-Winter-Eichberg motor. The current corresponding to the brushes B_3, B_4 in Fig. 206, instead of being fed directly into the armature, is supplied to it by the secondary of a transformer whose primary is in series with the stator winding. The secondary of this transformer is, as shown in the sketch, arranged to have a variable number of turns.

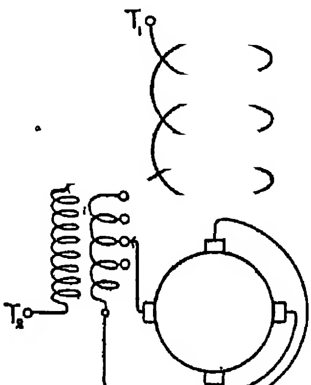


FIG. 207—Connections of Latour-Winter-Eichberg Motor.

§ 189. Theory of Latour-Winter-Eichberg Motor

The stator winding acts as the primary of a transformer towards the circuit closed by the brushes B_1, B_2 in Fig. 206, inducing a current in this circuit current. The primary and secondary currents will, as in a transformer, be nearly in phase opposition.

Under normal working conditions, the secondary circuit (*i.e.* the

circuit closed by the brush pair B_1B_2) will be the seat of two e.m.f.s which, by reason of the smallness of the resistance and leakage reactance drops in this circuit, will nearly balance each other. One of these e.m.f.s is the transformer e.m.f., due to the alternating flux common to stator and rotor; while the other is a rotation e.m.f., due to the rotation of the armature in the field produced by the current flowing between B_3 and B_4. Now, so far as the magnitude and phase of the secondary current are concerned, we may imagine that the rotation e.m.f. is annulled, and that for it there is substituted between B_1 and B_2 a non-inductive resistance (in place of the short-circuiting cable), such that, with the given armature current, it will absorb a p.d. exactly equal to the rotation e.m.f. This will leave the magnitude and phase of the armature current unaltered, since the rotation e.m.f. is necessarily nearly in phase with the stator current, and hence also with the armature current. By means of this device, we have transformed the stator and armature circuits into the primary and secondary respectively of a transformer working on a *non-inductive* load. Under these conditions, the power factor of the transformer is nearly unity (§ 92). Thus the power factor of the stator winding will be nearly unity.

Considering now the circuit between the brushes B_3 and B_4, we have in it an e.m.f. induced by the alternating flux, this e.m.f. being in quadrature with the flux, and hence nearly in quadrature with the current. Now, if this were the only existing e.m.f., it is obvious that the power factor of the B_3B_4 circuit would be extremely low—this circuit forming simply a choking coil. There is, however, another e.m.f. present, due to the rotation of the armature in the flux along the line B_1B_2—*i.e.* the transformer flux common to stator and rotor. Now, this e.m.f. is in phase opposition to the transformer flux, and in quadrature with the stator current. We thus have two e.m.f.s present in the B_3B_4 circuit, both of which are in quadrature with the current. Now, these e.m.f.s are (like the two e.m.f.s in the armature circuit) in phase opposition, the rotation e.m.f. tending to neutralize or compensate the reactance e.m.f. At low speeds the compensation is imperfect, the rotation e.m.f. being too feeble. But as the speed increases, the rotation e.m.f. rises, and at a certain speed completely wipes out the reactance e.m.f. The compensating circuit B_3B_4 then behaves as if it were a non-inductive resistance, and its power factor rises to unity. But since the power factor of the stator winding is also nearly unity, we see that the power factor of the entire motor will have the same value. At still higher speeds, over-compensation will take place, the power factor will decrease, and the motor will take a *leading* current.

From what has been said it follows that when the motor is at rest, and a suitable starting p.d. is applied to it, this p.d. is mainly

concentrated on the circuit B_3B_4, owing to its high reactance; the stator or transformer winding behaving like the primary of a short-circuited transformer (*i.e.* like a practically non-inductive resistance). The power factor at starting will thus be low. As soon as the motor begins to run, however, the p.d. across the stator winding increases, while that across B_3B_4 decreases, the power factor rising rapidly at the same time.

§ 190. Vector Diagrams of Latour-Winter-Eichberg Motor

The relations explained in the preceding paragraph will be more easily followed by a consideration of the vector diagram connecting the various quantities.

In Fig. 208 is shown the vector diagram relating to the transformer axis (*i.e.* the brush line B_1B_2 in Fig. 206). OA represents the transformer flux common to stator and rotor, while OB is the e.m.f. in the B_1B_2 circuit induced by the rotation of the armature in the field along the brush line B_3B_4 (Fig. 206). OA and OB must, as we have seen (§ 189), be nearly in quadrature with each other. The transformer e.m.f. in the armature winding lags 90° behind OA, and when compounded with the rotation e.m.f. OB, which is in phase with the flux along B_3B_4, yields the resultant e.m.f. along B_1B_2. This latter gives rise to a current, which, owing to the leakage reactance of the armature, lags behind the resultant e.m.f. by a certain angle.

For the sake of completeness, the current along the brush line B_3B_4 of Fig. 206 is also indicated in the diagram of Fig. 208. This current, which is identical with the stator current, is nearly in phase with the flux along B_3B_4, being but slightly in advance of it by an angle depending on the core loss due to this current.

In order to represent correctly by means of a second diagram the phase relations along the brush line B_3B_4 of Fig. 206, it becomes necessary to examine a little more closely into the phase relation of the rotation e.m.f. along this axis to the transformer flux. For this purpose, we may consider the changes taking place in the various quantities, while the transformer flux vector moves in the first quadrant, the projections of the various vectors on the vertical axis giving, as usual, the instantaneous values of the quantities. The instantaneous transformer flux during the quarter-period under consideration will clearly steadily increase, and will have a positive value. Let, in Fig. 209, a *downward* direction for this flux, as shown by OA, be regarded as *positive*. Let us assume a clockwise direction of rotation for the armature, as shown by the curved arrow

in Fig. 209. Since the transformer flux is increasing, the current induced by it will oppose it (in accordance with Lenz's law), and hence will have the direction indicated by the inner circle of conductors, in which a dot and a cross represent, as usual, an advancing and a receding current respectively. In accordance with the vector diagram of Fig. 208, the direction of the secondary current shown in Fig. 209 must be regarded as negative, since the projection of the armature-current vector on the vertical axis is negative during the greater part of the ½-period considered. Now, in order that the torque may have a positive or clockwise direction when the transformer flux

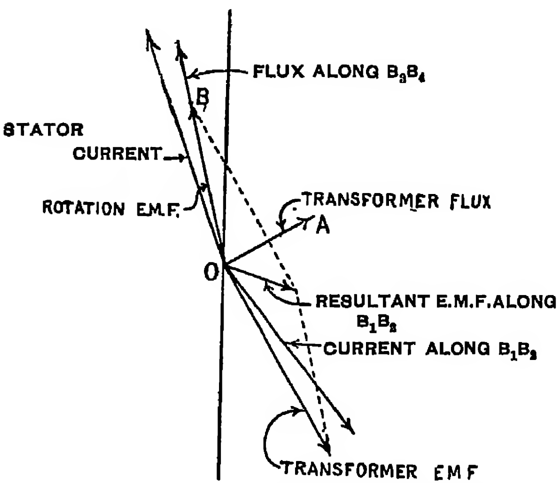


FIG. 208.—Vector Diagram for Transformer Axis of Latour-Winter-Eichberg Motor.

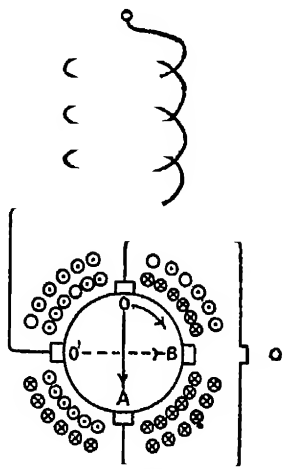


FIG. 209.—Distribution of Instantaneous Currents in Armature of Latour-Winter-Eichberg Motor.

has the direction OA, the current along B_3B_4 must have the direction indicated by the outer circle of conductors, and must, in accordance with Fig. 208, be reckoned *positive*. Now, the rotation of the armature in the transformer flux OA, will give rise to an e.m.f. in the B_3B_4 circuit which is *opposed* to the instantaneous current, and must, therefore, be reckoned *negative*. In order, then, to represent the rotation e.m.f. in the B_3B_4 circuit correctly in the diagram of Fig. 208, the vector of this e.m.f. must be drawn so as to be in *phase opposition* to OA. This has been done in Fig. 210 (*a*), where OR is the rotation e.m.f. vector.

The remaining quantities to be considered along the B_3B_4 axis are now easily dealt with. The reactance e.m.f. OT lags 90° behind the stator current OS. The resultant of the rotation and reactance e.m.f.s is represented by OE. In order to balance this, a component OF must be provided by the p.d. across the B_3B_4 brushes, and in order to make up for the resistance drop a further small component, FG, in phase with the exciting current, must be provided. The total p.d. across the B_3B_4 brushes is thus OG, and the power factor of this part of the circuit is cos χ.

In Fig. 210, the rotation e.m.f. is shown smaller than the reactance e.m.f., and the current OS lags behind the p.d. OG. Compensa-

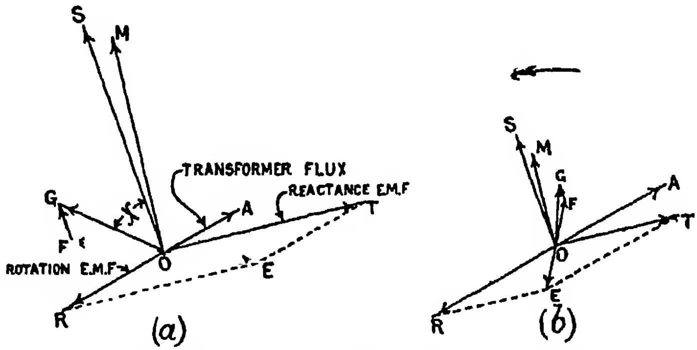


FIG. 210.—Vector Diagrams for B_3B_4 Axis of Latour-Winter-Eichberg Motor.

tion is here imperfect. As the rotation e.m.f. increases, χ decreases and finally vanishes, giving a power factor of unity for the B_3B_4 circuit. A still further increase in the rotation e.m.f. relatively to the reactance e.m.f. causes the current to lead, as shown in Fig. 210 (*b*), the power factor decreasing. Fig. 210 (*b*) corresponds to *over-compensation,* and shows clearly how a motor of this type may be made to take a leading current when running above a certain speed.

The power factor of the motor depends almost entirely on that of the B_3B_4 part of the circuit, since the stator part always has a power factor closely approaching unity, as already explained (§ 189).

A glance at Fig. 209 will explain why the Winter-Eichberg motor has been termed a compensated "repulsion" motor. It will be noticed that a superposition of the two armature currents results in a more or less complete neutralization of the currents in the belt of conductors lying in the first and third quadrants, while addition of the two currents takes place in the belt occupying the second and fourth quadrants. Hence the torque is practically provided by

this last belt of conductors, which is equivalent to a coil inclined at an angle of 45° to the inducing field provided by the stator winding. Such a coil will, in accordance with the principle of "electromagnetic repulsion" (§ 182), tend to place itself with its plane parallel to the inducing field—*i.e.* it will give rise to a torque in a clockwise direction.

§ 191. Series-Repulsion Motor

A type of motor which has to some extent been used for traction purposes is the *series-repulsion* or *doubly-fed* motor, represented diagrammatically in Fig. 211. T_1, T_2 are the secondary terminals of the transformer supplying the motor. The junction of the stator and armature circuits is connected to an intermediate point of the transformer winding whose position may be varied. If this point is made to coincide with T_2, we obtain an ordinary repulsion motor, and the machine may be started as such. With increasing speed the ratio of the stator to the armature voltage is reduced (as is the case with a plain series motor), and the machine begins to approximate more and more closely to a series motor. Owing to this fact, there is no difficulty (such as arises with a plain repulsion motor) in running the motor at speeds above synchronism.

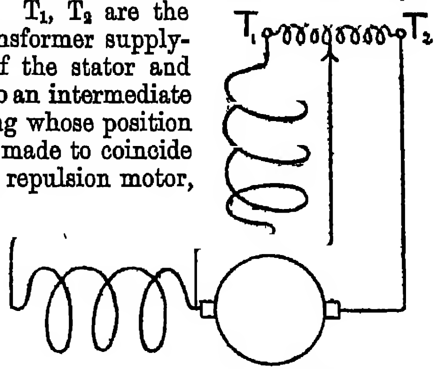


Fig. 211.—Series-Repulsion Motor.

§ 192. Mixed-action Commutator Motors

The single-phase commutator motors which we have so far considered are chiefly of interest in connection with railway work; they are variable-speed traction motors. For stationary work requiring approximately constant speed, the simple single-phase induction motor is capable of giving good results. The inherent weakness of this motor is its poor starting torque. Now, the repulsion motor is capable of exerting a powerful torque. If the good starting qualities of the repulsion type of motor could be combined with the satisfactory running qualities of the induction type, a motor would be obtained satisfactory from every point of view. Several forms of such compound or mixed-action commutator motors have been devised.

In the Wagner motor,* the rotor is provided with an ordinary continuous-current winding having a commutator, and at starting the brushes short-circuit the winding as in a repulsion motor. A powerful starting torque is thereby obtained. When the speed exceeds a certain limit, a centrifugal governor mounted on the motor shaft causes a collar to slide against the brush rocker ring, lifting the brushes clear of the commutator; † while at the same time a short-circuiting ring is forced into contact with the inner surfaces of the commutator segments, completely short-circuiting the individual armature coils, and thereby converting the motor into a single-phase, constant-speed induction motor.

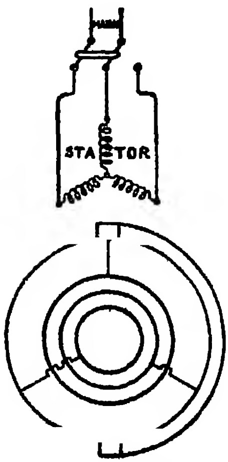


FIG. 212.—Diagram of Schüler Motor.

The Schüler motor (Fig. 212) is provided with an ordinary star-connected stator winding, of which only two phases (in series with each other) are in use at a time. By interchanging one of the active phases and the idle phase (by means of the switch shown in the diagram) the motor may be reversed. The rotor has a continuous-current winding, fitted with a commutator on one side and three slip-rings on the other. At starting, the slip-ring circuits are open, and the motor starts as a simple repulsion motor. By closing the slip-ring circuits through starting resistances such as are ordinarily used in connection with three-phase induction motors, and gradually short-circuiting the resistances, the motor is transformed from the repulsion to the induction type.

* *The Electrician*, vol. li. p. 743 (1903).

† The commutator bearing surface is vertical, forming an annular ring; a displacement of the brush rocker along the shaft will, therefore, move the brushes away from the commutator.

CHAPTER XIX

§ 193. Voltage regulators—§ 194. Instrument transformers—§ 195 Induction type measuring instruments—§ 196. Power factor meters—§ 197. Frequency meters—§ 198. Unbalanced systems and unbalance factor—§ 199. Power factor of unbalanced system—§ 200 Phase converters.

§ 193. Voltage Regulators

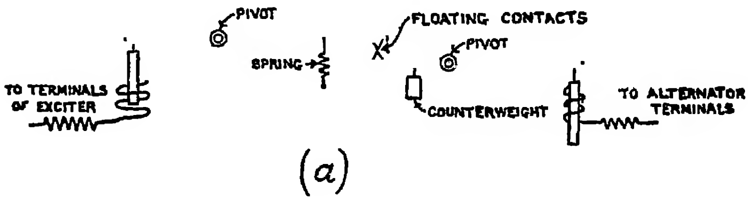
In the early days of alternating currents, the usual practice was to construct alternators having very close regulation, so as to avoid the necessity of using special devices for maintaining the voltage constant. Such machines were very expensive; and as the size of the generating units steadily increased, it was found that generators having very close regulation were liable to be wrecked by an accidental short-circuit, owing to the enormous short-circuit currents and resulting mechanical stresses (§§ 50 and 111). On account of these two disadvantages, modern generators are designed to have a relatively large reactance drop; and voltage regulation is secured by the use of devices external to the machine and known as *voltage regulators.* One of the most successful of these is the Tirrill regulator.

The principle of this regulator is as follows:—In the field circuit of the exciter which supplies current to the field-magnet of the alternator is included a comparatively high resistance, whose value is such that the alternator p.d. would be only about 35 per cent. of its normal value if the resistance were left permanently in the exciter field circuit. This resistance is, by means of suitable controlling electromagnets, periodically short-circuited, and so the exciter field strengthened to the required amount.

The arrangement of connections is shown in Fig. 213, (*a*) and (*b*). In the upper part (*a*) of the diagram will be seen the two main or "floating" contacts, each of which is mounted at the end of a lever controlled by a coil and plunger mechanism. The lever carrying the upper floating contact is fitted with a spring which tends to depress the contact, and this tendency is opposed by the *downward* pull exerted by a coil on a core suspended from the other end of the lever. This coil is connected in series with a suitable resistance and then

across the exciter terminals. The bottom floating contact is carried by a lever provided with a counterweight which partly counterbalances the weight of the core suspended from the other end of the lever. The coil acting on this core is connected in series with a resistance and then (generally through a transformer) across two of the alternator terminals. When there is no current through this coil, the weight of the core is sufficient to press the bottom floating contact against the top one. But when the current reaches a certain value, the core is pulled *upwards*, and the floating contacts are separated.

In the lower part (*b*) of Fig. 213 is shown the arrangement for



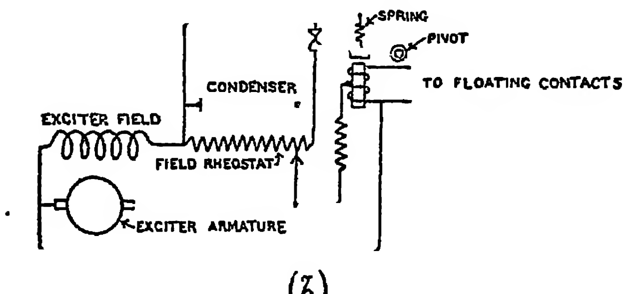


Fig. 213.—Connections of Tirrill Regulator.

periodically short-circuiting and open-circuiting the resistance in the exciter field-circuit. One of the short-circuiting contacts is fixed, while the other is carried by a lever provided with a spring which tends to maintain the contacts closed. This lever is also fitted with an armature acted on by an electromagnet. The middle point of the winding of this magnet is connected, through a suitable resistance, to one of the exciter terminals; one end of the winding is in permanent connection with the other terminal of the exciter, while the remaining end is periodically so connected through the floating contacts. When these contacts are closed, the magnetic effects of the currents

flowing through the two portions of the electromagnet winding neutralize each other, and the resistance in the exciter field circuit remains short-circuited. But as soon as the floating contacts separate, the magnet is strongly excited, and open-circuits the resistance. The condenser shown connected across the resistance is intended to reduce the sparking at the contacts.

The main feature to which the Tirrill regulator owes its success is undoubtedly the fact that it is constantly in a state of rapid vibration, and is thus able to respond very rapidly to any changes in the alternator p.d. The amount of resistance included in the exciter field circuit is so great that while the alternator p.d. would be far too low with this resistance in circuit, it would be far too high if the resistance were permanently short-circuited. No state of equilibrium is thus possible for the regulator, and the function which is really performed by it is to vary the relative duration of the periods of short-circuit and open-circuit of the resistance in the exciter circuit.

It might, perhaps, at first sight be thought that the arrangements adopted in the Tirrill regulator are unnecessarily complicated, and that a very much simpler design, consisting of a single simple control electromagnet which periodically short-circuits and un-short-circuits the field resistance, would be sufficient. As a matter of fact, such a simple arrangement has been in use in connection with very small continuous-current machines. With machines of larger size it is desirable to eliminate all sparking at the control magnet contacts, and hence the addition of a *relay* which takes the wear arising from sparking. With still larger machines, a single control magnet operated by the generator voltage would not be satisfactory, for the following reason. Assume the field resistance of the exciter to have been short-circuited. The exciter voltage begins to rise, and so does the generator exciting current. Owing, however, to the comparative slowness with which, in a large machine, the field current responds to changes of excitation voltage, by the time the generator voltage has assumed its normal value, the exciter voltage will be higher than that desired, and the generator voltage will continue to rise even after the exciter field resistance has been un-short-circuited and the exciter voltage has begun to fall. A similar effect takes place when the generator voltage is falling, and owing to this excessive "overshooting" the limits of fluctuation of the generator voltage will be too wide for satisfactory regulation. To reduce these limits and so get closer regulation, *two* control magnets, one actuated by the alternator voltage and the other by the exciter voltage, have been adopted.

In the case of large exciters, the field resistance is divided into several sections, each of which is short-circuited by a separate relay, the various relays being acted on by a common exciting coil.

In many cases, it may be desirable to maintain a constant voltage,

not at the terminals of the generator, but at the end of a long transmission line or feeder. This may be done by providing the regulator with a *line drop compensator*. The general principle of action of such a compensator is as follows. The voltage at the end of the line or feeder is obtained by subtracting *vectorially* from the generator voltage the line drop. Now imagine that into the circuit supplying the alternating current control magnet of a Tirrill regulator there is injected an e.m.f. whose ratio to the total e.m.f. of the control circuit is equal to the ratio of the line drop to the generator p.d., and whose phase relatively to the total e.m.f. is the same as that of the line drop to the generator p.d. Then the *resultant* e.m.f. in the control circuit will be maintained constant by the action of the regulator; and since this resultant e.m.f. is proportional to the p.d. at the far end of the line, the compensator will maintain this p.d. constant.

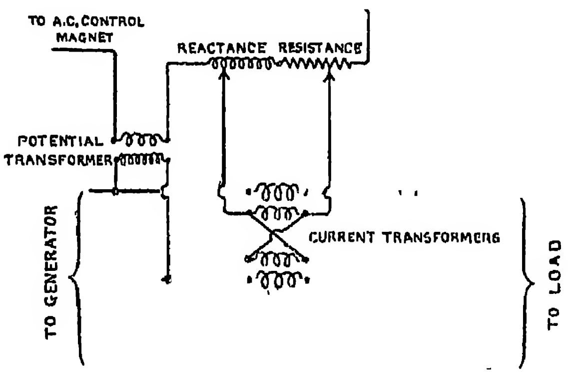


FIG. 214.—Connections of Line Drop Compensator for Tirill Regulator

The problem, then, is to obtain a voltage of suitable magnitude and phase, determined by the magnitude and phase of the line drop, for injection into the alternating current control magnet circuit.

The primary of the transformer which supplies the control magnet is connected across two alternator terminals, and connected to these terminals are also two line wires. The line drop in the loop formed by these wires is the vector *difference* of the drops along the individual wires. The vector *difference* (and not sum) of the drops is to be taken because according to the usual convention in the case of a three-phase line the positive directions along the three wires are all either outwards or inwards; whereas, in going around a loop, we proceed outwards along one conductor and inwards along the other. Now, since the drop along each conductor is the product of its impedance into the current, and since the impedances of the

conductors are all equal, the drop around the loop formed by two conductors is the product of the impedance of a single conductor into the vector difference of the currents in the two conductors. In order to obtain the vector difference, two current transformers are used as shown in Fig. 214, the secondaries of which are connected in parallel, but with one secondary *reversed.* The result of such reversal is to produce in the circuit external to the secondaries a current which is proportional to the vector difference of the two line currents—which is what is required. This current is allowed to flow through an adjustable reactance and resistance, the values of these latter being so adjusted that the phase angle between the voltage drop across them and the secondary p.d. across the potential transformer which supplies the control magnet is equal to the phase angle between the line drop and the generator p.d. (or secondary transformer p.d. if step-up transformers are used), and that the ratio of the voltage drop across the resistance and reactance to the secondary p.d. of the potential transformer is equal to the ratio of the line drop to the p.d. at the sending end of the line. A diagram of connections for the line drop compensator is given in Fig. 214.

§ 194. Instrument Transformers

In an *ideal* instrument transformer, the *ratio* of the voltages (or currents) is constant, and the secondary voltage (or current) is in direct phase opposition to the primary voltage (or current).

In any *actual* instrument transformer, the ratio of the voltages (or currents) is not constant; and the secondary voltage (or current) is not directly opposed in phase to the primary voltage (or current) for all values of the load.

The maker of the transformer gives a certain value for the *ratio* (of voltages or currents) which is taken to be the rated or standard ratio. The transformer may have this particular ratio when carrying a certain load; at other loads, the actual ratio will be different. The difference between the actual ratio and the rated ratio, generally expressed as a percentage of the latter, is termed the *ratio error* of the transformer.

The departure of the secondary voltage (or current) from exact phase opposition to the primary voltage (or current), generally expressed in degrees and minutes, is termed the *phase error* of the transformer.

We shall next show that a transformer having the following properties :—

(1) Infinite permeability of core,
(2) Zero core losses,

(3) Zero copper losses (*i.e.* negligible resistances of windings),
(4) Zero magnetic leakage,

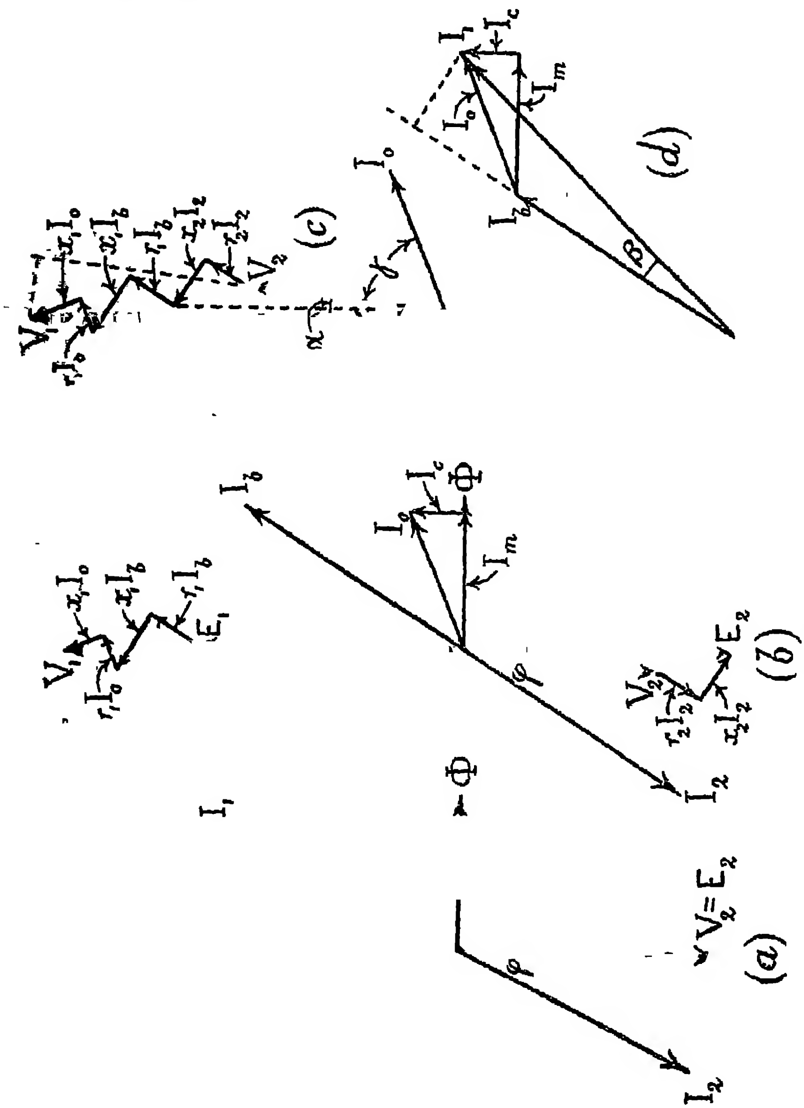


FIG. 215.—Vector Diagrams of Instrument Transformers.

will be an *ideal* instrument transformer, in the sense defined above.

In Fig. 215 (a) is shown the vector diagram of such a trans-

former. For the sake of simplicity, it is convenient to assume equality of primary and secondary turns, and to start with the voltage and current in the secondary in constructing the diagram, deducing from them the primary voltage and primary current. In order to adapt the results obtained to any other ratio $s = \frac{\text{primary turns}}{\text{secondary turns}}$, we have merely to *multiply* the primary voltage by s, and *divide* the primary current by the same quantity.

Let the secondary current I_2 lag behind the secondary p.d. by an angle ϕ, so that the power factor of the secondary load is $\cos\phi$. Since the secondary has, in accordance with assumptions (3) and (4) above, neither resistance nor leakage reactance (§ 11), there will be no "internal drop" in the secondary, and the secondary p.d. will be identical with the secondary e.m.f. E_2. The latter being proportional to (*minus*) the rate of change of flux, the flux vector Φ will be 90° ahead of E_2. Since the core, according to (1), has infinite permeability, the magnetizing current will be zero; similarly, by reason of (2), there will be no core loss component of primary current; the primary current I_1 must therefore be equal and directly opposed in phase to the secondary current at all loads—*i.e.* such a transformer will form an *ideal current* transformer.

Considering next the primary p.d., since there is neither resistance nor leakage reactance drop in the primary, the whole of the primary p.d. must be used up in balancing the e.m.f. Hence the primary p.d. E_1 must always be equal and opposite in phase to E_2, and hence the transformer will be an *ideal voltage or potential* transformer.

We thus see that the imperfections of ordinary instrument transformers are due to the non-fulfilment of the conditions laid down in (1) to (4) above; and the nearer we can approach to these ideal conditions, the better will be the behaviour of the transformer.

(1) demands that the core of the transformer should be built up of sheets having the highest attainable permeability over the working range; (2) requires fine lamination of the core and the use of a material having low hysteresis loss and high resistivity; also, the use of very low inductions in the core; to approach (3), the current density in the windings must be kept low; and (4) demands as close interpenetration of the two windings as possible—a condition which, unfortunately, is quite incompatible with the practical necessity of providing good insulation between them.

We shall now consider the vector diagram of an ordinary transformer not fulfilling the requirements laid down in (1) to (4). This is shown in Fig. 215 (*b*). As before, the secondary current I_2 lags behind the secondary p.d. V_2 by the angle ϕ. On account of the presence of resistance r_2 and leakage reactance x_2 in the secondary, we have

to add the vectors r_2I_2 and x_2I_2 to the vector V_2 in order to obtain the secondary e.m.f. E_2. This latter, as before, lags 90° behind the flux Φ. Considering next the primary current, we see that this must be made up of the following components: a component I_b equal and opposite to I_2 which balances the secondary ampere-turns; a magnetizing component I_m which maintains the flux through the core; and a component I_c which contributes the power represented by the core losses. The components I_m and I_c may be combined into a single component I_0 which may be termed the no-load component of the primary current: it represents the current that would be taken by the primary if the secondary were opened and the primary p.d. adjusted to give the same flux through the core as that originally present.

Lastly, considering the primary p.d. V_1, this is made up of the following: a component E_1 equal and opposite to E_2; the resistance and leakage reactance drop components r_1I_b and x_1I_b respectively due to the current I_b; and the resistance and leakage reactance drop components r_1I_0 and x_1I_0 respectively arising from the current I_0.

We shall now consider the application of the vector diagram of Fig. 215 (*b*) to the special case of a *potential transformer*, and shall obtain expressions for the ratio $\frac{V_1}{V_2}$, and the angle a between V_1 and V_2 reversed.

For the sake of clearness, the secondary voltage vectors of Fig. 215 (*b*) have been shown reversed in 215 (*c*). The angle between V_1 and V_2 (which is the phase angle of the transformer) being very small, V_1 will be practically equal to its projection on V_2; and this projection may be obtained by taking the sum of the projections of the various sections of the broken line representing the different components of which V_1 is made up. We thus get

$$V_1 = V_2 + r_2I_2 \cos \phi + x_2I_2 \sin \phi + r_1I_b \cos \phi + x_1I_b \sin \phi + r_1I_0 \cos \gamma + x_1I_0 \sin \gamma$$

where γ is the angle between V_2 reversed and I_0. Putting for the sake of brevity $r_1 + r_2 = r$, and $x_1 + x_2 = x$, we find, since numerically $I_b = I_2$,

$$V_1 = V_2 + rI_2 \cos \phi + xI_2 \sin \phi + r_1I_0 \cos \gamma + x_1I_0 \sin \gamma$$

and hence

$$\text{Ratio} = \frac{V_1}{V_2} = 1 + \frac{1}{V_2}\{(r \cos \phi + x \sin \phi)I_2 + (r_1 \cos \gamma + x_1 \sin \gamma)I_0\} \quad . \quad . \quad . \quad (1)$$

Again, the sine of the phase angle a may be obtained by taking

the sum of the projections of the various components of V_1 on a line normal to V_2 and dividing this sum by V_1. We thus get

$$\sin a = \frac{1}{V_1}\{(x\cos\phi - r\sin\phi)I_2 + (x_1\cos\gamma - r_1\sin\gamma)I_0\} \quad . (2)$$

The angle a being small, its value in radians is practically equal to its sine. Hence the right-hand side of (2) may be taken to give the phase angle in radians. To reduce to minutes, we have to multiply by 3438.

We shall next take the case of the *current* transformer. The diagram of currents is shown in Fig. 215 (*d*), where I_b is I_1 reversed. Denoting the angle between I_1 and I_2 reversed by β, we have approximately, projecting the various current vectors first on I_b and then on a line normal to it,

$$I_1 = I_2 + I_m \sin\phi + I_c \cos\phi$$

and
$$I_1 \sin\beta = I_m \cos\phi - I_c \sin\phi$$

Hence

$$\text{Current ratio} = \frac{I_1}{I_2} = 1 + \frac{1}{I_2}(I_m \sin\phi + I_c \cos\phi) \quad . \quad . \quad (3)$$

and

$$\text{Phase angle} = \beta = \frac{1}{I_1}(I_m \cos\phi - I_c \sin\phi) \quad . \quad . \quad . \quad (4)$$

the phase angle being expressed in radians.

The above formulæ are in the first instance applicable only to a transformer with a unity turn ratio. Taking now the general case of a transformer whose turn ratio is $s\left(= \frac{\text{primary turns}}{\text{secondary turns}}\right)$, if we denote the primary resistance and reactance by ${}_1r$ and ${}_1x$ respectively, and if we suppose that the primary turns are now (with the same total amount of copper) made equal to the secondary turns, then, r_1 and x_1 denoting, as above, the primary resistance and reactance respectively of the equivalent unity ratio transformer, we must put (§ 91)—

$$r_1 = \frac{1}{s^2} \times {}_1r, \text{ and } x_1 = \frac{1}{s^2} \times {}_1x$$

so that
$$r = \frac{{}_1r}{s^2} + r_2, \text{ and } x = \frac{{}_1x}{s^2} + x_2$$

also we must put $I_b = sI_2$, and $I_0 = s_sI_0$, where ${}_sI_0$ is the no-load current of the transformer with a turn ratio s.

Further, the value of the voltage ratio obtained for a potential transformer must be *multiplied* by s, and the value of the current ratio obtained for a current transformer *divided* by s. Making these

changes in the formulæ (1) to (4), we obtain, for the general case of a transformer having a turn ratio s,

$$\text{voltage ratio} = s\left[1 + \frac{1}{V_2}\left\{\left(\frac{{}_1r}{s} + r_2\right)I_2 \cos\phi + \left(\frac{{}_1x}{s} + x_2\right)I_2 \sin\phi + \frac{{}_1r \cos\gamma + {}_1x \sin\gamma}{s}\,{}_sI_0\right\}\right] \quad . \quad . \quad (5)$$

$$a = \frac{1}{V_2}\left\{\left(\frac{{}_1x}{s} + x_2\right)I_2 \cos\phi - \left(\frac{{}_1r}{s} + r_2\right)I_2 \sin\phi + \frac{{}_1x \cos\gamma - {}_1r \sin\gamma}{s}\,{}_sI_0\right\} \quad . \quad . \quad (6)$$

$$\text{Current ratio} = \frac{1}{s}\left\{1 + \frac{s}{I_2}({}_sI_m \sin\phi + {}_sI_c \cos\phi)\right\} \quad . \quad . \quad (7)$$

$$\beta = \frac{1}{{}_sI_1}({}_sI_m \cos\phi - {}_sI_c \sin\phi) \quad . \quad . \quad (8)$$

In (7) and (8), ${}_sI_m$ and ${}_sI_c$ stand for the magnetizing and core loss currents respectively of the transformer with a turn ratio s.

If the various quantities which occur on the right-hand sides of equations (5) to (8) were known, then we could predetermine the behaviour of the transformer—*i.e.* we could predict the values of the ratio and phase angle corresponding to any *given* values of V_2, I_2 and ϕ (or $\cos\phi$).

Thus, in the above equations, V_2, I_2 and ϕ are given; s must also be given; $r = \frac{{}_1r}{s^2} + r_2$ and $x = \frac{{}_1x}{s^2} + x_2$ may be determined by the short circuit test (§ 91); ${}_1r$ and r_2 may also be measured in the ordinary way; ${}_1x$ and x_2 cannot be determined separately, but it is generally permissible to assume that $\frac{{}_1x}{x} = \frac{{}_1r}{r}$, so that ${}_1x = \frac{{}_1r}{r}x$, and, ${}_1x$ and x being known, we find x_2 from the relation $x = \frac{{}_1x}{s^2} + x_2$. Thus, ${}_1r$, r_2, ${}_1x$ and x_2 being known, we can find the values of $\frac{{}_1r}{s} + r_2$ and $\frac{{}_1x}{s} + x_2$ which occur in (5) and (6); so far as these two equations are concerned, the only remaining unknown quantity is now γ. This may be determined approximately by the *open-circuit* test (§ 91). The secondary being open-circuited, the normal p.d. is applied to the primary, and the primary p.d., current and power are measured. This enables us to find the power-factor of the open-circuited transformer. If we refer to the vector diagram of Fig. 216 (*c*), it will be readily seen that when the secondary is open-circuited V_1, V_2 and E_1 become practically coincident and normal to

Φ; and the open-circuit power-factor is practically the cosine of the angle between E_1 and I_0; now this differs so little from the angle γ between V_2 and I_0 in the loaded transformer that we may approximately put $\cos\gamma$ = power factor of transformer on open circuit (with normal p.d. across primary). The whole of the quantities on the right-hand sides of (5) and (6) thus become known.

Considering next the case of a *current* transformer, we see that in order to be able to utilize (7) and (8) we have to determine the two components of ${}_sI_0$, viz. ${}_sI_m$ and ${}_sI_c$. It is to be noted that these are variable with the load, instead of being practically constant as in the case of a potential transformer. Further, the use of a wattmeter for determining ${}_sI_m$ and ${}_sI_c$ in the forms ${}_sI_m = {}_sI_0 \sin\gamma$ and ${}_sI_c = {}_sI_0 \cos\gamma$ as in the case of a potential transformer is attended with serious difficulties, owing to the low voltages across the windings. We first of all take a series of readings connecting the various values of I_2 with the secondary p.d.s by using a suitable low-reading voltmeter. The values of ${}_sI_m$ and ${}_sI_c$ may then be very conveniently determined by a method due to P. G. Agnew,* which involves the use of a *phase-shifter* (Appendix VI.). The arrangement of connections is shown in Fig. 216. In series with the primary of the current transformer is connected a known non-inductive resistance R. By means of the two-way switch S_1, the fine-wire circuit of a delicate electrodynamometer (wattmeter) ED may be connected either across the primary or across R. The current coil of the electrodynamometer is joined in series with a known non-inductive resistance r and the ammeter A, and is supplied with current from the secondary of a phase-shifter. The p.d. across the primary of the current transformer is first adjusted until the desired secondary voltage is obtained (by taking the reading of the low-reading voltmeter V). By means of the switch S_2, the primary is then connected across the fine-wire dynamometer circuit, and the phase-shifter is adjusted so as to reduce the dynamometer reading to

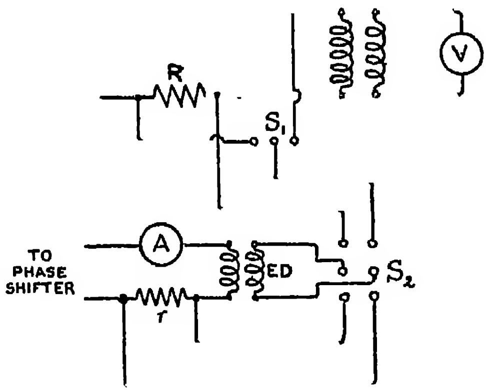


FIG. 216.—Method of determining Magnetizing and Core Loss Components of Transformer Current.

* *Bulletin of the Bureau of Standards*, Washington, vol. vii p 433 (1911).

zero. This means that the current through the dynamometer current coil is in quadrature with the primary p.d. of the transformer. The switch S_1 is then thrown over so as to connect the dynamometer fine-wire circuit across R, and the dynamometer reading, which is

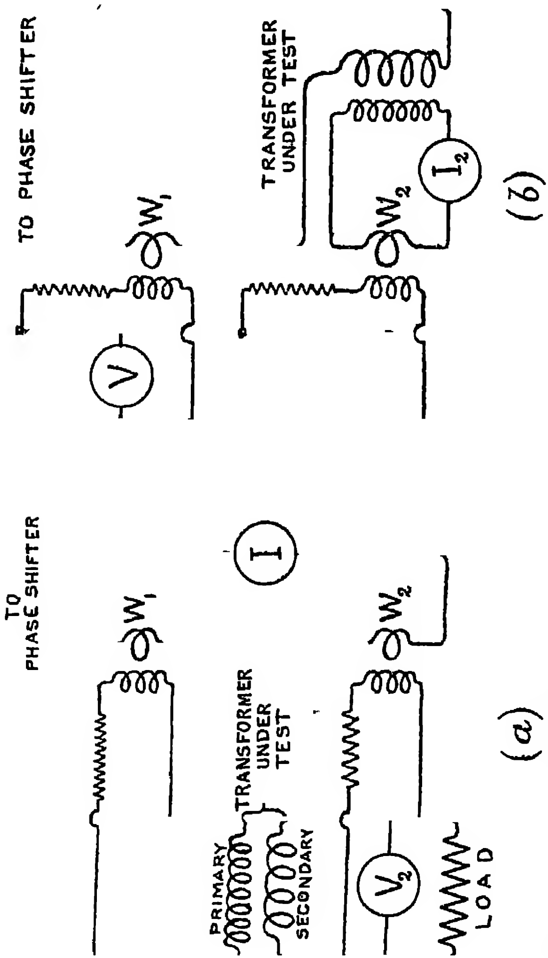


Fig. 217.—Experimental Method of Determining Ratio and Phase Angle of Instrument Transformers.

proportional to ${}_sI_m$ is taken. The phase of the secondary phase-shifter current is then altered by 90° (by turning the secondary through 90 electrical degrees), and the new reading, which is proportional to ${}_sI_0$ is noted. Lastly, the absolute values of the dyna-

mometer readings are determined by throwing over the switch S_2 to r and taking the corresponding reading.

Direct Experimental Determination of Ratio and Phase Angle.—Although, as explained above, it is possible to *calculate* the ratio and phase angle of an instrument transformer from certain data, a direct *experimental* determination of these quantities may be considered more satisfactory. A great deal of ingenuity has been displayed in devising various methods for this purpose. Those aiming at the highest degree of accuracy involve the use of special apparatus not ordinarily available.

(*a*) *Ratio Determination by Voltmeters and Ammeters.*—This is theoretically the simplest method for determining the ratio, and good results may be obtained by the use of suitable high-grade instruments. The method suffers from the disadvantage that unless *simultaneous* readings of the two voltmeters (in the case of a potential transformer) or ammeters (in the case of a current transformer) are taken, the accuracy will be affected by fluctuations in the voltage or current.

(*b*) *Ratio and Phase Angle Determination by means of Electrodynamometer Wattmeters.**—The following comparatively simple method may be used for determining both the ratio and the phase angle.

Taking first the case of a potential transformer, the connections are arranged as in Fig. 217 (*a*). Two electrodynamometers, W_1 and W_2, are used, with their current coils connected in series and to the secondary of a phase-shifter. The potential coil of W_1 is across the primary, and that of W_2 across the secondary, of the transformer under test. The phase of the phase-shifter current is first adjusted to give maximum reading of W_1; this will also correspond, to all intents and purposes, to maximum reading of W_2. If I be the phase-shifter current and w_1, w_2 the readings of the dynamometers (in watts), then

$$w_1 = V_1 I, \text{ and } w_2 = V_2 I \cos a$$

where a is the phase angle. Hence

$$\frac{V_1}{V_2} = \frac{w_1}{w_2} \cos a$$

But since $\cos a$ is practically indistinguishable from unity, we may write

$$\text{ratio} = \frac{V_1}{V_2} = \frac{w_1}{w_2}$$

* Methods depending on the use of electrodynamometers have been described by L. T. Robinson (*Proceedings of the American Institute of Electrical Engineers*, vol. xxviii. p. 996 (1909)) and by C. H. Sharp and W. W. Crawford (*Trans. Am. Inst. El. Engineers*, vol. xxix. p. 1521 (1912)).

The phase-shifter current is next altered in phase (without change of magnitude) until the reading of W_1 becomes zero. If w_2' is the new reading of W_2, we have

$$w_2' = V_2 I \cos (\alpha \mp 90°)$$

and hence $$\sin \alpha = \alpha = \frac{w_2'}{V_2 I}$$

It will be noticed that the determination of α involves the use of high-grade instruments for the measurement of V_2 and I.

A similar method may be applied to current transformers. The connections for this case are shown in Fig. 217 (*b*). If, as before, the readings of W_1 and W_2 when W_1 is at its maximum be denoted by w_1 and w_2, we have

$$\text{ratio} = \frac{I_1}{I_2} = \frac{w_1}{w_2}$$

and if when the reading of W_1 is zero that of W_2 is w_2',

$$\text{phase angle} = \beta = \frac{w_2'}{VI_2}$$

Constructional Features of Instrument Transformers.—In the general arrangement of core and coils, instrument transformers do not differ greatly from ordinary power transformers. The commonest type of construction is that in which the core is of the type shown in Fig. 100, preferably without any joints in the magnetic circuit, and with the windings either arranged on one limb of the core or evenly distributed over both limbs. A special type of construction for *current* transformers is that in which the primary is threaded through the central opening of the core, forming a *single-turn* winding; this is a very convenient form for heavy currents, but is generally incapable of giving as high a degree of accuracy as the other type.

Precaution to be observed in using Current Transformers.—When using current transformers, it must be carefully borne in mind that such transformers should on no account have their secondary open-circuited, even momentarily, while the primary remains in circuit. If it is desired to disconnect them, the secondary should first be short-circuited. For this purpose, every current transformer is provided with a special short-circuiting switch. Should by a mishap the secondary be open-circuited while current is passing through the primary, the core induction will rise to a very high value, and when the transformer is disconnected the core *may* be left with a very strong magnetic "set" or residual induction, which the subsequent weak magnetizations when the transformer is once more put into use will be unable to wipe out, with the result that

both the magnitude and phase of the no-load current will be altered, and both the ratio and phase angle will suffer change. The only method of restoring the transformer to its original condition is to demagnetize it carefully by the method of reversals.

§ 195. Induction Type Measuring Instruments

The principles underlying the action of induction motors are also usefully applied in the construction of a certain class of measuring instruments known as *induction* instruments.

Imagine a tiny two-phase two-pole stator, and for the rotor of the ordinary induction motor let there be substituted a *fixed* cylindrical core and a light movable cylindrical shell of aluminium mounted on a shaft and provided with a controlling spring and pointer (the aluminium shell being placed in the annular air-gap between the stator and the central core). If two-phase currents be sent through the stator winding, the aluminium shell or drum (which represents the rotor winding) will experience a torque, and the pointer of the instrument will be deflected.

In order to produce a deflection of the pointer, it is not necessary to have currents through the two stator windings which are in exact quadrature: a torque will be obtained so long as there is a phase difference between the two currents, even if this is considerably less than 90°. For, if the two currents be denoted by I_1 and I_2, and if their phase difference is θ, we may resolve I_2 into two components, $I_2 \sin \theta$ and $I_2 \cos \theta$, of which the first is in quadrature with I_1, and the second co-phasal with I_1. We may then replace the given system by the following equivalent system:—(1) a true two-phase system, represented by currents, each of magnitude $I_2 \sin \theta$, in quadrature with each other; (2) a system consisting of currents $I_1 - I_2 \sin \theta$ through the first coil, and $I_2 \cos \theta$ through the second, which are in phase with each other. The first system will produce a rotating field and torque on the aluminium drum, the second system will produce a simple alternating field and no torque.

In order to construct an *ammeter* of this type, the total current to be measured must be split up into two currents differing in phase, and these currents sent through the stator windings of the instrument. Similarly, in order to construct a *voltmeter*, the currents through the parallel-connected paths containing the stator windings must be arranged to differ in phase. The requisite phase difference is in either case easily obtained by making one of the parallel paths have a larger ratio of reactance to resistance than the

other. The general arrangement of such an instrument is shown in Fig. 218.

A somewhat different arrangement, involving the same principle, is to be found in the so-called *shaded pole* type of instrument. Let a **C**-shaped alternating current electromagnet have a metal disc placed between its poles, the disc being controlled by a spring and provided with a pointer. Let the poles of the magnet be divided into two parts by slots, and let a band of copper be fitted into each

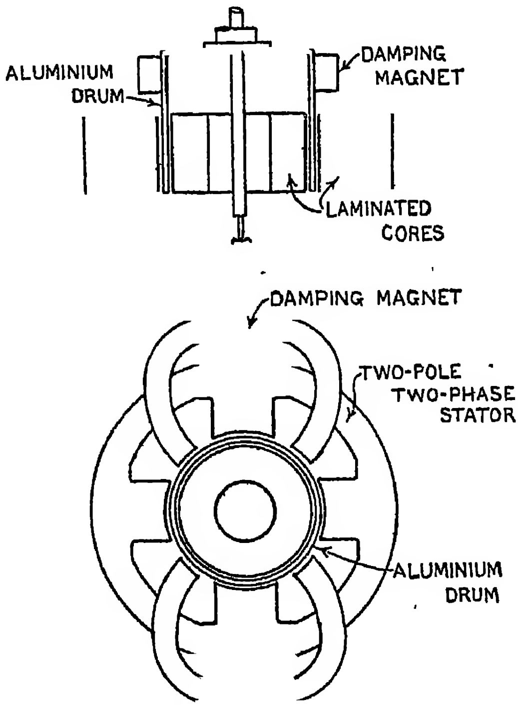


FIG. 218.—Induction Type Instrument.

slot so as to surround one half of the magnet pole, as shown in Fig. 219. When an alternating current is sent through the magnet coil, secondary currents (which are nearly in direct phase opposition to the primary) will be induced in the short-circuited copper bands. The flux through those portions of the poles which are surrounded by the bands or *shaded*, being due to the resultant of the primary and secondary ampere-turns, will be nearly *in quadrature* with the primary current; while the flux through the unshaded portion will

be nearly *in phase* with the primary current. We thus get over two adjacent areas magnetic fluxes, which are in approximate time-quadrature with each other, and the resultant is a shifting or travelling flux, which pulls the disc along and causes a deflection of the instrument.

The main advantage of voltmeters and ammeters of the induction type is that they may be made to have a very long, open scale. On the other hand, their readings are affected by changes of frequency,* and their cost is relatively high. For these reasons, they have not been able to compete seriously with other types, and have only been used to a limited extent.

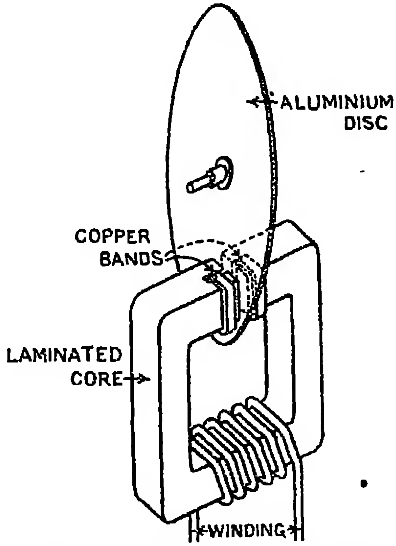


Fig. 219.—Shaded Pole Instrument.

It is otherwise, however, with induction type *integrating wattmeters*, or energy meters, as these are used more than any other type on alternating current circuits. The conditions to be satisfied by an induction type energy meter are more difficult to realize than those in the case of an ammeter or voltmeter. In the latter instruments, the consideration of the power factor of the load does not enter into the problem. An energy meter, on the other hand, must exert a torque which is proportional to the true power in the circuit, irrespective of the power-factor of the load.

The moving element of an integrating wattmeter of the induction type consists of a disc mounted on a spindle, which is geared to the counting mechanism.† Acting on this disc are two torques: the driving torque provided by the alternating current electromagnets, and the resisting torque due to a permanent magnet. Since this latter torque is proportional to the speed, and since the driving and resisting torques are equal when the speed has become steady, the driving torque is also proportional to the

* Various ingenious compensating devices have been employed to render the indications of such instruments practically independent of frequency changes within certain limits.

† In some types of induction meter the spindle carries *two* discs: one of these being acted on by the alternating-current electromagnets and providing the *driving* torque, while the other is acted on by a permanent magnet and furnishes the *braking* torque.

speed. Thus the number of revolutions during any time will be proportional to the energy, provided the driving torque is proportional to the power.

The alternating current electromagnets consist of a series and a shunt magnet. The former represents the ammeter, and the latter the voltmeter part of the instrument. The series magnet carries the load current, and the shunt magnet is connected across the mains. These magnets are diagrammatically shown in Fig. 220. Let Φ_1 and Φ_2 denote the fluxes due to the shunt and series magnets respectively, and let a be their time phase-difference. The flux Φ_1 induces a system of eddy-currents in the disc, which are proportional to Φ_1 and in quadrature with it, and these are acted on by the field Φ_2.* The phase difference between the eddy-currents and Φ_2 being $\frac{\pi}{2} - a$, the torque exerted on the disc is proportional to $\Phi_1\Phi_2 \cos\left(\frac{\pi}{2} - a\right)$. Now, if V stand for the p.d. of the mains, I for the load current, and $\cos\theta$ for the power-factor, then the power is given by $VI\cos\theta$. In order that the torque may be proportional to the power, we must have $\Phi_1\Phi_2 \cos\left(\frac{\pi}{2} - a\right) \propto VI\cos\theta$. But since $\Phi_1 \propto V$, and $\Phi_2 \propto I$, it follows that we must have $a = \frac{\pi}{2} - \theta$. Hence, when $\theta = 0$, *i.e.* when the power factor of the load is unity, $a = \frac{\pi}{2}$, or the shunt and series fluxes must be in quadrature with each other. If this condition is satisfied, we get a resultant shifting or travelling flux similar to that which occurs in an ordinary induction motor.

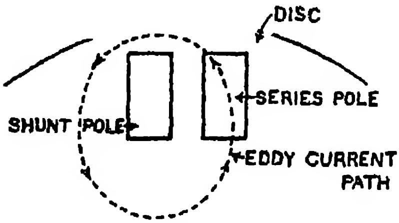


FIG. 220.—To illustrate Principle of Induction Energy Meter.

Now the flux Φ_2 is always in phase with the load current I. Hence when the load is non-inductive, Φ_1—which is required in this case to be in quadrature with Φ_2, and therefore with I—must be in quadrature with V (since V and I are in phase with each other). If the resistance of the shunt magnet coil be small in comparison with its reactance, this condition will be nearly but not quite fulfilled,

* Similarly, the flux Φ_2 induces a system of eddy-currents in the disc which are acted on by the flux Φ_1, and the torque so produced is *added* to that arising from the eddy-currents induced by Φ_1 and acted on by Φ_2.

the flux lagging behind $\bar{V}$ by an angle slightly less than $\frac{\pi}{2}$. In order to increase the lag to exactly $\frac{\pi}{2}$, numerous ingenious devices have been proposed and used. One of the best known of these consists in providing the shunt magnet with a secondary winding, which is closed through a suitably adjusted resistance. The way in which such a closed secondary alters the angle between Φ_1 and V will be best understood by a reference to the diagrams, Figs. 221 (*a*) and

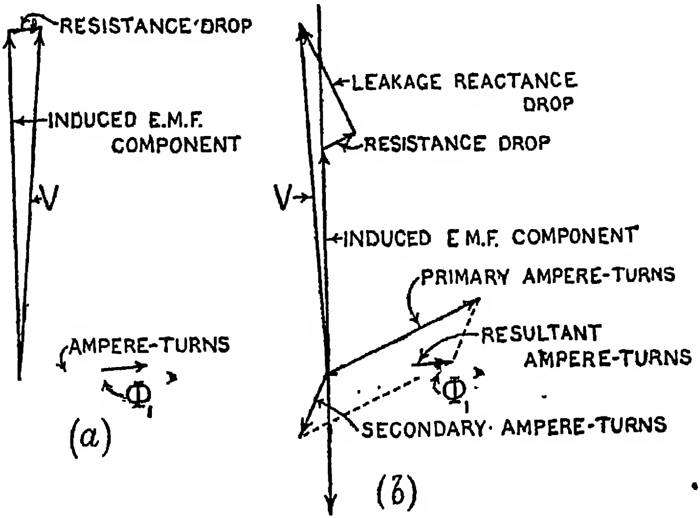


FIG. 221.—Vector Diagrams to show Phase Displacement by use of Secondary Winding.

(*b*), the first of which shows the conditions when there is no closed secondary, and the second when such a secondary is present. The flux Φ_1 passing through the disc is the main or resultant flux, which is due to the joint action of the primary and secondary ampere-turns. In addition to the flux Φ_1 (which is nearly but not quite in phase with the resultant ampere-turns) the primary is linked with a certain leakage flux, which is in phase with the primary ampere-turns. As will be seen from the diagram, by suitably controlling the leakage reactance drop we can cause V to become coincident in direction with its induced e.m.f. component (due to Φ_1), and so bring it into exact quadrature with Φ_1.

§ 196. Power Factor Meters

We have already (§ 41) given a description of a power factor indicator suitable for use on a balanced three-phase circuit. We now propose giving a brief account of an instrument suitable for a single-phase circuit.

In such an instrument, the fixed system consists of a coil carrying the load current, while the movable system is represented by two coils at right angles to each other and conveying equal currents in quadrature with each other. This quadrature relationship of the two currents is obtained by connecting one of the coils in series with a high non-inductive resistance, and the other in series with a reactance. The general arrangement of the coils and their connections are represented in Fig. 222 (*a*) and (*b*). The movable

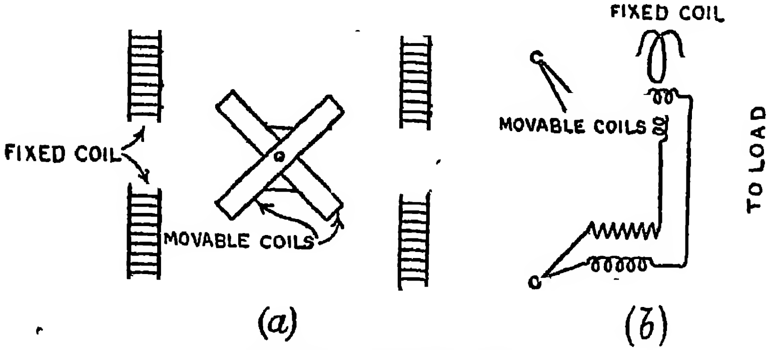


FIG. 222.—Power Factor Meter.

system is not subject to any fixed control, and will remain at rest in any position when the instrument is not in use.

The action of the instrument is as follows: The currents through the movable system give rise to a rotating field which rotates synchronously in a definite direction (clockwise or counter-clockwise). The alternating field due to the fixed coil may be resolved (§ 21) into two oppositely rotating fields; one of these has a direction of rotation opposed to that of the movable system field, and is incapable of interacting with it, the mean value of the torque between the two fields being zero; while the other rotates in the same direction as the movable system field, and hence is capable of interacting with it. As a result, the movable system takes up a position corresponding to coincidence of the fields having the same direction of rotation—in which position the torque vanishes. If now the phase of the load

current changes, the rotational component of the alternating field which has the same direction of rotation as the movable system field will become displaced relatively to the latter, and the resulting torque will cause the movable system to move into a new position of coincidence of the fields. Thus for each value of the phase angle of the load current there is a definite position of the movable system, and the instrument may be used as a power factor meter.

In order that the instrument may be reliable under all conditions, it is important that the movable coils should be as nearly alike as possible. The condition is difficult to secure when, as is often done, the one coil is slipped inside the other. In the type of instrument manufactured by the Weston Electrical Instrument Co., practical interpenetration of the coils at the crossing-points is secured by a special method of winding: the first layer of the first coil having been wound, the first layer of the second coil is wound next, then the second layer of the first coil, followed by the second layer of the second coil, and so on, the layers of the two coils alternating with each other at the crossing-points, and practical similarity of size and shape of the two coils being thereby obtained.

From what has been said regarding the action of the instrument, it will be evident that in the case of the power-factor indicator for balanced polyphase circuits described in § 41, it is not really necessary to provide polyphase windings for both the fixed and movable coils: either winding may be replaced by a single-phase winding connected to one phase only.

§ 197. Frequency Meters

Modern instruments for determining the frequency of a current are of two kinds: vibrational and indicating. In the former, the frequency is found by noticing which of a series of tuned vibrating steel strips has the greatest amplitude of vibration; in the latter, the frequency is indicated by a pointer moving over a scale.

In Frahm's frequency meter, illustrated in Fig. 223, an alternating current electromagnet having a laminated core (consisting of a bundle of iron wires in the type illustrated) and laminated pole-pieces acts on a series of tuned steel springs. These are accordingly thrown into vibration, and that particular spring whose natural frequency of vibration most nearly synchronizes with the pull of the magnet exhibits the greatest amplitude of vibration. In order to render the relative amplitudes of vibration clearly visible, the ends of the springs are bent at right angles and painted white, so that the observer sees a series of white rectangular patches arranged around

the circumference of a circle and standing out clearly against the dark background of the interior of the instrument. The patch or patches corresponding to the spring or springs for which the condition of resonance is most nearly fulfilled appear more or less elongated in a radial direction as compared with the others. Opposite each patch is a scale mark giving the corresponding reso-

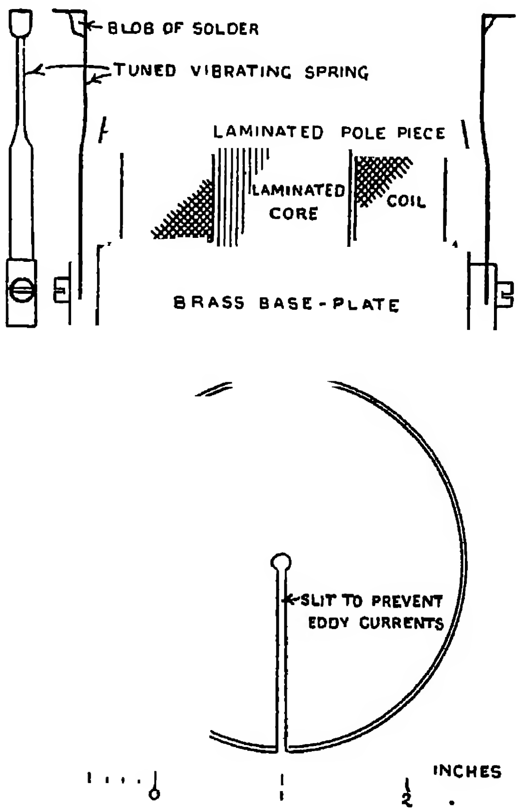


FIG. 223.—Frequency Meter.

nance frequency. The springs are tuned by adjusting the mass of the blob of solder at the bend.

Instead of arranging the springs around the circumference of a circle, they may be arranged in a straight line, the magnet being provided with a rectangular instead of a circular pole.

As an example of the indicating type of frequency meter we shall

take the Weston instrument, the general arrangement of which is diagrammatically represented in Fig. 224 (*a*), the connections being shown in Fig. 224 (*b*). The working parts of the instrument consist of two flat coils arranged at right angles to each other and a soft iron needle which moves inside the coils and whose spindle carries the pointer. Each coil is wound in two sections to allow of the introduction of the needle into the interior of the coils, and one of the coils is slipped inside the other as shown in Fig. 224 (*a*), the outer coil being of greater depth. In order to understand the working of the instrument, we may consider two extreme cases, viz. those of zero frequency and of infinite frequency. When the frequency is zero, practically the entire current will flow through the left-hand coil, in Fig. 224 (*b*), since the resistance of the inductive path is negligible in comparison with that of the shunting resistance. On the other hand, when the frequency is infinite, the entire current will pass through the right-hand coil, the non-inductive resistance path being of negligible impedance compared with that of the shunting reactance. Hence as the frequency changes from zero to infinity, the soft-iron needle moves from a position along the magnetic axis of one of the coils to a position along the axis of the other, *i.e.* through a right angle. It will be seen from this that the useful range of the instrument corresponds to a relatively small angle.

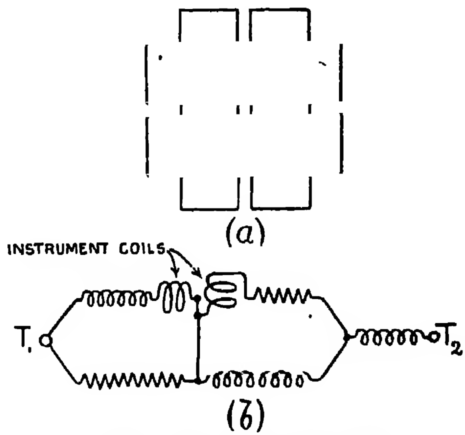


FIG. 224.—Weston Frequency Meter.

The additional reactance (one end of which is in connection with terminal T_2) is intended to suppress any higher harmonics which may be present in the p.d. wave.

§ 198. Unbalanced Systems and Unbalance Factor

Problems relating to unbalanced three-phase systems not infrequently arise in practice, and in connection with such problems a certain artifice, to be considered presently, is very useful. This

artifice consists in substituting for the unbalanced system two balanced systems of opposite phase sequence.* One of these, which may be termed the *positive* or *direct phase sequence component*, has the same phase sequence as the original unbalanced system; the other, which we may call the *negative* or *reverse phase sequence component*, has a phase sequence opposed to that of the original unbalanced system.

Let in Fig. 225 (*a*) A, B and C represent the conductors of a three-phase system, and let the positive directions around the mesh be counter-clockwise as shown, v_1, v_2 and v_3 denoting the instantaneous values of the line p.d.s. Let the triangle ABC in Fig. 225 (*b*) represent the line voltage vectors. Returning now to Fig. 225 (*a*) since $v_1 + v_2 + v_3 = 0$, we may put $v_3 = -v_1 - v_2$, and so for Fig.

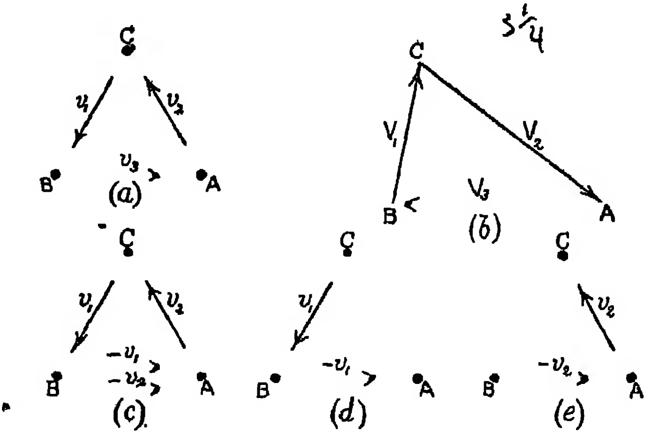


FIG. 225.—Transformations of Unbalanced System.

(*a*) we may substitute the equivalent Fig. (*c*). Now Fig. (*c*) may be analyzed into the two component systems (*d*) and (*e*).

Considering first the system Fig. 225 (*d*), the voltages across BC and AB are, in accordance with Fig. 225 (*b*), represented by the long vectors marked BC and AB respectively in Fig. 226 (*a*). Now for each of these vectors there may be substituted two components making angles of 30° with the long vectors, and so we obtain the equivalent system of four short vectors marked BC and AB. Again,

* See L. G. Stokvis, *Comptes Rendus*, vol. clix. p. 46 (1914); P. Muller, *Elektrotechnische Zeitschrift*, vol. xxxix. p 343 (1918); R. E Gilman and C. le G. Fortescue, *Transactions of the American Institute of Electrical Engineers*, vol. xxxv p. 1329 (1916); C. L. Fortescue, *ibid.*, vol. xxxvii p. 1027 (1918); C. J. Fechheimer, *ibid.*, vol. xxxix. p. 1469 (1920).

referring to Fig. 225 (*d*), for the zero p.d. across CA there may be substituted two equal and opposite p.d.s, represented by the vectors marked CA in Fig. 226 (*a*), each of these being equal in magnitude to each of the four vectors just mentioned. We thus get the six vectors D_1', R_1', D_1'', R_1''', D_1''' and R_1''. Denoting the instantaneous projections of these vectors by corresponding small letters, we may for Fig. 225 (*d*) substitute Fig. 226 (*b*). Again, for this latter we may substitute the two balanced three-phase systems represented in Figs. 226 (*c*) and (*d*), the first of which has the same phase sequence as the original unbalanced system, while the second has an opposed phase sequence.

We may deal in a precisely similar manner with the system of

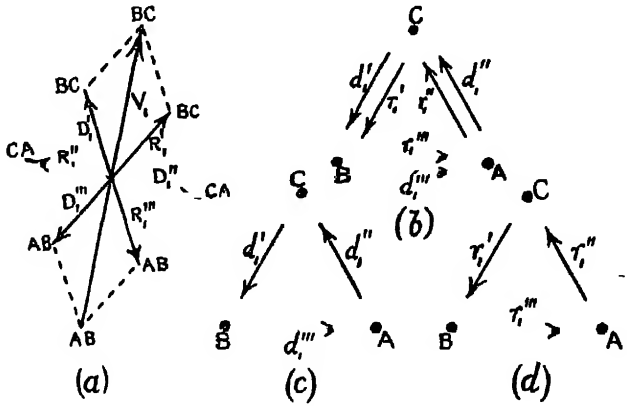


FIG. 226.—Analysis of Unbalanced into Two Balanced Systems.

Fig. 225 (*e*), the corresponding set of diagrams being shown in Fig. 227, Fig. 227 (*c*) and Fig. 227 (*d*) giving the direct and reverse phase balanced components respectively of the system Fig. 225 (*e*).

Thus for Fig. 225 (*a*) we may substitute the four systems represented by (*c*) and (*d*) in Figs. 226 and 227.

Taking next the two systems (*c*) of Figs. 226 and 227, which have the same phase sequence, we may combine them into the single system represented by the three vectors marked AB, BC and CA in Fig. 228 (*a*). Similarly, the systems corresponding to Figs. 226 (*d*) and 227 (*d*), both of which have reverse phase sequence, may be combined into a single system represented by the vectors marked AB, BC and CA in Fig. 228 (*b*).

We have thus shown that the unbalanced system represented by the three vectors of Fig. 225 (*b*) may be analyzed into the two balanced systems corresponding to Fig. 228 (*a*) and (*b*). Of these

(a) represents the positive or direct phase sequence component, and (b) the negative or reverse phase sequence component.

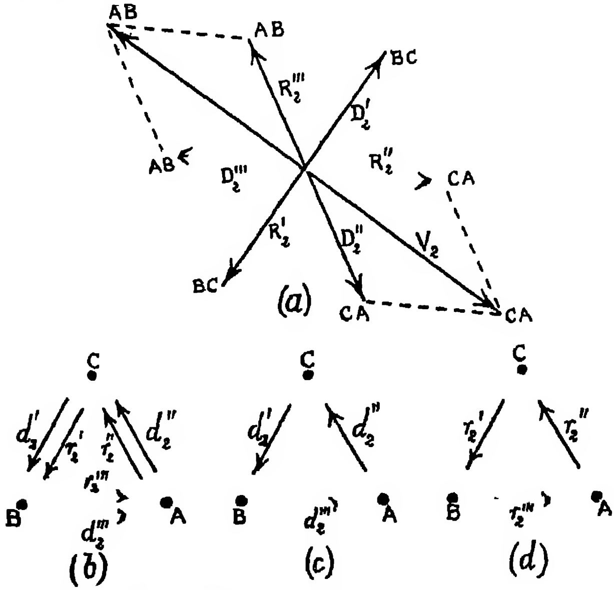


FIG. 227.—Analysis of Unbalanced into Two Balanced Systems.

The procedure adopted above might also be used as a means of actually finding the magnitudes and phase relationships of the two

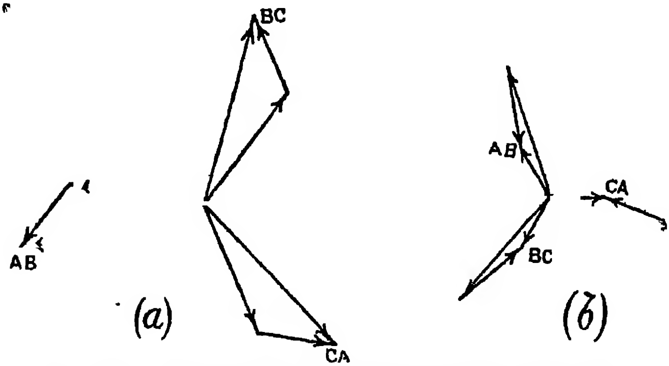


FIG. 228.—Analysis of Unbalanced into Two Balanced Systems.

balanced components of an unbalanced system. But once the possibility of such a transformation has been established, the

two components may be found more simply by the following method.

Let the three vectors marked AB, BC and CA in Fig. 229 represent the voltage vectors of the unbalanced system (they correspond to the three sides of the triangle of vectors shown in Fig. 225 (*b*)). Each of these vectors is shown as the resultant of two components, such that one set of components forms a balanced system of direct phase sequence, and the other a balanced system of reverse phase sequence. The analysis into the two balanced systems is provisionally assumed to have been effected, so that the two component systems are known. Thus, the vector AB of the original system is shown as consisting of two components, one of which is equal to the vector AB of Fig. 228 (*a*), and the other to the vector AB of Fig. 228 (*b*). Similarly, the vectors lettered BC and CA in Fig. 229 are built up of the similarly lettered vectors in Fig. 228 (*a*) and (*b*).

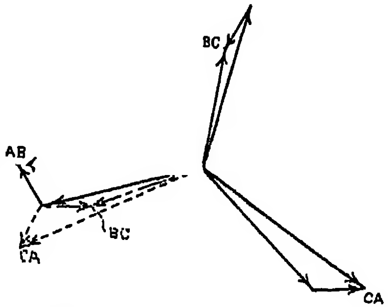


FIG. 229.—Unbalanced System of Vectors and its Balanced Components.

Suppose now that in Fig. 229 the triangle containing the BC vector is rotated through an angle of 120° in a counter-clockwise direction, so as to assume the position indicated by the chain-dotted lines, and that the triangle containing the CA vector is rotated through

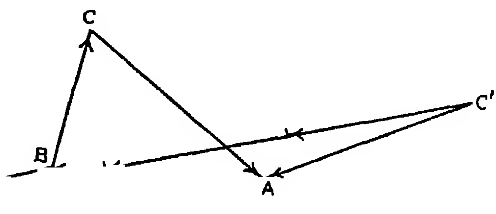


FIG. 230.—Method of finding Direct Phase Sequence Components.

the same angle but in a clockwise direction. Then the three vectors which formed the direct phase sequence component system become coincident, and their resultant is equal to thrice the vector which forms the direct phase sequence component of the undisplaced vector AB; while the reverse phase sequence component of AB together with the displaced reverse phase sequence components of BC and CA form a system of three equal vectors spaced 120° apart

whose resultant is zero. From this it is evident that the resultant of AB and the displaced vectors BC and CA (shown dotted) is a vector which is equal to the direct phase sequence component of AB magnified threefold.

We thus arrive at the following very simple method of determining the direct phase sequence system, illustrated in Fig. 230. Draw the triangle ABC of voltage vectors. In order to find the direct phase sequence component of any vector, say AB, keep this vector fixed, and rotate the remaining two sides of the triangle through 120° *outwards,* so as to open the triangle. This gives us the three vectors C′A, AB and BC″. Find their resultant C′C″; one-third of this represents the direct phase sequence component of AB. The remaining two direct phase sequence components (those of BC and CA) immediately become known, since they make angles of 120° with the component already found.

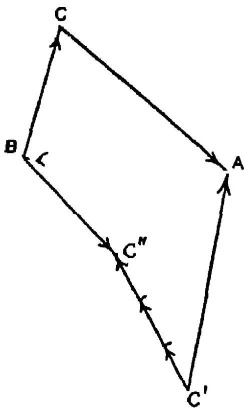


FIG. 231.—Method of finding Reverse Phase Sequence Components.

The direct phase sequence component of AB having been found, the reverse phase sequence component also becomes known, since it is represented by the vector difference between AB and its direct phase sequence component. But there is a very simple method of determining the reverse phase sequence component independently. This is illustrated in Fig. 231, and is as follows. Keeping AB fixed as before, rotate the remaining two sides of the triangle *inwards* (*i.e.* towards AB) through 120°. Then by using a construction similar to that of Fig. 229 * it is easy to show that the resultant of C′A, AB and BC″ in Fig. 231 represents the reverse phase sequence component of AB magnified threefold.

When dealing with unbalanced three-phase systems, it is useful to adopt some method of expressing the extent or degree of unbalance, and for this purpose a quantity known as the *unbalance factor* has been introduced. *The unbalance factor is defined as the ratio of the magnitude of the negative or reversed phase sequence vectors to the magnitude of the positive or direct phase sequence vectors.*

All that has been said above with regard to unbalanced *voltages* is obviously also applicable to unbalanced *currents.*

* But rotating the BC triangle *clockwise* and the CA triangle *counter-clockwise.*

§ 199. Power Factor of Unbalanced System

In a single-phase circuit in which the voltages and currents obey the simple sine law, the power is given (§ 7) by VI cos θ, where V, I and cos θ stand for the p.d., the current and the phase difference between these respectively; and cos θ is defined to be the power factor of the circuit.

The product VI cos θ may be regarded as the projection on the horizontal axis of a vector of length VI (the volt-amperes or apparent power) which makes an angle θ with that axis. The projection of the same vector on the *vertical* axis is VI sin θ, and is spoken of as the *wattless power*. Thus the true or *wattful power* and the *wattless power* may be regarded as the two components of a vector representing the *apparent power* or volt-amperes.

It will be evident that in a single-phase circuit the power factor has a perfectly definite meaning, and that it might be defined in a number of different ways:

(1) As the cosine of the angle of phase difference between the p.d. and current.

(2) As the ratio of the true or wattful power to the apparent power or volt-amperes.

(3) As the cosine of an angle whose tangent is equal to the ratio of the wattless to the wattful power.

All these definitions lead to the same result, and this remark also holds good for a *balanced* polyphase circuit.

In the case of an *unbalanced* polyphase load, however, the term "power factor" ceases to have any physical meaning, and we find that it may be defined in several ways which lead to results *inconsistent* with each other. This accounts for the fact that there exists at the present moment no generally accepted definition of the term in its application to an *unbalanced* polyphase system.

We shall illustrate the above remarks by considering the simple case of a two-phase system, but the same procedure is applicable to a three-phase system.

Using the suffixes 1 and 2 to distinguish the two phases, we have

true or wattful power in phase 1—$V_1I_1 \cos \theta_1$
" " " " 2—$V_2I_2 \cos \theta_2$

Similarly, the wattless powers are $V_1I_1 \sin \theta_1$ and $V_2I_2 \sin \theta_2$ respectively.

Now there are a number of possible ways of defining the power-factor of the system:

(*a*) As the mean of the power-factors of the phases, *i.e.* $\frac{1}{2}(\cos \theta_1 + \cos \theta_2)$. A knowledge of this would not convey

much useful information, and the value of $\cos\theta_2$ and hence also $\frac{1}{2}(\cos\theta_1 + \cos\theta_2)$ become indeterminate when only the first phase is loaded.

(*b*) As the ratio of the total power to the *arithmetic* sum of the volt-amperes of the two phases. This gives for the value of the power factor

$$\frac{V_1I_1\cos\theta_1 + V_2I_2\cos\theta_2}{V_1I_1 + V_2I_2}$$

(*c*) As the cosine of the angle whose tangent is

$$\frac{\text{total wattless power}}{\text{total wattful power}}$$

This leads to the value

$$\frac{(V_1I_1\cos\theta_1 + V_2I_2\cos\theta_2)}{\{(V_1I_1\sin\theta_1 + V_2I_2\sin\theta_2)^2 + (V_1I_1\cos\theta_1 + V_2I_2\cos\theta_2)^2\}^{\frac{1}{2}}}$$

Now it is easy to see that the denominator of the above fraction is the resultant of two vectors, V_1I_1 and V_2I_2, making angles θ_1 and θ_2 respectively with the horizontal axis. Hence our definition may be put into the following alternative form: The power factor of an unbalanced system is the ratio of the total power to the *vector* sum of the volt-amperes.

The three definitions (*a*), (*b*) and (*c*), while yielding the same value for the power-factor in the case of a balanced system, lead to totally different results in the case of unbalance of the phases. A considerable amount of controversy has arisen in connection with the best way of defining the power-factor of an unbalanced system; the weight of opinion being in favour of the adoption of definition (*c*).

§ 200. Phase Converters

When a polyphase system is seriously unbalanced, various difficulties arise. The supply is unsatisfactory for a lighting load, owing to the different values of the voltages across different phases; the overload capacity and the efficiency of motors are reduced; and the output of the generating plant is reduced in a similar way. The reduction in output is immediately obvious from the fact that the windings of the three phases will have different currents flowing through them, and that one phase may have reached the safe temperature limit while the others are still considerably below it. In order to deal with these difficulties, special correcting devices are sometimes used which are known as *phase converters*.

One type of phase converter which has been adopted in practice is the *shunt synchronous phase converter*.* In order to understand

* *General Electric Review*, vol. xx. p. 479 (1917).

the action of such a machine, we may suppose that across the terminals of the generator supplying the unbalanced load there is connected an ideal synchronous motor whose windings are of negligible impedance. Since the e.m.f.s generated by the three phases of this motor are equal, and since there is no drop in the machine itself, its p.d.s will be equal to the e.m.f.s, and will therefore form a balanced system. Hence such a machine—if one could be obtained—would necessarily impress a *balanced* system of voltages on the mains. But if the terminal voltages across the generator become balanced, the *currents* which the generator supplies to the network must also be balanced. Now since the load currents are unbalanced, it follows that the currents in our ideal motor must form a balanced system of opposite phase sequence to that of the currents supplied by the generator. By impressing a balanced system of voltages on the mains, the ideal motor acts as a *phase voltage balancer*. The generator phase sequence currents when added vectorially to the ideal motor currents of reversed phase sequence yield the unbalanced load currents, and in spite of an unbalanced load the generator voltages and currents remain constant, and the generator maintains its full overload capacity.

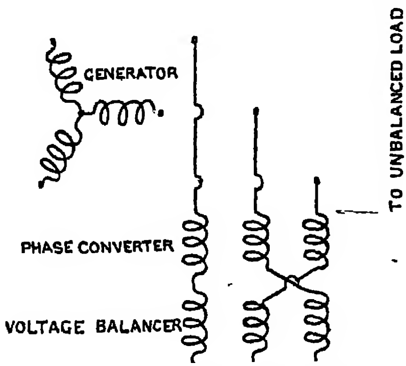


FIG. 232.—Connections of Shunt Phase Converter.

Now although a synchronous motor with negligible impedances of its windings is an impossibility, its equivalent may be obtained by using an ordinary synchronous machine and injecting into its windings e.m.f.s *exactly* sufficient to neutralize the drops. But since the drops are due to currents of opposite or reversed phase sequence (as compared with the generator currents), it follows that the injected e.m.f.s must also be of reversed phase sequence. Such e.m.f.s are provided in practice by an auxiliary generator which has been termed a *voltage balancer*, and whose windings are connected in series with the windings of the phase converter. The voltage balancer is driven by the converter, to which it is mechanically coupled. The e m.f.s generated by the voltage balancer must obviously be sufficient not only to compensate for the drops in the phase converter, but also for those in the voltage balancer itself. The general arrangement of connections of such a phase converter set is shown in Fig. 232.

The *shunt* type of phase converter just described is suitable for

securing balanced voltages and generator currents for an entire system of distribution. Where it is desired to prevent the unbalancing effect due to a large *localized* single-phase load, another type of synchronous converter, termed the *series phase converter*, is more suitable. This type of converter consists of a two-phase synchronous machine one of whose windings is connected *in series* with the single-phase load, the two-phase currents required for the converter being obtained by the use of a T-connection in conjunction with an auto-transformer. The arrangement of connections will be understood by reference to Fig. 233 (a), the topographic and voltage vector diagram being shown in Fig. 233 (b). Phase OB of the converter contains 1·732 times the number of turns in phase OD, so that the voltage of phase OB is $\sqrt{3}$ times that of phase OD. The p.d. across the load is equal to the line p.d. The positive directions of the p.d.s and currents being those indicated by the arrow-heads in Fig. 233 (a), the correct direction of the arrow-heads in the voltage vector diagram are those shown in Fig. 233 (b). Let i^{A}, i_B and i_C stand for the three line currents, i for the load current, and i_{AO} for the current flowing through the half AO of the auto-transformer winding, instantaneous values being understood. Then we must have the relations

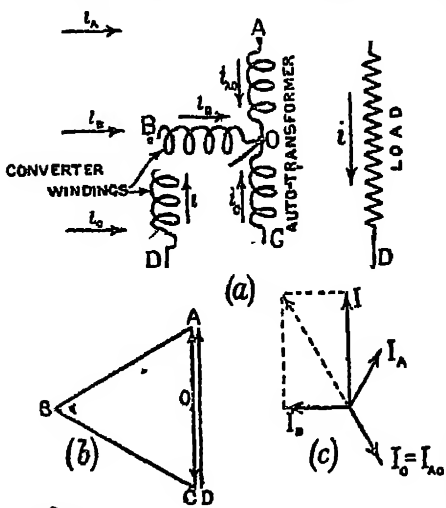


FIG. 233.—Connections and Vector Diagrams of Series Phase Converter.

$$i_A + i_B + i_C = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$i_{AO} + i_B + i + i_C = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Further, if we neglect the magnetizing current of the auto-transformer, we have approximately

$$i_{AO} = i_C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Let us assume, for the sake of simplicity, that the load is a non-inductive one. Then i will be in phase with the p.d. across AD, and will therefore be represented by a vector I (see Fig. 233 (c)) parallel to DA in Fig. 233 (b). Now a reference to Fig. 233 (b) shows that a current in phase with DA will be in phase opposition

to OC; this means that i flows in a direction opposed to the p.d. across DO, and hence in the direction of the e.m.f. (the p.d. and e.m.f. balancing each other if the drop in the winding be neglected). But if i flows in the direction of the e.m.f. generated by the phase DO of the converter winding, this phase must act as a *generator*, supplying power to the load. Hence, neglecting losses, the phase BO, which must act as a *motor*, receives an amount of power equal to that in CO; but since the p.d. across BO is $\sqrt{3}$ times that across CO, it follows that the current in BO is $\frac{1}{\sqrt{3}}$ times that in CO. Further, the phase BO being the motor phase and hence absorbing power, the current in it will be in phase with the p.d. (and not with the e.m.f. as in phase CO), and hence will be represented by a vector I_B (Fig. 233 (*c*)), parallel to the vector OB in Fig. 233 (*b*) and equal to $\frac{1}{\sqrt{3}}I$. Next, using equations (2) and (3) above, we have

$$i_{AO} = i_C = -\tfrac{1}{2}(i + i_B)$$

which means that in the vector diagram $i_{AD} = i_C$ is represented by a vector I_C equal and opposite to the resultant of I and I_B. This is easily shown to be a vector equal in length to I_B and 120° ahead of it. Lastly, equation (1) shows that i_A is represented by a vector I_A equal in length to I_C and I_B and making angles of 120° with them.

It is thus seen that the arrangement shown in Fig. 233 enables us to deal with a single-phase load without disturbing the balance of the three-phase system.

The phase converters so far described are machines of the *synchronous* type. Instead of these, *induction* or synchronous machines might be used. An interesting form of *induction phase converter*, which is in actual use in practice, is the single-phase to three-phase converter. It is employed in connection with electric railways for obtaining a three-phase supply for the motors (which are three-phase induction motors) from a single-phase high-voltage line. The converter is primarily a single-phase to two-phase converter, but a three-phase supply is obtained by the use of the Scott T-connection (§ 66). The general arrangement of this type of converter is shown in Fig. 234 (*a*), in which S_1S_2 denotes the secondary winding of a single-phase step-down transformer (D being the middle point of the winding), and AB and CD are the two-phase stator windings of an induction machine provided with the ordinary type of short-circuited rotor. The topographic diagram of the system is shown in Fig. 234 (*b*). A converter of the type considered, which transforms single-phase into two-phase currents, is sometimes termed a *quadrature phase converter*.

The action of the converter depends on the fact that a rotating

squirrel-cage transforms an alternating magnetic field into a rotating one. This action may be explained as follows. Consider first a squirrel-cage which is rotating very slowly in a steady magnetic field occupying a fixed position in space. Owing to the low frequency of the e.m.f.s induced in the rotor conductors, the currents in these conductors will be practically in phase with their e.m.f.s, and hence the magnetic axis of the rotor currents will be in space quadrature with the field; in other words, the rotor currents will give rise to a cross-field, but will exert no direct magnetizing or demagnetizing action on the impressed field.

Consider now the other extreme—that in which the rotor is running at a very high speed in a steady field fixed in space. Owing to the high frequency of the e.m.f.s in the rotor conductors, the currents will now practically be in time quadrature with their e.m.f.s, and the magnetic axis of the rotor currents will be coincident in position with that of the impressed field and in direct opposition to it. We now have no appreciable cross-field, but only a direct demagnetizing effect.

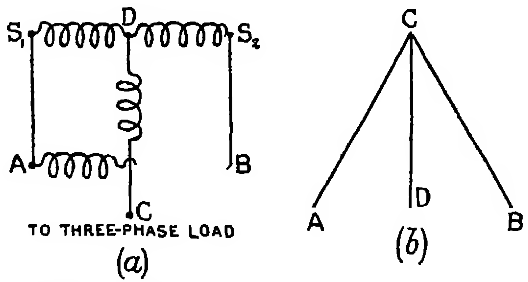


FIG. 284.—Phase Converter for obtaining Three-Phase Currents from Single-Phase Supply.

To sum up, then, we may say that slow rotation of a squirrel-cage in a steady magnetic field produces no appreciable demagnetizing, but only a distorting, effect on the field; while rapid rotation results in practical wiping out of the impressed field.

Since these effects depend on the relative motion of the rotor and the field, they will obviously continue to exist if, the relative velocity remaining unaltered, a velocity of the same magnitude and direction be impressed on both rotor and field.

Hence if a squirrel-cage rotor is made to rotate in a *rotating* magnetic field at a speed not differing greatly from that of the field itself, the slow relative motion will give rise to a cross-field; but the magnitude of the original rotating field will not be appreciably affected. On the other hand, if the rotor speed be made to differ

greatly from that of the field—corresponding to a high relative speed between the two—the original field will be largely wiped out.

We may now pass to the case of a squirrel-cage rotating in an *alternating* impressed field. As is well known, such a field may be replaced by two equal and oppositely rotating fields. If now the rotor is made to run in the direction of one of the component fields at a speed not differing greatly from that of this component, the value of the component will remain practically unaltered; while the other component, whose direction of rotation is opposed to that of the rotor and which consequently has a high speed of rotation relatively to it, will be practically wiped out. The final result, then, is a conversion of the original impressed alternating field into a more or less pure rotating field which has the same direction of rotation in space as the rotor.

From the above it will be evident that in the case of a single-phase induction motor the field when the motor is running at its normal speed is practically a *rotating* field, and if the stator is provided with a second winding arranged in space quadrature to the first winding, an e.m.f. will be induced in the winding which is in time quadrature with that in the first winding. By connecting a two-phase load across the two windings, a two-phase supply becomes available.

A reference to the topographic diagram of Fig. 234 (*b*) shows that if the triangle ABC is to be an equilateral one, the e.m.f. induced in the winding AB must be to that in the winding CD as $2 : \sqrt{3}$. Accordingly, the number of turns in CD is $\frac{\sqrt{3}}{2}$ times that in AB.

APPENDICES

APPENDIX I (see p. 47)

Flux Waves due to Various Types of Polyphase Windings

The shape of the resultant travelling wave produced by a polyphase winding depends on the number of slots per pole per phase. In order to explain the method to be used in determining the nature of the resultant wave, we shall take the simple case of one slot per pole per phase. We shall suppose the currents traversing the windings to vary in time according to the simple harmonic law; so that the instantaneous currents may be represented by the projections on a fixed straight line of three rotating vectors making angles of 120° with each other. We shall further assume that the air-gap of the machine is uniform along the entire circumference, and that the reluctance of the iron path is negligible; so that the induction or flux density at any point of the air-gap circumference is proportional to the magnetic p.d. across the gap at that point. Lastly, we shall assume the slots in which the winding is arranged to be infinitely narrow.

Consider first the conductors belonging to one phase. With one slot per pole, we have an arrangement of (infinitely narrow) groups of conductors the direction of the current in which changes sign as we pass from one group to another. Corresponding to these space reversals of current we have space reversals of the flux; and from the symmetry of the arrangement it is not difficult to see that the points of flux reversal lie on the middle planes of the (infinitely narrow) slots. Referring to Fig. A, let AA′ represent the middle line of a slot belonging to a certain phase (the slots belonging to the other phases are not shown in the diagram). Then along AA′ the tangential component* of the magnetic force is everywhere zero. Consider now any point P in the air-gap, and draw a radial line through P. This line represents the line of force passing through P. Connect the extremities of this line with A and A′ by means of any lines (shown dotted in diagram) drawn arbitrarily inside the iron, and consider the m.m.f. around the closed path APA′A so formed. Since the drop of magnetic potential along AA′ is zero (the tangential magnetic force being zero at every point of it), and since similarly the drop along the dotted portions of the path is zero (owing to the assumed infinite permeability of the iron), the entire m.m.f. is represented by the magnetic potential

* *I.e.* the component along AA′.

difference across the opposing iron surfaces at P. But the m.m.f. is equal to $4\pi \times$ total c.g.s. current linked with the closed curve. Hence

$$\left.\begin{array}{r}\text{Magnetic p.d. between}\\ \text{iron surfaces at P}\end{array}\right\} = 1{\cdot}257 \times \tfrac{1}{2}\text{ ampere conductors per slot}$$
$$= {\cdot}628 \times \text{ampere-conductors per slot.}$$

Now it is obvious that this magnetic p.d. will, with the one slot per pole per phase arrangement assumed, remain constant for all positions of P until the next slot is reached. On passing this slot, we see that the magnetic p.d. changes sign (consider chain-dotted lines and closed curve AP′A′A), and again remains constant until the next slot (belonging to the same phase) is reached; and so on.

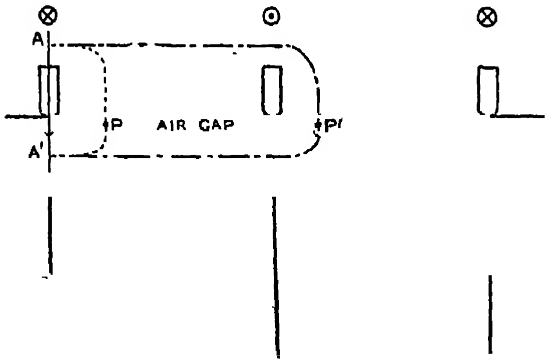


FIG. A.—Showing Distribution of Magnetic P.D. due to One Phase of Unislot Three-Phase Winding.

The magnetic p.d. may therefore be represented by the rectangular curve shown in the lower part of the diagram; and since the magnetic force = magnetic induction = $\dfrac{\text{magnetic p.d.}}{\text{length of gap}}$, the same curve will also, to a different scale, represent the distribution of the magnetic force or induction in the gap due to the current in one phase.

We are now in a position to consider the resultant effect due to the joint action of all three phases. In Fig. B (*a*) are shown the positions of the infinitely narrow slots of all three phases, the different phases being indicated by full, interrupted, and dotted lines. The next diagram (*b*) shows the various component magnetic p.d.s and the resultant p.d. * (or, to a different scale, resultant induction) due to the joint action of the three phases at the instant when the current in the full-line phase has reached its maximum positive value. The succeeding diagrams (*c*), (*d*), (*e*), and (*f*) show the components and the resultant at successive instants separated by intervals of $\frac{1}{24}$th of a period (corresponding to advances of the rotating vectors through 15°).

* Indicated by a heavy line.

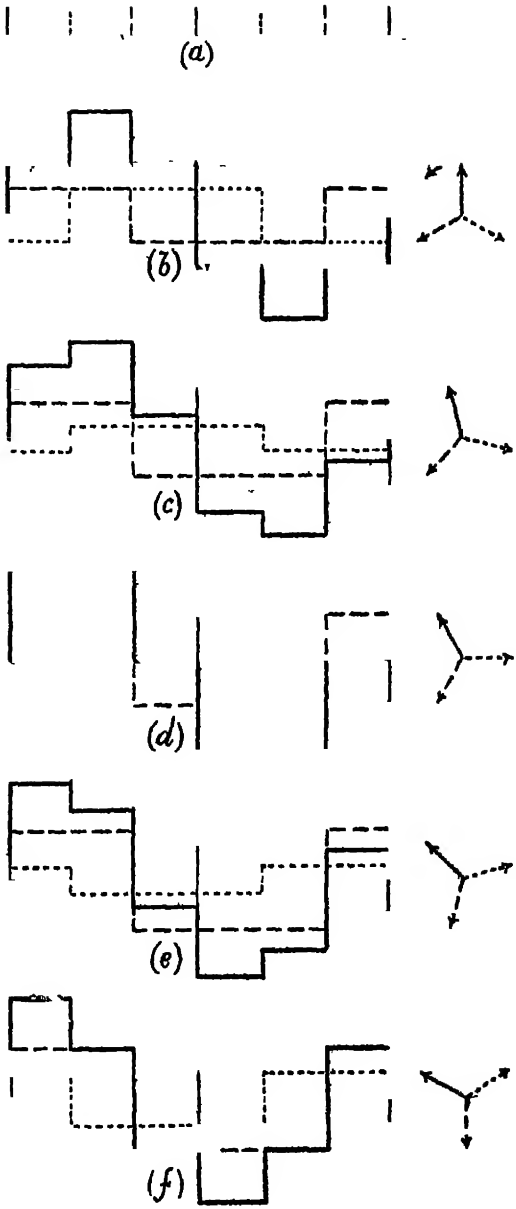


FIG. B.—Progression of Wave due to Unislot Three-Phase Winding.

It will be seen that during the $\frac{1}{6}$th of a period corresponding to the diagrams the resultant wave changes its shape considerably; this change of shape is accompanied by a variation in the total flux per pole amounting to about 18 per cent. After $\frac{1}{6}$th of a period the wave returns to its original shape, after having advanced through $\frac{1}{6}$th of a wave-length.

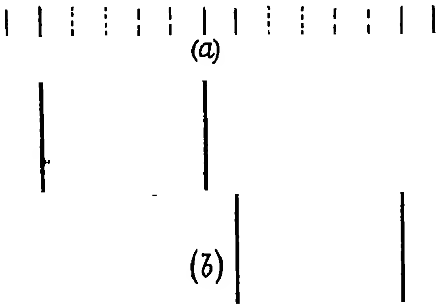


FIG. C.—Showing Distribution of Magnetic P.D. due to One Phase of a Three-Phase Winding having Two Slots per Pole per Phase.

The principle underlying the construction of the diagrams of Fig. B may be applied to more complicated cases—*e.g.* to that of two slots per pole per phase. In Fig. C (*a*) is shown the arrangement of slots for this case, while Fig. C (*b*) gives the shape of the component due to the full-line phase alone. By drawing the components due to the other phases and finding the resultant, and repeating the construction for instants separated by equal successive intervals of time as in Fig. B, it will be found that the resultant wave changes its shape much less, that it forms a much closer approximation to a sine wave, and that the flux per pole varies to a much smaller extent than with one slot per pole per phase.

In general, it will be found that the more the winding is distributed, the closer is the approach to a sine wave, and the less the variation of wave-shape and flux per pole.

APPENDIX II (see p. 47)

TOPOGRAPHIC METHOD AND PHASE SEQUENCE

A method which is extremely helpful in problems relating to polyphase systems is that known as the *topographic method*, so called by its originator, C. P. Steinmetz.

The method is essentially a method of representing the varying potentials and potential differences of alternating current systems.

Consider any point P of a circuit, and suppose that its potential varies according to the simple harmonic law between a certain maximum and a certain minimum value. Then the simplest graphical method of representing the variations of potential is as follows. Describe a circle having its centre on the vertical axis at a distance above the origin equal to half the algebraic *sum* of the maximum and minimum values of the potential,

and of radius equal to half the algebraic *difference* of those values. Let a point move round this circle at a constant speed corresponding to the frequency (one revolution per cycle); then the vertical distance from the horizontal axis of the point in the circle gives the instantaneous value of the potential of P.

Take, *e.g.*, the case represented in Fig. D (*a*). Here we suppose that AB is a winding in which an alternating e.m.f. is being generated, the end A of the winding being *earthed*, so that the potential of A is always zero, and in the topographic diagram will be represented by the origin. The potential of B will fluctuate between a certain maximum positive value and an equal negative value, and in the topographic diagram will be represented by the ordinate of the point B which describes a circle about the origin, of radius equal to the maximum numerical value of the potential of B. Similarly, the potential of any other point such as P in the winding is represented by a corresponding point P in the topographic diagram.

If now we suppose both A and B insulated, but the mid-point M of the winding earthed, then the topographic diagram assumes the form shown in Fig. D (*b*).

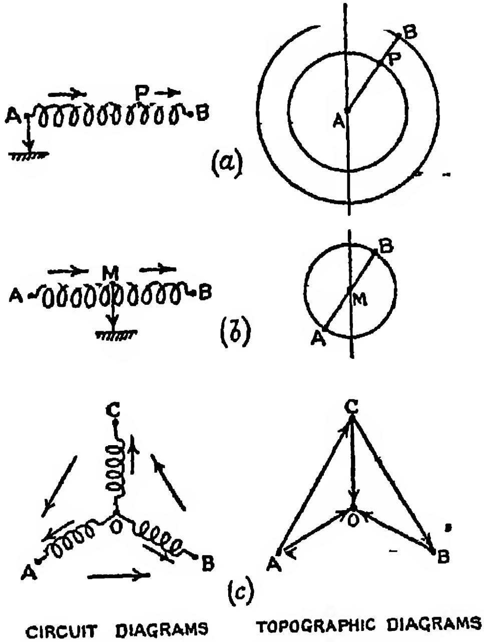


FIG. D.—To illustrate Principle of Topographic Method.

The topographic diagram is essentially a diagram of *points*: corresponding to each point in the circuit diagram (left-hand diagrams in Fig. D) there is a certain point in the topographic diagram. The assemblage of points which form any portion of a circuit corresponds to an assemblage of points in the topographic diagram which form a continuous curve.

If we suppose that the e.m.f.s induced in the various portions of the circuit AB in Fig. D (*a*) are in phase with each other, the potentials of the various points will reach their maxima values *simultaneously*, and hence in the topographic diagram the corresponding points must lie on the same straight line AB. The topographic diagram of the circuit is thus in this case a straight line. In general, the topographic diagram of a circuit is represented by a curve.

The most valuable feature of a topographic diagram is the fact that the r.m.s. p.d. between any two points of a circuit is proportional (and, to a suitable scale, equal) to the length of the line joining the corresponding points in the topographic diagram.

Take, for example, the points P and B in the left-hand or circuit diagram of Fig. D (*a*). Corresponding to these we have the points P and B in the right-hand or topographic diagram. Since the instantaneous potentials of P and B are given by the distances of their vertical projections from the origin, the instantaneous p.d. between them is the difference of these distances, and this is equal to the projection on the vertical axis of the straight line joining P and B. Hence, to a suitable scale, the length of PB (which gives the maximum value of the p.d.) may be made to represent the r.m.s. value of the p.d.

The above property of the topographic diagram allows of its being readily transformed into an ordinary vector diagram of voltages. We have merely to affix an arrow-head to the straight line joining any two points of the diagram in order to obtain the ordinary voltage vector which represents the p.d. between those two points (and whose projection on the vertical axis determines the instantaneous value of the voltage). In so transforming a topographic into a vector diagram, it is necessary, in order to avoid confusion, to correlate the vector diagram with the circuit diagram in a perfectly definite manner, as follows.

We first draw the *circuit diagram.* This is a diagram showing the connections of the circuit, and also, by means of arrows, *the directions which are assumed as positive.*

We next construct the topographic diagram, and in transforming it into a vector diagram *we have to choose the directions of the arrow-heads so as to make the vector diagram consistent with the circuit diagram.*

This general principle will be best understood by the consideration of a particular case, and we shall select as our example a star-connected three-phase circuit with earthed neutral. The diagrams relating to this case are shown in Fig. D (*c*). In the circuit diagram, we have assumed the directions of the star p.d.s positive when acting outwards, and the directions of the line p.d.s positive when acting in a counter-clockwise direction. Considering next the topographic diagram of the four points O, A, B, C in the circuit diagram, it is obvious that, owing to the earthing of the neutral, the point O will be represented by the origin in the topographic diagram; and owing to the symmetry of the three-phase system, the points A, B and C of the circuit diagram will be represented by three points equidistant from the origin and situated at the corners of an equilateral triangle. The numerical values of the star and line p.d.s will be represented by the six lines joining the corresponding points in the topographic diagram. In order to complete the conversion of the topographic into a vector diagram, the lines must be provided with arrow-heads. This is easily done as follows. Taking the particular position of the (rotating) topographic diagram shown in the figure, and considering the star p.d. across OC, we notice that since C is above O in the topographic diagram, it is at a higher potential than O, so that at the instant considered the p.d. across

OC in the circuit diagram acts in the direction C to O, and is therefore *negative*. Hence the (rotating) vector in the vector diagramw hich represents the p.d. across OC must at the instant considered give a *negative* projection on the vertical axis, and hence in the topographic diagram the arrow-head attached to OC must point *downwards*. Similar considerations enable us to determine the correct directions of the arrow-heads for the other lines, which are as shown in the figure.

Determination of Phase Sequence.

We shall illustrate the use of the topographic diagram by applying it to the solution of what is in many cases an important problem—that of determining the *phase sequence* of the phases of a three-phase system. In such a system the potentials of the three line wires reach their maxima positive values in a definite order. Denoting the line wires by A, B and C, this order may be either ABC or ACB.

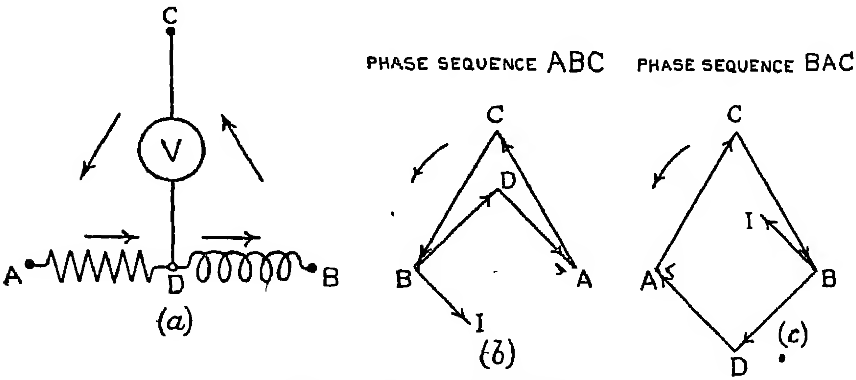


Fig. E.—Method of determining Phase Sequence.

One method of determining the phase sequence is to use the arrangement shown in Fig. E (*a*).* Here A, B, C are the line wires, AD a non-inductive resistance, and DB a pure reactance equal to the resistance. V is a voltmeter connected between the junction of the resistance and reactance and the remaining line wire. The voltmeter reading is compared with the line p.d., and from their relative magnitudes the phase sequence may be inferred, as will be shown presently.

Assume first that the phase sequence is ABC. Then the corresponding points in the topographic diagram must reach their highest positions in the above order, and the topographic diagram of the mains is as shown in Fig. E (*b*). The positive directions being assumed to be those shown in Fig. E (*a*), the arrow-heads of the three line voltage vectors are those shown in diagram (*b*). Assume, for the sake of simplicity, that the voltmeter V is an electrostatic one or that the current taken by it is negligible,

* W. V. Lyon, *Electrical World*. vol. lxix. p. 968 (1917).

so that the current in AD is identical with that in DB. Then this current must (owing to the equality of resistance and reactance) lag behind the voltage BA by 45°, and may be represented by the vector I. Again, since the voltages across AD and DB in diagram (*a*) must be equal and in quadrature with each other, and since their resultant must equal BA in diagram (*b*), the point D in the topographic diagram (*b*) must be equidistant from B and A, and must lie on a circle described on BA as diameter. Now there are two possible positions of D which satisfy the above requirements, one being above BA and the other below. To determine the correct position, we notice that the voltage vector representing the p.d. across AD must be *parallel* to I: hence D must lie *above* BA as shown, and the voltage CD read by V will be *less* than the line voltage, and equal to about 37 per cent. of that voltage.

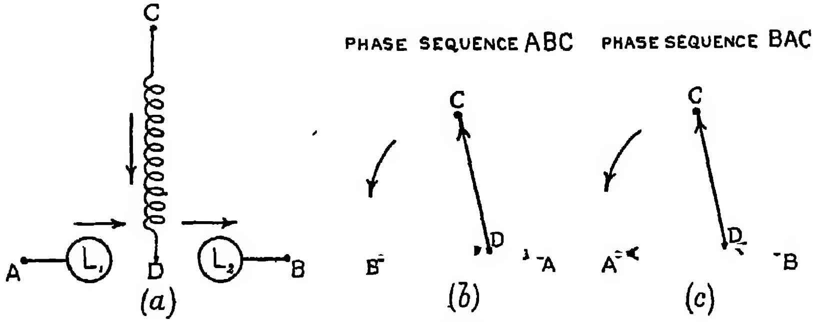


FIG. F.—Method of determining Phase Sequence.

If the phase sequence is BAC, the topographic and vector diagrams are as shown in (*c*), the point D_1 being now *below* AB, and the voltage CD *greater* than the line voltage (being about 137 per cent. of that voltage).

Another method of determining the phase sequence in a three-phase system is shown in Fig. F (*a*).* The only apparatus required are two incandescent lamps L_1 and L_2 and a reactance connected as shown. The normal voltage of the lamps should be half the line voltage, so that when connected in series across AB, with the reactance disconnected, they would be equally and fully incandesced. The topographic diagram of the mains would be the usual three vertices of an equilateral triangle ABC, while the point corresponding to the junction D of the lamps would be represented by the mid-point of the straight line joining A and B. If we now transform the topographic into a vector diagram, then the p.d. across DC will be represented by a vector normal to AB (so long as the points C and D are not connected through a reactance). If next we suppose a reactance connected across CD, and if in the first instance we neglect the shift of the point D in the topographic diagram which results from such connection, then since the current in the reactance lags 90° behind the p.d., this current will be *added* to that in one lamp, and subtracted from that in the other.

* T. W. Waley, *Electrical World*, vol. lxix. p. 466.

One lamp will therefore glow more brightly than the other, and from the topographic diagrams (*b*) and (*c*) of Fig. F it will be seen that with a phase sequence ABC the lamp connected to B will be the brighter, whereas with a phase sequence BAC that connected to A will be the brighter. As a consequence of the increase in one lamp current and decrease in the other, the point D in the topographic diagram gets shifted to the right, and as this alters the phase of the vector DC, the phase of the reactance current will also change, so that D will also suffer an upward displacement, and occupy the position shown in the diagram.

The two methods described are by no means the only ones which may be employed for determining phase sequence, but they have been selected on account of their extreme simplicity.

By some writers the term *phase rotation* is employed to denote phase sequence.

APPENDIX III (see p. 60)

Two-Wattmeter and Other Methods of Measuring Power in Balanced Three-Phase Circuit

The variations in the readings of the two wattmeters used in the two-wattmeter method of power measurement as applied to a balanced load of variable power factor are shown in Fig. G. A reference to Fig. 43 will show that of the two currents, I_1 and I_3, which flow through the current coils of the wattmeters, I_1 leads and I_3 lags. We may therefore speak of the current coil corresponding to I_1 as being in the *leading phase*, and of that corresponding to I_3 as being in the *lagging* phase, and the curves of Fig. G marked "leading phase" and "lagging phase" refer to the wattmeter readings corresponding to I_1 and I_3 respectively.

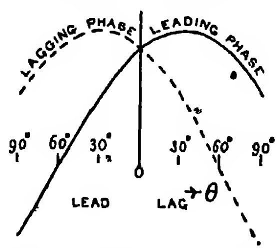


Fig. G.—Balanced Load Readings of Wattmeters in Two-Wattmeter Method of Power Measurement.

In Fig. H is shown an alternative method of measuring the power in a *balanced* three-phase circuit. A single wattmeter is used, with its current coil in one of the line wires. A reading is first obtained with the voltage coil connected as shown by the full lines in Fig. H (*a*), and then the voltage circuit is transferred to the position shown dotted. The first reading is VI cos $(30°-\theta)$, and the second VI cos $(30°+\theta)$, as will be evident from Fig. H (*b*). The sum of the two readings therefore gives the total power (§ 30).

When—as is usually the case—potential and current transformers are used in connection with the wattmeter, then a *single* wattmeter is sufficient

for the measurement of the total power, provided the load is *balanced*. The method of connections used in this case has been termed the *reversed-V* method.

There are two varieties of the reversed-V method: in the one, two

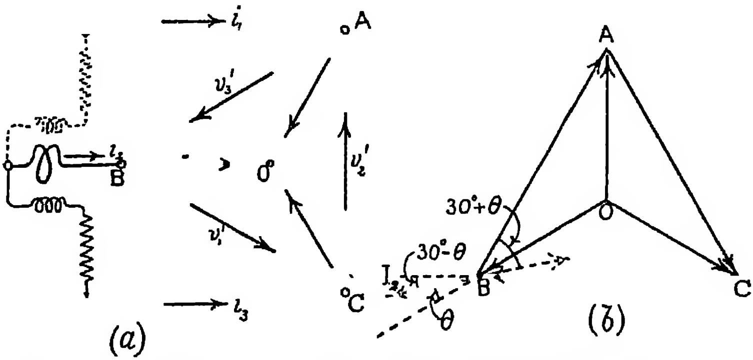


Fig. H.—Power Measurement in Case of Balanced Three-Phase Load.

potential transformers and one current transformer are used; and in the other, two current transformers and one potential transformer.

Referring to Fig. H (*b*), consider the two sides AB and BC (which together form a "V") of the triangle of vectors, and suppose that the vector BA is *reversed*. Then the resultant of OB and BA reversed is a vector parallel to OB and equal to $\sqrt{3}V$, where V is the line voltage. If,

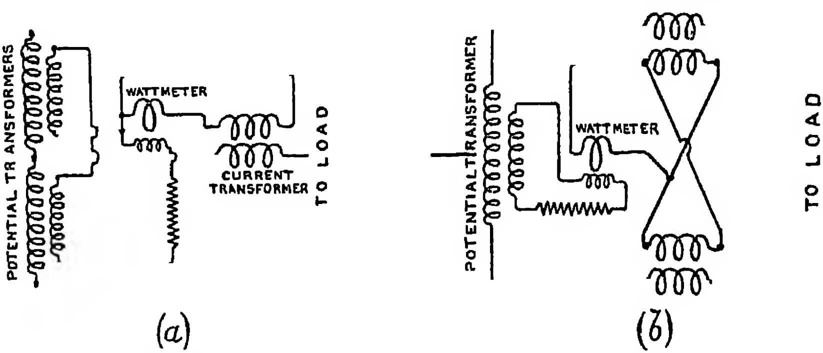


Fig. I.—Single-Wattmeter (Reversed-V) Methods of Power Measurement in Balanced Three-Phase Circuit.

then, the current coil of a wattmeter is traversed by the current I_2 (Fig. H), and if its potential circuit is acted on by a p.d. which is the resultant of V_1' and V_3' reversed, then the reading of the wattmeter will be equal to $\sqrt{3}V\ I\cos\theta$, which gives the total power (I denoting the value of the line current).

The resultant of CB and BA reversed is easily obtained by the use of potential transformers, the primaries of which are connected across AB and BC, while the secondaries are connected so as to give the required resultant. The arrangement of connections is shown in Fig. I (*a*).

Considering next the second variety of the reversed-V method, suppose that we send a current through the wattmeter current coil which is the resultant of I_2 and I_3 reversed. Then it is easy to show that the magnitude of this current is $\sqrt{3}I$, and that it lags behind CB by the angle θ (see Fig. H). If, then, the potential coil is subjected to the action of V_1', and if the current coil is traversed by a current which is the resultant of I_3 and I_2 reversed, the wattmeter reading will be $\sqrt{3}VI \cos\theta$, which again represents the total power. The arrangement of connections for this case is shown in Fig. I (*b*).

APPENDIX IV (see p. 126)

The Magnetic Properties of Lohys and Stalloy

Stalloy has not only a much lower hysteresis and eddy-current loss than Lohys, but—for the range of inductions employed in transformer work—a higher permeability. The following table contains corresponding values of B and H for Lohys and Stalloy :—

H =	0·5	1·0	1·5	2·0	2·5	3·0	3·5	4·0	4·5	5·0	5·5
B Lohys	900	4,000	6,200	7,450	8,350	9,000	9,500	10,000	10,500	10,950	11,340
B Stalloy	2,500	5,000	6,600	7,900	9,000	9,850	10,250	10,600	10,850	11,100	11,450

The following table shows the loss, in watts per lb. of material, for various thicknesses of sheets, at an induction of 10,000 and a frequency of 50.

Thickness of sheets, inches	0·014	0·016	0·018	0·020	0·022	0·024	0·026
Loss in watts/lb. Lohys .	1·35	1·44	1·55	1·67	1·81	1·96	2·13
Loss in watts/lb. Stalloy .	0·72	0·755	0·788	0·82	0·852	0·886	0·92

The relation connecting the loss per lb. with B, for stalloy sheets 0·018 inch thick, at a frequency of 25, is as follows :—

B	3,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000	11,000	12,000	13,000
Loss in watts/lb	0·047	0 070	0·100	0·140	0·180	0·230	0·290	0 350	0·41	0 49	0·57

APPENDIX V. (see p. 130)

Methods of Cooling Transformers and Prevention of Sludging

Methods of cooling which involve the use of running machinery are unsuitable where large transformers are required to work without frequent supervision—as in the case of outdoor substations. This has led to a development of the self-cooling method which has made it applicable to transformers of very large output (up to about 8000 k.v.a.). The difficulty of applying the simple self-cooling method to large transformers arises from the fact that the cooling surface of the containing tank, even when provided with ribs, is insufficient for the purpose. It accordingly becomes necessary to increase it, and this is done by attaching to the tank a number of radiators (from 3 to 24, according to the size of the transformer). This arrangement is shown in Fig. J. Each radiator consists of a number of steel tubes of elliptic cross-section welded to two headers. Both the tank and the headers are provided with standard pipe flanges, and the radiators are bolted to the tank through cast-iron connectors as shown in Fig. J. The hot oil which rises to the top of the tank passes through the connectors into the top headers of the radiators, thence down the radiator tubes, where it is rapidly cooled, and finally, through the bottom headers and connectors, back into the tank.

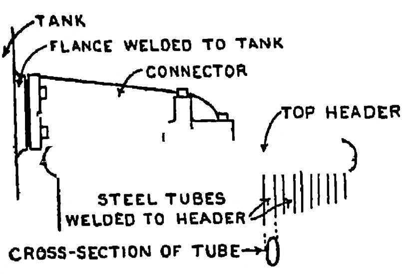


Fig. J.—General Arrangement of Radiator Transformer Tank.

One of the most serious troubles connected with the use of oil as a cooling and insulating medium in transformers is that known as "sludging." This consists in the formation of a deposit or sludge on the core, windings,

and containing tank, which prevents the free passage of heat from the core and windings to the oil, and by blocking the oil circulation channels impedes the free circulation of oil—both effects resulting in a greatly increased temperature rise of the windings and core. The nature of the deposit or sludge varies greatly: sometimes it is a soft light-coloured mud, at other times a hard dark-coloured substance. Experiment seems to indicate * that the formation of sludge is due to the oxidizing action of the air with which the oil is in contact, and that this action is accelerated if the oil is in direct contact with bare copper conductors.

In order to prevent sludging as far as possible, a device known as an "oil conservator" has been fitted to some large modern transformers.†

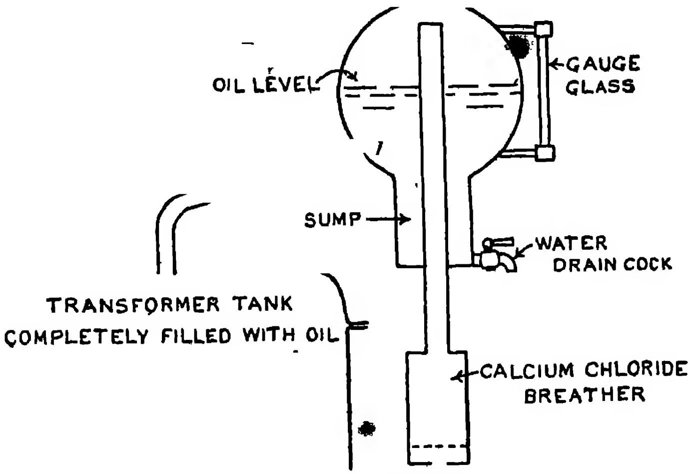


Fig. K.—Oil Conservator.

The object of the "conservator" is to reduce greatly the actual area of contact between oil and air. The arrangement will be understood by reference to Fig. K. An ordinary transformer tank cannot obviously be filled completely, as allowance must be made for the changes of volume due to temperature changes. In the "oil conservator" method the tank is completely filled, and the small free surface of the oil is in the cylindrical vessel which forms the oil conservator. As the volume of the oil expands or contracts, air is expelled from or sucked into the upper part of the conservator. In order to prevent moisture from passing in with the air, a vessel filled with calcium chloride is provided, and the "breathing" of the transformer takes place through the calcium chloride.

* A. C. Michie, *Journal of the Institution of Electrical Engineers*, vol. li. p. 213 (1913).

† *General Electric Review*, vol. xxi. p. 556 (1918).

APPENDIX VI. (see p. 134)

Phase-Shifting Transformers

In certain important classes of measurements, it is necessary to have some means of changing the phase relations of one polyphase (or single-phase) system relatively to another in a continuous manner. For this purpose, a *phase-shifting transformer* or *phase-shifter* is used. The general principle of such a transformer is easily understood. The transformer has two laminated cylindrical cores, one inside the other, as shown diagrammatically in Fig. 31 on p. 41 (the arrangement resembles the stator and rotor cores of an ordinary polyphase induction motor—see Chapter VIII.). The "stator" carries a polyphase primary winding, and the "rotor" a polyphase secondary (of which only one phase may be used if desired). The "rotor" is not allowed to rotate when the transformer is in use, but may be placed in various positions relatively to the "stator" (by means of suitable worm gearing actuating the "rotor"). The stator currents give rise to a rotating field which sweeps across the secondary and induces e.m.f.s in it. At a certain instant, the e.m.f. in a certain phase of the secondary will reach its maximum value. If now we rotate the secondary in the direction of field rotation into a new position, it is evident that the maximum e.m.f. in the secondary phase under consideration will occur *later*, *i.e.* the e.m.f. of that phase has been retarded. Similarly, the e.m.f.s of the remaining phases of the secondary have also been retarded. By rotating the secondary against the direction of field rotation, phase acceleration of the secondary e.m.f.s is obtained.

In order to avoid the complications which would arise from the presence of higher harmonics, the primary winding of each phase is distributed in the core slots in such a manner as to give rise to a practically pure sine wave distribution of the magnetic flux in the air gap.

APPENDIX VII (see p. 174)

Weston Synchroscope

One of the most recent forms of synchronizer is the Weston Synchroscope. This consists of a wattmeter movement, the fixed element of which is connected in series with a non-inductive resistance and across the bus bars (generally through a potential transformer), while the movable element is connected in series with a non-inductive resistance and condenser, and across the terminals of the incoming machine (through a potential transformer). The arrangement of connections is shown in

Fig. L (*a*), where FC denotes the fixed coil or element, and MC the movable coil. This latter is, as in a wattmeter, controlled by a spring and carries a pointer.

Imagine the FC and MC circuits connected in parallel to the *same* source of e.m.f.—say across the bus bars. In Fig. L (*b*) is shown the voltage vector V which acts on both circuits. The current in FC lags (owing to the self-inductance of FC) behind V by a small angle, and is represented by the vector F; while the current in MC falls short of exact quadrature relation with V by the same angle—a result which may be obtained by suitable adjustment of the resistance in the MC circuit.

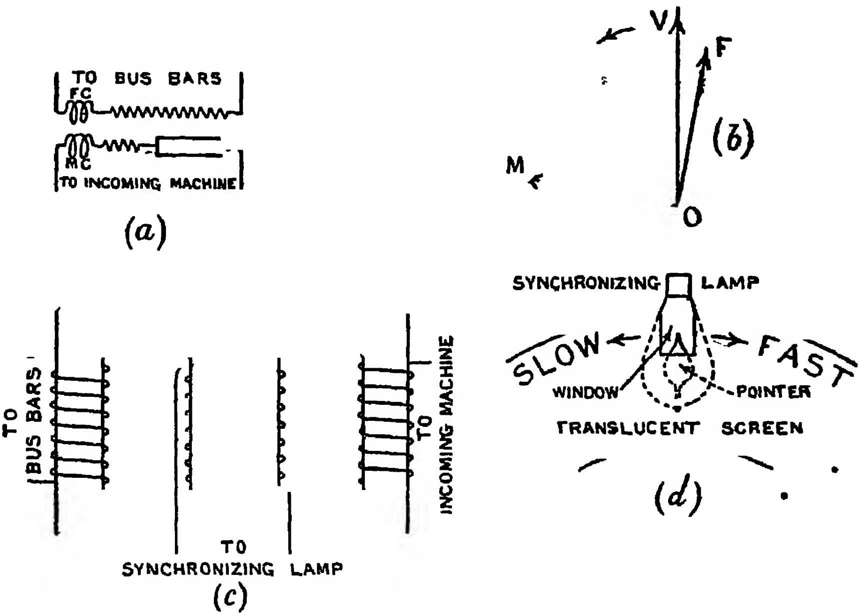


Fig. L.—Weston Synchroscope.

Hence F and M will be in exact phase quadrature, and the pointer will stand at zero, there being no deflecting torque.

If now we imagine the MC circuit transferred to the incoming machine, the pointer will still remain at zero if the incoming machine is in proper phase relation and has the correct frequency. But if the frequency differs slightly from that of the bus bars, the vector M in the vector diagram will revolve slowly relatively to the vector F, and as it does so the pointer will slowly oscillate to and fro. The greater the frequency difference, the more rapid will be the oscillations of the pointer.

The instrument in this form would be practically useless, as it would not be capable of indicating the correct instant of closing the main

switch of the incoming machine. It is true that—as we have seen—when the incoming machine is in correct phase relation, the pointer will be at zero; but unfortunately it will also be at zero when the incoming machine is in direct opposition to its correct phase.

This difficulty is overcome in a most ingenious manner by an arrangement which renders the pointer *invisible* during the half-oscillation (from one extreme position to the other) in the course of which the pointer passes through its zero position when the machine is directly opposed to its correct phase, and only allows the pointer to be seen during the half-oscillation when the pointer passes through zero with the incoming machine in its correct phase.

The suppression of the visibility of alternate half oscillations is effected in a very simple manner, by the use of a small synchronizing transformer of the type shown in Fig. L (*c*). The connections of this transformer are arranged so that at the instant of correct phase relation the magnetic fluxes due to the primaries both pass through the middle core on which the secondary is wound, and the lamp connected across the secondary is fully incandesced. On the other hand, when there is direct opposition to correct phase relation, there is no flux through the middle core, and the lamp is dark. This lamp is fixed inside the synchroscope, behind a translucent glass screen which takes the place of the usual scale. The pointer is arranged to move *behind* the screen, and between the lamp and the screen, so that when the lamp is bright the *shadow* of the pointer is thrown on the screen. To mark the zero position of the pointer, the translucent screen is provided with a small opening or window, shown in Fig. L (*d*), and when there is no deflecting couple acting on the movable coil the pointer stands in the middle of the window. The correct instant for closing the switch is when the pointer, strongly illuminated by the synchronizing lamp, is seen in the middle of the window. But the instrument does more than merely indicate the instant of correct phase relation; it shows whether the incoming machine is fast or slow. Owing to the suppression of the visibility of alternate swings, the swings will all have the same direction (from left to right or right to left, as the case may be), and the illusion will be produced that the pointer is *rotating* one way or the other. It is not difficult to see, by a consideration of the relative motion of the vectors F and M in Fig. L (*b*), that when M is gaining on F the pointer will appear to rotate one way, whereas if M is falling behind F, the pointer will appear to rotate the other way. Two arrows, marked "fast" and "slow" respectively, indicate the direction in which the speed of the incoming machine must be altered in order to secure exact synchronism.

APPENDIX VIII (see p. 191)

Predetermination of Alternator Regulation according to the Standardization Rules of the American Institute of Electrical Engineers

For the purpose of carrying out the method recommended by the American Institute of Electrical Engineers, two experimental curves are required, viz. (1) the *open-circuit* curve, connecting terminal voltage on open circuit with exciting current; and (2) the full-load zero power-factor curve, connecting exciting current with terminal p.d. when the armature current is maintained constant at its full-load value and lags 90° behind the p.d.

From the above two curves, a third curve, which is the load curve (exciting current—p.d.) for the given value of the power factor, may be derived as follows. Draw a horizontal line OA (Fig. M), and below it lay off a line OB making an angle ϕ with it, where $\cos\phi$ is the given value of the power factor. Next, lay off a horizontal distance OR equal to the armature resistance drop, and at R erect a perpendicular RS. Assuming any value of the exciting current, and referring to the two curves previously determined (exciting current—p.d. (1) on open circuit and (2) with full-load current at zero power factor), we take the difference of the two ordinates, and, using this as radius, we strike an arc with O as centre. Using the intersection X of this arc with the perpendicular RS as centre, we strike an arc with a radius equal to the open-circuit p.d. corresponding to the assumed value of the exciting current. The intersection V of this arc with OB determines OV, which is the terminal p.d. corresponding to the assumed excitation. By taking various values of the excitation we determine a number of points which enable us to draw the load curve for full-load current and power factor $\cos\phi$. This load curve, together with the open-circuit curve, immediately enables us to find the regulation of the machine.

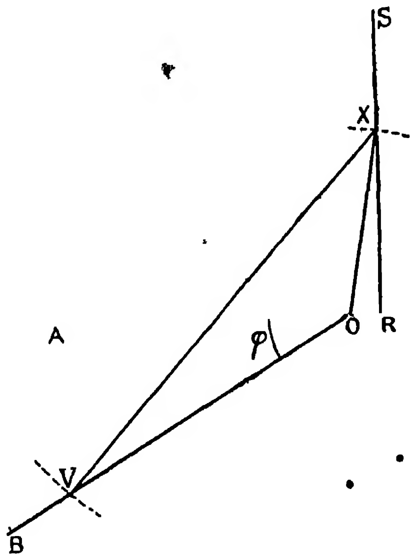


Fig. M.—Construction for determining Regulation of Alternator.

APPENDIX IX (see p. 203)

Heating Tests of Alternators and Transformers

The following modification of Goldschmidt's method is used by the Westinghouse Co. (U.S.A.) in carrying out the heating tests of large three-phase generators.* The generator armature windings are connected Δ-fashion. The generator is run at its normal speed, and is supplied with the normal full-load exciting current, so as to produce the normal field copper losses and the normal iron losses. One corner of the Δ having been opened, an alternating e.m.f. is impressed on the local circuit of the Δ, of sufficient magnitude to produce the full-load current in the armature windings. This e.m.f. may be conveniently obtained from a small transformer, and so the necessity of providing a continuous current machine or secondary battery (which are required in Goldschmidt's method) is done away with.

The balance of e.m.f.s in the Δ may also be disturbed by another method, originally described by A. F. Gustrin.† This consists in leaving out a suitable number of turns or coils in one phase of the Δ, as shown in Fig. N. Gustrin's method was originally intended to be used in connection with the heating tests of three-phase transformers (or of three single-phase transformers connected to three-phase mains).

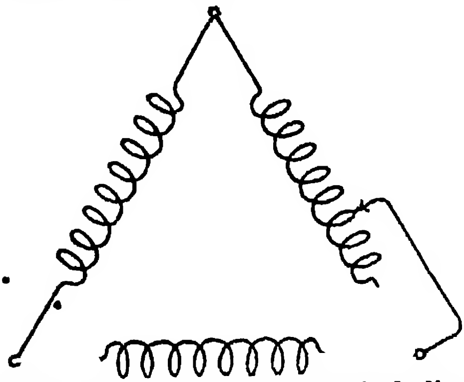


Fig. N.—Gustrin's Method of artificially loading Three-Phase Transformers.

As regards the heating tests of single-phase transformers, the most convenient method, when two similar transformers are available, is to employ the arrangement of connections shown in Fig. 138 (p. 196). This method of coupling transformers is originally due to Dr. Sumpner.

If only one single-phase transformer is available for the purposes of the test, a method similar to that used by Hobart and Punga (§ 109) may be employed. The transformer core and copper losses having been determined (§ 103), the transformer is run on open and on short-circuit alternately for equal successive periods, the p.d.s in the two cases being so adjusted that the iron loss in the first case and the copper loss in the second are equal to the sum of the normal core and copper losses. One inconvenience of this method is the necessity of constantly having to switch over from the one arrangement of connections to the other;

* *Elektrotechnische Zeitschrift*, vol. xxviii p. 911 (1907).

† *Ibid.*, vol. xxviii. p. 574 (1907); or *The Electrician*, vol. lxiv. p. 1029 (1910).

another is the necessity of running the transformer during the open-circuit periods at a p.d. which may be as much as 50 per cent. in excess of the normal p.d.

For mesh-connected three-phase transformers, Gustrin's test is most convenient. If the transformer windings are star-connected, and if two similar transformers are available, the following modification of Goldschmidt's method, due to G. Molnár,* may be used. The connections are shown in Fig. O. The primaries of the transformers are connected in

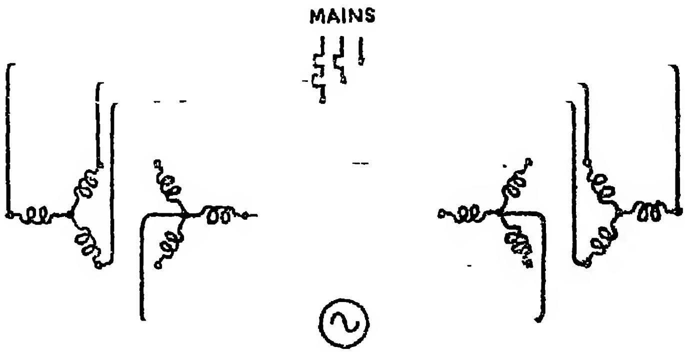


FIG. O.—Molnár's Method of loading Star-connected Three-Phase Transformers.

parallel across the mains. The secondaries are connected so as to oppose each other. An auxiliary source of (single-phase) alternating e.m.f. is then connected across the neutral points of the transformer secondaries, and the currents in the windings are adjusted to their normal full-load values by suitably altering the single-phase e.m.f. Equivalent full-load currents, of course, appear in the primaries.

APPENDIX X (see p. 230)

DETERMINATION OF FRICTIONAL LOSS OF INDUCTION MOTOR

An example of the method explained in § 124 is given in Fig. P, which refers to a 186-volt, three-phase, 70 b.h.p. induction motor having a synchronous speed of 750 r.p.m. at 25 ~ By producing the curve backwards, the frictional loss is found to amount to about 600 watts. This value was confirmed by the method explained towards the end of § 128. The motor when running light at a p.d. of 24·5 volts was found to take 661 watts. On suddenly open-circuiting the rotor, the power

* *Elektrotechnische Zeitschrift*, vol. xxx. p. 450 (1909).

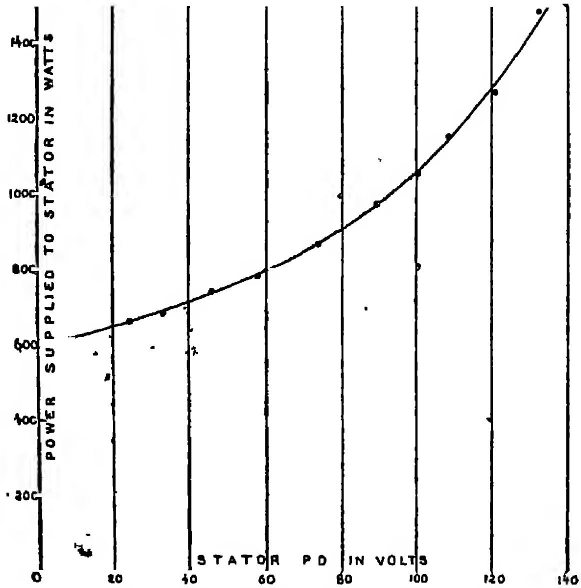


FIG. P.—Relation connecting Power with P.D. in Induction Motor running light.

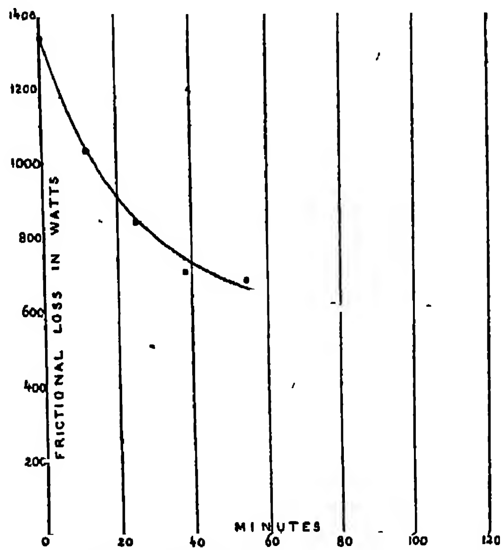


FIG. Q.—Variation of Frictional Loss with Time of Running.

dropped to 58 watts, the frictional loss being thus $661-58=603$ watts. It may be mentioned that when using this second method of determining the frictional loss, it is desirable to run the motor at a voltage considerably below the normal, as the second reading of the wattmeter will in that case be very small—as in the example cited—and greater accuracy will be obtained in the determination of the loss.

When determining the frictional loss of a motor, it is important to bear in mind that this loss does not assume a constant value until after the motor has been kept running for a sufficiently long time. If the loss be determined immediately after starting up the motor, a totally wrong value will be obtained. This is well illustrated by the curve of Fig. Q, which shows the variation of the frictional loss with time in the case of the 70 b.h.p. motor already referred to. It will be seen that at starting the frictional loss has the very high value of 1340 watts, and that this steadily decreases, owing to the heating up of the lubricating oil and a reduction in its viscosity, until after a run of about $1\frac{1}{2}$ hours the loss assumes the constant value of about 600 watts.

APPENDIX XI (see p. 241)

Measurement of Slip

The following method, due to J. Poupelet-Laganterie,* is characterized by great simplicity. Owing to magnetic irregularities in the rotor, when there is relative motion of the rotor and the rotating field, the irregularities travel relatively to the field at a speed corresponding to the slip, and the few magnetic lines which cross and re-cross the shaft of the motor, while the rotating field travels relatively to the rotor, induce in the shaft alternating e.m.f.s of slip frequency. Hence all that is necessary in order to determine the slip is to hold the wires connected to a suitable low-reading instrument (milliammeter or shunted galvanometer) against the ends of the shaft, and to count the number of complete vibrations of the pointer; this (in the case of polarised instruments) gives the slip frequency (if the instrument is non-polarized, *half* the number of complete vibrations gives the slip frequency).

* *Association des Ingenieurs Électriciens, Bulletin,* vol. xiv. p. 167 (1914).

APPENDIX XII (see p. 289)

Starting of Rotary Converters. Self-synchronizing Converter

Some details of the last method mentioned in § 156 are shown in Figs. R and S. Fig. R gives the connections of the two triple-pole double-throw switches used for varying the voltage across the slip-rings. The permanent connections of these switches are shown in Fig. R (*a*), where the switch blades are omitted. The contact set C_1, C_2, C_3 is connected to C_1', C_2', C_3', and to nothing else. The contact set R_1, R_2, R_3 is in connection with three of the converter slip-rings, the diametrically opposite (in an electrical sense) set of slip-rings being permanently connected to one set of the transformer secondary terminals (such as R_1'). The

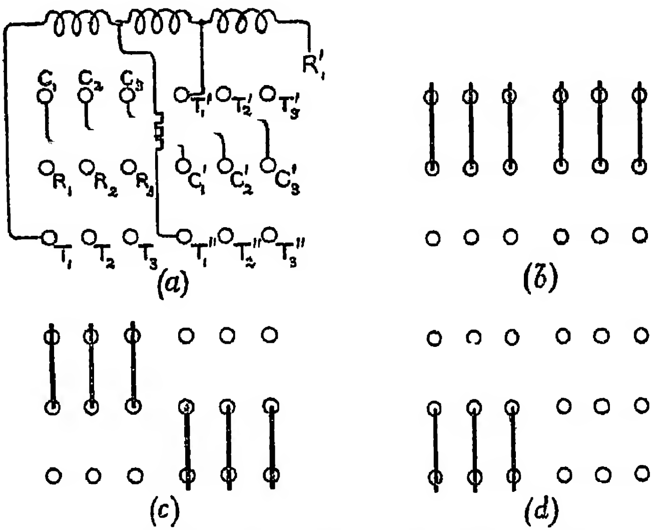


Fig. R.—Starting Switch Connections for Rotary Converter.

remaining set of transformer terminals is connected to T_1, T_2, T_3, while the contacts marked T_1', T_2', T_3' and T_1'', T_2'', T_3'' are in connection with transformer tappings giving one-third and two-thirds of the full voltage respectively. For the sake of clearness, only one of the transformer secondaries is shown in Fig. R (*a*). Figs. R (*b*), (*c*), and (*d*) show the three positions of the switch blades corresponding to one-third, two-thirds, and full voltage respectively across the slip-rings.

Fig. S shows the connections of the field break-up and reversing switch. This also is a double-throw switch, and the number of poles is greater by unity than the number of sections into which the field winding

is broken up. The switch shown in Fig. S is a five-pole one, corresponding to four sections of the field. The connections are so arranged that in the upper or normal position of the switch (full lines) the field rheostat is included in the circuit, while in the reverse position (dotted

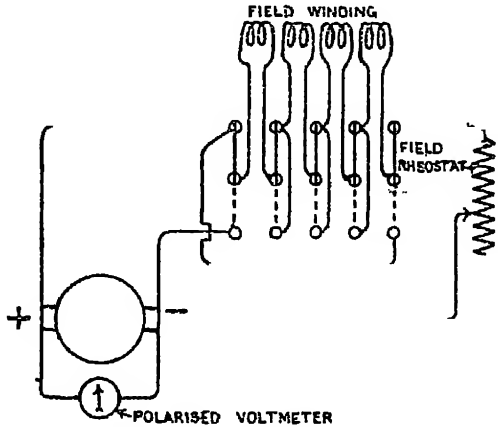


Fig. S.—Connections of Field Break-up and Reversing Switch.

lines) it is cut out, as this position is only a temporary one, and as it is then desirable to get as large a current as possible through the field.

The actual procedure adopted in starting up a converter by this method is as follows. A polarized voltmeter being connected across the brushes, and the field break-up switch being in the "off" position, the starting switches are thrown into the positions shown in Fig. R (*b*), one-third of the normal voltage being thereby applied to the slip-rings. The hysteresis and eddy-current torque is sufficient to start the machine, although a heavy current is drawn from the mains. The polarized voltmeter connected across the brushes is thrown into vibration, the frequency of which steadily decreases as the speed of the machine increases. This is due to the fact that the speed of the rotating field *in space* steadily decreases with increasing speed of armature rotation, so that the frequency of the alternating p.d. between the brushes decreases. Finally, when the machine becomes locked into synchronism, the rotating field becomes stationary in space, and the brush voltage is no longer an alternating but a continuous one. If the voltmeter indicates normal polarity, the field switch is thrown into its normal position, and the starting switches are successively thrown into the positions R (*c*) and R (*d*), when the starting process is completed. If, however, when synchronous speed is reached the voltmeter indicates a reverse polarity, the field switch is thrown into the lower or reverse position. The result is that the brush p.d. now tends to wipe out the existing field, and as the field gets weaker the torque acting on the armature decreases, and momentary retardation

takes place. During this momentary retardation, the rotating field slowly travels *in space* in a direction opposed to that of the armature rotation. At a certain instant, the rotating field will have come into a position such that it simply tends to produce a cross-flux in the converter field-magnet, but no main flux either one way or the other. At this instant, the brush p.d. becomes zero, as indicated by the voltmeter, and it is necessary that the field switch should now be thrown into the normal position, as further retardation of the armature causes the rotating field to act inductively on the converter field in the right direction (so as to impart to it its normal polarity); and if the field switch were allowed to remain in the reverse position, it would prevent the field from building up. When the field switch has been restored to its normal position, a field of the correct polarity begins to appear, the driving torque increases, and the armature regains synchronous speed. The starting process is then completed as already explained, by raising the p.d. across the slip-rings to two-thirds of and finally to full normal voltage.

A disadvantage of the above method of starting rotary converters is the uncertainty of the polarity acquired by the machine when it has become synchronized: if the polarity is wrong, it must be reversed as described, by the operation of "slipping a pole," and this involves loss of time. This disadvantage is overcome in a method devised by Dr. E. Rosenberg,* in which an auxiliary starting motor of the induction type is used. Instead, however, of connecting the starting motor across the supply mains, it is connected *in series* with the converter. The arrangement of connections for a diametrally connected six-phase converter is shown in Fig. T, where, for the sake of simplicity, the connections of only one of the transformer secondaries are indicated. The auxiliary motor is generally provided with a squirrel-cage rotor, and is designed so that when the starting switch is closed the starting current does not exceed about 30 per cent. of the converter full-load current, the p.d. across the slip-rings being only about 6 per cent. of the normal. Owing to this low current, the rotating field of the converter armature is unable to reverse the residual field of the converter, and the machine always comes up with the right polarity and automatically drops into synchronism when the speed has nearly reached its synchronous value. The field of the converter may remain connected across the brushes

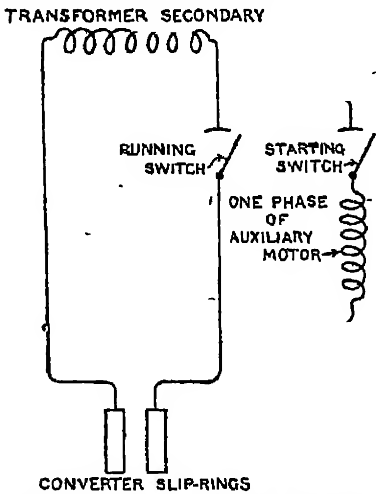


FIG. T.—Connections of Rosenberg's Self-synchronizing Converter.

* *Journal of the Institution of Electrical Engineers*, vol. li. p. 77 (1913).

during the entire period of starting, or it may be closed after the machine has run up to nearly synchronous speed. The polarized voltmeter across the brushes of the converter will indicate fluctuations before synchronism is reached, but its reading will become steady as soon as the machine has dropped into synchronism. The voltage is then adjusted to its normal value, and the running switch is closed, the starting motor being thereby short-circuited. The starting motor may have either the same number of poles as the converter or a smaller number.

The main advantages of Rosenberg's arrangement are: (1) smallness of starting current; (2) quickness and ease of starting up; (3) absence of destructive sparking at the brushes during the starting period. This last advantage, which results from the low value of the starting current, is particularly striking when compared with the violent sparking which occurs when using the method previously described—in which one-third of the normal voltage is applied directly to the slip-rings, no auxiliary motor being used. In this latter case, the sparking trouble is very serious in converters provided with interpoles, and such converters are generally fitted with brush-lifting gear, so that during the starting period all the brushes may be lifted off the commutator except two, which are connected to a polarized voltmeter for the purpose of indicating the polarity of the machine. Where no brush-lifting gear is available, a short-circuiting switch for the interpole windings should be used, so that the currents induced in these windings may damp the field under the interpoles.

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